Relaxing Convergence Assumptions for Continuous Adaptive Control

by

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Dedication

I dedicate this thesis

in memory of my father who started this study with me

and in honor of my mother for her prayers

also this thesis dedicated to my wife and my kids for their understanding

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Table of Contents

Dedication **ii**

Acknowledgments **iii**

List of Figures **vi**

Notation **vii**

Abstract **ix**

1 Introduction **1**
1.1 Brief Review of Adaptive Control Methods 1
1.2 Stability of Switched Systems 7
1.3 Motivation 10
1.4 Outline of the Thesis 14

2 Basic Concepts **15**
2.1 Preliminaries 15

3 Unfalsified Adaptive Control **24**
3.1 Definitions in Unfalsified Adaptive Control Theory 24

4 Results **28**
4.1 Problem Formulation and Preliminary Results 28

4.2 Main Result 36

5 Performance Criterion **38**
5.1 Performance Criterion Example 38

6 Comparison **48**
6.1 Compactness Property 52
6.2 Optimality 54
6.3 Convergence 56
6.4 Performance Improvement 64
6.4.1 Performance In the Presence of Disturbances 66
6.4.2 Performance In the Absence of Disturbances 67
6.5 Summary 68

7 Conclusion and Future Direction **69**
7.1 Summary 69
7.2 Future Directions 70

Bibliography **72**

List of Figures

1.1 Gain adaption MRAC. (From Ref. [Cha87]) 4

1.2 Switched closed-loop system 9

2.1 General adaptive control system $\Sigma(\mathcal{P}, \hat{k}_L)$ 16

4.1 Unfalsified adaptive control system $\Sigma(\mathcal{P}, \hat{k}_L)$ 29

5.1 Unfalsified adaptive control system $\Sigma(\mathcal{P}, \Theta_L)$ 39

5.2 Control configuration 40

5.3 Fictitious reference generator 42

5.4 Control configuration example 43

6.1 Supervisory control block diagram 57

Notation

P_τ: Linear truncation operator.

$\mathbb{R}_+$: (0 , ∞).

$\mathbb{R}$: (-∞, ∞).

x_τ: truncated signal.

y_p: plant output in MIT algorithm.

y_m: model reference output in MIT algorithm.

r: reference signal.

u: plant input.

y: plant output.

$z : (u, y)$.

$\mathbb{Z}$: set of all possible z in the interval time 0 to ∞.

z_τ: experiment data in time interval (0,τ).

e: error signal.

K_D: disturbance gain.

K_C: adjustable control gain.

K_m: model reference gain.

σ: discrete switching events.

k: a controller.

k_f: final controller in the closed-loop.

$\hat{k}_L$: the controller in the feedback loop.

K: controller set.

V: performance criterion.

$\hat{k}$: optimal controller.

t: time.

$\Sigma(P, \hat{k}_L)$: closed-loop system.

Abstract

Adaptive control convergence has been proved for long time by using slow switching schemes through separating the two successive switching events by a positive time interval (e.g., dwell-time, average dwell-time, hysteresis switching technique). This thesis addresses the inherent limitations of some logic-based switching among infinite (i.e. continuum) set of candidate controllers. In this thesis, we examine adaptive control convergence in the context of well-known hysteresis switching algorithm by relaxing the usual requirement that the hysteresis constant is strictly positive. Relaxing this constraint allows the adaptive controller to converge to a unique optimum in the case of an infinite (continuum) candidate controller set, provided that at least one controller in the controller set has the ability to satisfy adaptive control performance.

Chapter 1

Introduction

1.1 Brief Review of Adaptive Control Methods

In recent years adaptive control has become a topic of active research. The concept of adaptive control is not new; control techniques based on switching between different controllers have been used since the 1950s [DL51, WYK58]. Adaptive control has a rich and varied literature; for more details, the reader can refer to textbooks such as [AW94, IS96, Cha87, NA89], which contain additional explanations of the different types of adaptive control theory.

An adaptive control technique was first proposed by Draper and Li in 1952

[DL51]. The next main step in adaptive control theory was taken in 1958 by Whitaker et al. [WYK58]. They were planning to design an aircraft flight control system when they recognized that a fixed gain feedback control does not help in this situation. Since the dynamics of the aircraft is changing from one operating point to another, it needs an advanced control system that has the ability to learn and tune its own parameters. At that time, model reference adaptive control (MARC) was proposed by Whitaker [WYK58] to deal with the varying system dynamics of the aircraft system. Their further work was reported in [Whi59a, Whi62, OWK61].

One of the earliest definitions of the term *"adaption"* was introduced by Drenick and Shahbender [DS57] in 1957:

> *adaptive systems in control theory are control systems that monitor their own performance and adjust their parameters in the direction of better performance.*

Adaptive control is usually used to control imprecisely known plants. The main goal of adaptive control is to achieve improved performance by choosing among a given set of candidate controllers using real-time data and prior information. Many studies have been published to achieve this goal. The MIT rule was suggested by Osbourne [OWK61] and Whitaker [Whi59b]. The general idea behind the MIT rule is to control a stable system with unknown gain by using a gradient descent algorithm to adjust a scalar parameter to reach zero output difference between the modeled linear system and the actual plant (system to be controlled), which is simply illustrated in Fig. 1.1. Both systems (reference model and actual plant) used with the MIT rule are derived from the same reference signal, r. The key idea is to minimize the integral of the square of error between y_m and y_p by adjusting K_c such that $K_c K_D$ will eventually be equal to the model reference gain K_m, more detail about this algorithm can be found in [Cha87].

Even with this straightforward method, which requires much prior information about the plant (e.g., knowing the exact transfer function of the plant, the plant has stable transfer function, knowing the gain's sign, etc.). The method may yield unpredictable poor performance or even instability in some circumstances, as shown in [Par66, HP73]. In the 1960s, which are considered to be a golden period for adaptive control, several studies and developments in adaptive control were introduced (e.g., stochastic control, state space techniques, Lyapunov stability theory, dual control, etc.). These studies played a crucial role in understanding and improving the concept of an adaptive control system. However, the stability and convergence of these developments were proved based on restricted plant assumptions, like linear time-invariance, minimum phase plant, no noise,

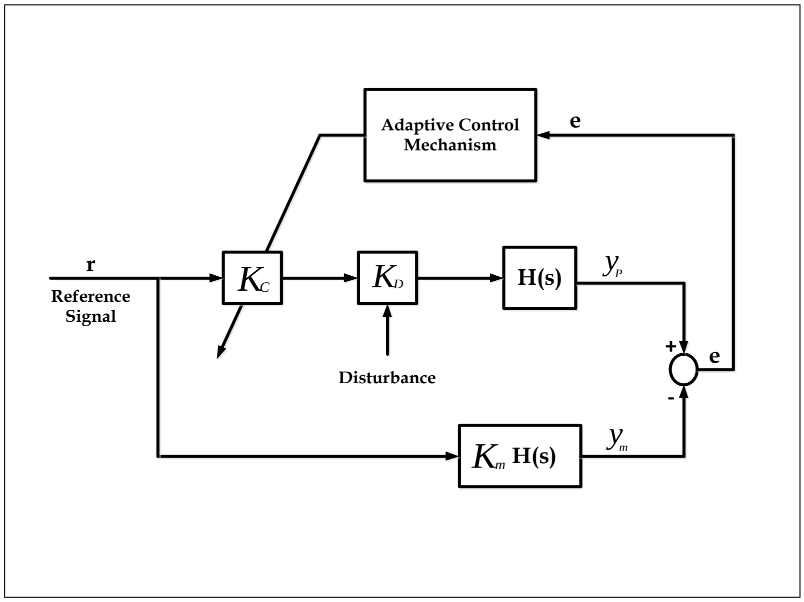

Figure 1.1: Gain adaption MRAC. (From Ref. [Cha87])

no time delays, known upper bound of the plant order, no external disturbances, and so on. When one or more of these assumptions fail to hold, which is the case in most practical situations, traditional adaptive control may not be able to cope with the system, as shown by Rohrs [RVAS85]. Since this list of *"unrealistic"* assumptions rarely holds in practice, a powerful and successful method was needed to deal with the lack of instability and fill the gaps in understanding the adaptive control weakness.

By the mid 1980s, a significant development effort had led to a new framework known as robust adaptive control. The robust adaptive control method successfully coped with a system in the presence of bounded disturbance characteristics and ensured system robustness under the assumption of some bounded disturbances. In the late 1980s and early 1990s, several studies were published on control theory that contributed to the development of robust adaptive control (e.g., [IS88, ID91, DI91]). The basic idea of robust adaptive control is to design a controller for a known nominal plant based on a given *"small"* bound of uncertainties around the nominal model by choosing the worst case scenario controller.

Although robust adaptive control theory has been used successfully in several applications, it has inherent limitations: in order to ensure the robustness properties by using this method, the system may fail to achieve the optimal performances. The other drawback of robust adaptive control is that the method requires prior information about the plant and assumes a sufficiently small bound on the uncertainties, while such information may not be available in real time or large uncertainties might arise in practice, which will cause the real system to lie outside the predicted uncertainty bound.

Since we are dealing with an unknown plant in most cases, these assumptions about the plant were easily violated, with the missing information making exact model-following impossible, as shown by a well-known example [RVAS85]. This situation was the motivation for several studies that searched for a smarter way to deal with adaptive control problems without requiring numerous assumptions about the plant and its structure. Subsequent research was directed toward relaxing many of the assumptions about the plant. Some of these studies succeeded to relax some, but not all, of the assumptions, as reported in [Mor96, ABLM01].

In 1986, proof of adaptive control stability under perhaps the weakest assumption in the history of adaptive control appeared in ([Mar86, FB86]). They showed that it is possible to design an adaptive controller that will converge under only a feasibility assumption (i.e., at least one controller in the candidate controller set has the ability to satisfy the adaptive control performance) using pre-routed switching among the candidate controllers until the stabilizing controller is found. Although this idea does not require many assumptions about the plant beyond feasibility it had few practical applications because of its shortcomings. This approach works by switching the candidate controllers one by one into the feedback loop until the control performance is satisfied by one of them, which can cause poor transient response. In addition, in the case of a large number of candidate controllers, the process may require a long time for the stabilizing controller to be switched into the loop.

A similar approach called data-driven unfalsified adaptive control was proposed by Safonov in [ST97]. This approach can overcome the above pre-routed shortcomings through directly validating the candidate controllers by using experimental data only, with no assumptions about the plant beyond feasibility. Other unfalsified adaptive control studies can be found in ([SWS04, WS05, BBMT09, VHDJS05, VHDJS08, ISP08, BHMT, WHK99]) and elsewhere. This algorithm has the ability to detect whenever an active controller fails to achieve the performance and to switch it out of the loop when the given data prove this failure. Sufficient conditions for the stability and convergence of unfalsified adaptive control were proven in [SWS04, WSS04] under a feasibility assumption and the cost detectability property of performance criterion. It has been found that if the system is cost detectable and the feasibility assumption holds, the unfalsified adaptive control approach always converges to a stabilizing controller.

1.2 Stability of Switched Systems

The main reason for introducing adaptive control is to ensure the satisfactory performance (e.g., regulation and tracking problems) of a closed-loop system by switching among a given set of candidate controllers when no single controller is capable of achieving the desired performance objectives. Therefore, this algorithm requires a system served by a multi-controller set (finite or infinite set of controllers) and we refer to such systems as multi-controller systems. If the switching between controllers happens to be in a discrete form, the multi-controller system is called a hybrid system because of the combination of discrete dynamics associated with switching events σ and the continuous dynamics system associated with the rest of the system. A well-defined performance criterion should be chosen to reflect the desired performance. The whole process is orchestrated by a smart unit called a supervisor, which is responsible for making a decision, at each instant of time, about when to switch and which controller should be used next, based on the available plant input/output data and performance criterion. A switched closed-loop system is shown in Fig. 1.2.

One serious challenge facing switching systems is occurrence of infinitely fast switching (chattering). Chattering phenomena can occur because of fast discontinuous switching and can cause unmodeled dynamics excitation and unacceptable system dynamics behavior. Therefore, the key method for avoiding these phenomena is to separate the two successive switching events by a positive time interval. These undesirable phenomena were the motivation for several studies (e.g., [Mor96, Mor97, HM99, MMG92, MGHM88]). The concept of dwell-time switching was studied in the context of supervisory control by Morse in [Mor96, Mor97]. In these studies, Morse showed how to introduce chatter-free switching by using a sufficiently large dwell time.

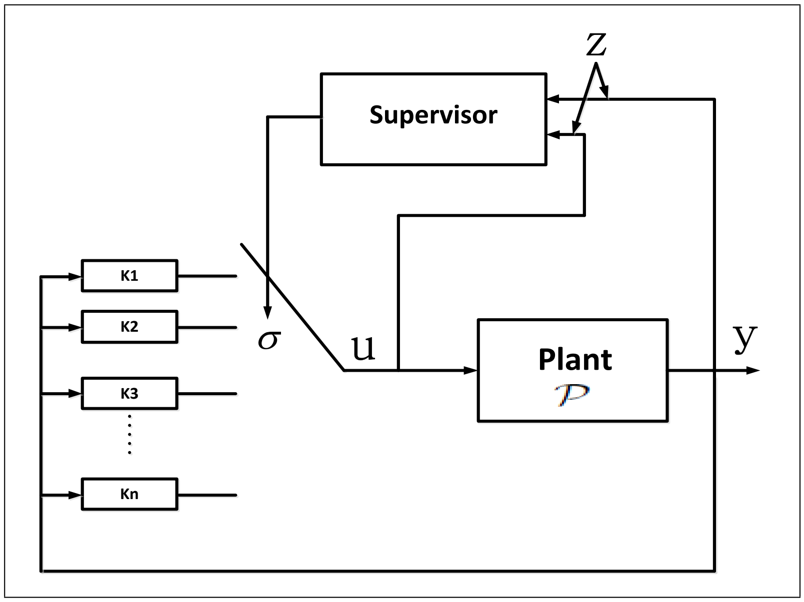

Figure 1.2: Switched closed-loop system

Although successful applications of switching control techniques have been reported by using the dwell-time algorithm [Mor96, Mor97], it is not capable of coping with the control of nonlinear systems because of the finite escape time possibilities [HM98]. A new concept called average dwell-time was introduced by Hespanha in [HM99]; this concept is an extension of the dwell-time technique. In [HM99], Hespanha proved that the average time period between successive switches should be greater than a sufficiently large "specified" constant to ensure the exponential stability of the switched system.

Fundamental contributions were made by Morse [MMG92] and Middleton [MGHM88]. In [MMG92], Morse and his co-workers proved that the stability and convergence of adaptive control can be achieved with a finite number of switches when using a hysteresis switching algorithm under certain plant assumptions. While, in [WPSS05], Wang showed that this algorithm can be more powerful when used with just the feasibility assumption, given that the used performance criterion has the cost-detectability property. Although the hysteresis switching algorithm has been applied to several successful applications (e.g., [MMG92, MGHM88, LHM00, HLM+01, SS08]), it has some drawbacks, especially in the case of an infinite set (e.g., containing a continuum) of candidate controllers, as we will discuss in later chapters, which is the main source of motivation for this work.

1.3 Motivation

According to Åström and Wittenmark in [AW94]:

In every language, 'to adapt' means to change a behavior to conform to new circumstances. Intuitively, an adaptive controller is thus a controller that can modify its behavior in response to changes in the dynamics of the process and the character of the disturbances.

When a fixed controller is not capable of coping with unknown or time varying plant parameters, switching among a set of candidate controllers algorithms is needed to ensure satisfactory performance. Two different techniques for switching between controllers have been used: continuous adaptive tuning and logic-based switching. In both cases, a primary goal of adaptive control is to ensure the stability and convergence of a controller to reach optimum performance. The switching process in adaptive control is orchestrated by a supervisory unit based on the given data and performance criterion. If there is no constraint on how the supervisor unit works, infinitely fast switching (chattering) may occur, which could cause an unbounded signal (instability).

Several techniques have been proposed to avoid this undesirable phenomenon. The main goal of these techniques is to ensures a non-zero dwell time by separating the two successive switching events by a positive time interval length. One of the most famous technique is the hysteresis switching algorithm reported in [MMG92, MGHM88]. Under certain assumptions, the convergence and stability of adaptive control systems by a finite number of switches have been proven when using the hysteresis switching algorithm. The beauty of the hysteresis switching algorithm is that it can cope with nonlinear systems unlike, the dwell-time algorithm.

Several successful applications have been reported in adaptive control systems for both finite and infinite sets of candidate controllers when using hysteresis switching techniques, which can be found in ([SS08, MMG92, MGHM88, WPSS05, HLM$^+$01, HLM03, LHM00]) and elsewhere. Using an infinite set of candidate controllers could create a better environment that would help the feasibility assumption to hold because of the cardinality difference between finite and an infinite sets of candidate controllers. Significant progress has been made using such an infinite set, with the help of the hysteresis switching algorithm [SS08, HLM03, LHM00, HLM$^+$01]. The authors of these studies succeeded in proving the stability and convergence of adaptive control systems using a strictly positive hysteresis constant.

Although the hysteresis switching algorithm can play an important role in the stability and convergence of adaptive systems, it has some drawbacks. One of the biggest obstacles facing this algorithm is that it does not ensure optimal performance, especially in the case of an infinite set (e.g., containing a continuum) of candidate controllers, which means some performance may be sacrificed. In this case, the hysteresis switching algorithm ensures the convergence of the adaptive controller to the neighborhood of the continuum controllers within a radius ϵ (hysteresis constant) far from the optimal controller.

In this thesis, we will consider the case in which the candidate controller set $\mathbb{K}$ has continuum controllers. In Chapter 4, we propose the above problem and establish a theoretical proof of adaptive control convergence to a unique optimum controller.

1.4 Outline of the Thesis

This thesis is organized as follows:

- Chapter 2 presents an overview of the preliminary definitions and notation.
- Chapter 3 introduces some unfalsified adaptive control definitions and concept needed in this thesis.
- Chapter 4 gives the problem formulation and results.
- Chapter 5 provides an example of a performance criterion satisfying sufficient conditions for convergence.
- Chapter 6 contains a comparison between the idea introduced in this thesis and local priority hysteresis switching logic [HLM+01].
- Conclusions follow in Chapter 7.

Chapter 2

Basic Concepts

2.1 Preliminaries

Consider a general adaptive control system $\Sigma(\mathcal{P}, \hat{k}_L)$ shown in Fig. 2.1 mapping $r \mapsto (u, y)$, where u and y are the measured plant input and output vector signals respectively, r is reference signal, $\mathcal{P}$ is a plant and $\hat{k}_L$ is the controller in the feedback loop. The input signal of supervisor is the measured data

$$z := \begin{bmatrix} u \\ y \end{bmatrix}$$

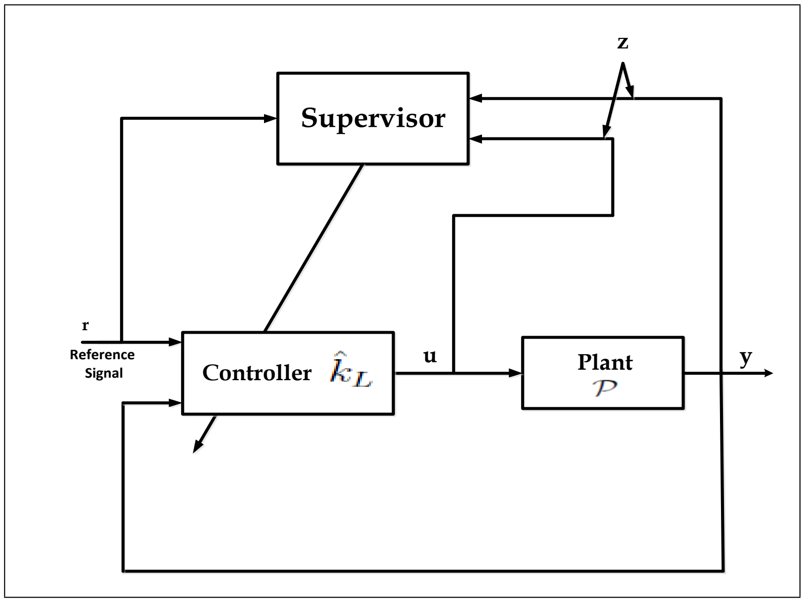

Figure 2.1: General adaptive control system $\Sigma(\mathcal{P}, \hat{k}_L)$

and the output of the supervisor is the chosen $\hat{k}_L$ where the adaptive control law has the general form

$$u := \hat{k}_L(t, z) \begin{bmatrix} u \\ y \end{bmatrix}$$

So at each time the supervisor will switch in the loop the best controller among the controller set K based in measured data z and performance criterion. Let's denote the final controller switched in the loop by $k_f(z)$ at the time $t_f(z)$.

Comment 2.1.1 *The controller set* K *may have finite or infinite (e.g containing a continuum) number of controllers. We limit our consideration in this thesis to the case in which the candidate controller set* K *has infinitely many controllers (typically, a continuum of controllers).*

Definition 2.1.1 [Saf80] *Linear truncation operator* $P_\tau : x \rightarrow x_\tau$ *is given by*

$$P_\tau x(t) = \begin{cases} x(t), & \text{if} \quad t \in [0, \tau] \\ 0, & \text{otherwise.} \end{cases}$$

and x_τ *refer to* $P_\tau x(t)$ *as shown below*

$$x_\tau(t) = \begin{cases} x(t), & \text{if} \quad t \in [0, \tau] \\ 0, & \text{otherwise.} \end{cases}$$

Let one possible experimental plant data for the switching adaptive system shown in Fig. 2.1 be $z = (u, y)$ and let $\mathbb{Z}$ represent the set of all possible z in the interval time 0 to ∞. z_τ represent the truncated signal z. Thus, z_τ is the experiment data in time interval (0,τ).

Definition 2.1.2 *$\ell_2 -$ norm of a truncated signal $P_\tau x$ is given as*

$$\|x\|_\tau = \sqrt{\int_0^\tau x(t)^T x(t) dt}.$$

Definition 2.1.3 [Ber99] *Let $C \subset \mathbb{R}^n$ be a convex set and let $f : \mathbb{R}^n \mapsto \mathbb{R}$ be differentiable over C then, f is strictly convex over C if*

$$f(y) \geq f(x) + (y - x)' \nabla f(x), \qquad \forall x, y \in C$$

Definition 2.1.4 [KS90] *If the function f is twice continuously differentiable, then f is strongly convex in k with parameter c if and only if $\nabla_k^2(V(k, z, t)) \geq c > 0$ for all k.*

Definition 2.1.5 [KS90] *Let $C \subset \mathbb{R}^n$ be a convex set and let $f : \mathbb{R}^n \mapsto \mathbb{R}$ be differentiable over C then, f is strongly convex (or uniformly convex) on C if and only if, for any $x, y \in C$.*

$$f(y) \geq f(x) + \nabla f(x)^T (y - x) + \tfrac{\alpha}{2} ||y - x||^2$$

where $\nabla^2 f(x) \geq \alpha > 0.$

Comment 2.1.2 *It is not necessary for a function to be differentiable in order to be strongly convex.*

Definition 2.1.6 [Ber99] *Let* $C \subset \mathbb{R}^n$ *be a convex set and let* $f : \mathbb{R}^n \mapsto \mathbb{R}$ *be twice continuously differentiable over* C*. then,* f *is strictly convex over* C *if* $\nabla^2 f(x)$ *is positive definite for every* $x \in C$*.*

Lemma 2.1.1 [Ber99] *Let* f *be a strongly convex function then, the local minimum of* f *is also a global minimum and there exists at most one minimum of* f*.*

Lemma 2.1.2 [Ber99] *Let* f *be a strongly convex function in* x *and let* x^* *be a local minimum of* $f : \mathbb{R}^n \longmapsto \mathbb{R}$*, then* $\nabla_x f(x^*) = 0$.

Definition 2.1.7 *A level set in* $\mathbb{R}^n$ *is defined as* $L(\alpha) \triangleq \{x \in \mathbb{R}^n | f(x) \leq \alpha\}$ *for some* $\alpha \in \mathbb{R}$.

Definition 2.1.8 $\nu(k, z, t)$ *is an* equi-quasi-positive definite (EQPD) *function in* k*, that is, there exists a continuous nondecreasing scalar function* ϕ *such that*

$\phi(0) = 0$ *and* $\nu(k, z, t) - \nu(\hat{k}(t), z, t) \geq \phi(\|k - \hat{k}(t)\|) > 0$ *for all* t, *all* $z \in \mathbb{Z}$ *and all* $k - \hat{k}(t) \neq 0$.

Remark *An equi-quasi-positive definite function has the same properties as positive definite function except that the minimum of equi-quasi-positive definite function occur at* $\Delta k = 0$ *(i.e.* $\Delta k = k - \hat{k}$*) while the minimum of positive definite function occur at* $k = 0$.

Definition 2.1.9 [IS96] *(Persistence of Excitation (PE)) A piecewise continuous signal vector* $\phi : \mathbb{R}_+ \mapsto \mathbb{R}^n$ *is* PE *in* $\mathbb{R}^n$ *with a level of excitation* $\alpha_0 > 0$ *if there exist constants* α_1, $T_0 > 0$ *such that*

$$\alpha_1 I \geq \tfrac{1}{T_0} \int_t^{t+T_0} \phi(\tau)\phi^T(\tau) d\tau \geq \alpha_0 I, \qquad \forall t \geq 0 \tag{I}$$

Although the matrix $\phi(\tau)\phi^T(\tau)$ *is singular for each* τ, *(I) requires that* $\phi(t)$ *varies in such a way with time that the integral of the matrix* $\phi(\tau)\phi^T(\tau)$ *is uniformly positive definite over any time interval* $[t, t + T_0]$. *If we express (I) in the scalar form, i.e.,*

$$\alpha_1 \geq \tfrac{1}{T_0} \int_t^{t+T_0} (q^T \phi(\tau))^2 d\tau \geq \alpha_0, \qquad \forall t \geq 0$$

where q is any constant vector in $\mathbb{R}^n$ with $|q| = 1$, then the condition can be interpreted as a condition on the energy of ϕ in all directions.

Definition 2.1.10 *(second-order Taylor-theorem expansion) Let $C \subseteq \mathbb{R}^n$ and let $f : \mathbb{R}^n \mapsto \mathbb{R}$ be twice continuously differentiable over C then,*

$$f(x) = f(a) + \nabla f(a)(x-a) + \nabla^2 f(\xi)$$
$$a \leq \xi \leq x \quad or \quad \xi = \alpha a + (1-\alpha)x \quad for \quad \alpha \in [0,1]$$

where the gradient $\nabla f(x)$ of the function $f(x)$ is a row vector of size n, i.e.,

$$\nabla f(x) = \left(\frac{\partial f}{\partial x_1}(x), \quad \frac{\partial f}{\partial x_2}(x), \quad \cdots, \quad \frac{\partial f}{\partial x_n}(x) \right)$$

the Hessian $\nabla^2 f(x)$ is an $n \times n$ matrix;

$$\nabla^2 f(x) = \begin{pmatrix} \frac{\partial^2 f}{\partial x_1^2}(x) & \frac{\partial^2 f}{\partial x_1 \partial x_2}(x) & \cdots & \frac{\partial^2 f}{\partial x_1 \partial x_n}(x) \\ \frac{\partial^2 f}{\partial x_2 \partial x_1}(x) & \frac{\partial^2 f}{\partial x_2^2}(x) & \cdots & \frac{\partial^2 f}{\partial x_2 \partial x_n}(x) \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial^2 f}{\partial x_n \partial x_1}(x) & \frac{\partial^2 f}{\partial x_n \partial x_2}(x) & \cdots & \frac{\partial^2 f}{\partial x_n^2}(x) \end{pmatrix}$$

and

$$x - a = \begin{pmatrix} x_1 - a_1 \\ x_2 - a_2 \\ \vdots \\ x_n - a_n \end{pmatrix}$$

Definitions of input-output stability related to the ℓ_{2e}-norm can be found in [Zam66], which pertains to the ratio between the norm of the output z to the norm of the input v. A slight generalization of the input-output stability of Zames [Zam66] has been developed by Willems [Wil73, Wil76]. The role of the α and $\tilde{\alpha}$ is to prevent the denominator from assuming values too close to zero.

Definition 2.1.11 *(Stability and Gain)* [WPSS05] *We say a system F with input v and output z is stable if for every input $v \in \ell_{2e}$-norm there exist constants β, $\alpha \geq 0$ such that*

$$\|z_\tau\| < \beta\|v_\tau\| + \alpha, \; \forall t > 0 \qquad (\star)$$

otherwise, it is said to be unstable. Furthermore, if the $(\star)$ equation holds with a single pair β, $\alpha \geq 0$ for all $v \in \ell_{2e}$-norm, then the system F is said to be *finite-gain stable, in which case the gain of F is the least such β.*

Definition 2.1.12 [WPSS05] *(Incremental Stability and Incremental Gain) We say that F is incrementally stable if, for every pair of inputs v_1, v_2 and outputs $z_1 = Fv_1$, $z_2 = Fv_2$, there exists constants $\tilde{\beta}$, $\tilde{\alpha} \geq 0$ such that*

$$\|[z_2 - z_1]_\tau\| < \tilde{\beta}\|[v_2 - v_1]_\tau\| + \tilde{\alpha},\ \forall t > 0; \qquad (\star\star)$$

and the incremental gain of F, when it exists, is the least $\tilde{\beta}$ satisfying $(\star\star)$ for some α and all v_1, $v_2 \in \ell_{2e}$.

Chapter 3

Unfalsified Adaptive Control

3.1 Definitions in Unfalsified Adaptive Control Theory

The beauty of using the Morse-Mayne-Goodwin [MMG92] hysteresis switching algorithm with unfalsified adaptive control is that it is possible to adjust the controller's parameters based only on the measured data without any assumptions about the plant beyond feasibility, with convergence ensured for any strictly positive hysteresis constant [SS08]. In this method, the potential performance of every candidate controller is evaluated directly from the measured data using some suitably defined performance criterion, without trying to identify the actual process. This algorithm is typically fast to converge because it does not require inserting controllers in the feedback loop to be falsified.

The main contribution of the unfalsified adaptive control algorithm is that, the algorithm does not require any assumption about the plant (i.e. plant-assumption-free method) in order to ensure the stability of the system, given the feasibility of the adaptive control problem and a cost detectable performance criterion. The feasibility is defined as the existence of at least one controller in the candidate controller set that has the ability to stabilize the system. The cost-detectability property is a condition of the performance criterion that ensures closed-loop stability for the switched multi-controller adaptive control (MCAC) system whenever stabilization is feasible. For this reason, an adaptive control system that employs cost-detectability has been called a "safe adaptive control systems" [WPSS05].

In this chapter, some important concepts and definitions of unfalsified adaptive control are presented. Further details about unfalsified adaptive control can be found in [SS08, WPSS05, JS99].

Definition 3.1.1 [JS99] *Given a set of measurements of I/O data* (u, y) *and a* *candidate controller $k_i \in K$ a fictitious reference signal for k_i is a hypothetical reference signal $\tilde{r}_i$ that would have produced exactly the same measurements data (u, y) had the candidate controller k_i been in the feedback loop with the unknown plant during the entire time period over which the measurements data (u, y) were collected.*

Definition 3.1.2 [SS08] *The adaptive control problem is said to be* feasible *if a candidate controller set K contains at least one controller that achieves stability and performance goals.*

Comment 3.1.1 *It is unknown a prior which controller k in a controller set* K *that achieves the satisfactory performance.*

Definition 3.1.3 [SS08] *A controller K is said to be a* feasible controller *if it satisfies given stability and performance constraints.*

Definition 3.1.4 [WPSS05] *Given V, K and a scalar $\gamma \in \mathbb{R}$, we say that a controller $k \in K$ is falsified at time τ with respect to cost level γ by past measurement information z_τ if $V(k, z, \tau) > \gamma$. Otherwise the control law k is said to* *be unfalsified by* z_τ*.*

Definition 3.1.5 [WPSS05] *Let* r *denote the input and let* $z := \begin{bmatrix} u \\ y \end{bmatrix}$ *denote the resulting plant data collected while* $\hat{k}_L(t,z)$ *is in the loop. Consider the adaptive control system* $\Sigma(\mathcal{P}, \hat{k}_L)$ *of Fig. 2.1 with input* r *and output* $z := \begin{bmatrix} u \\ y \end{bmatrix}$*. The pair* (V, K) *is said to be cost detectable if, without any assumptions on the plant* $\mathcal{P}$ *and for every* $\hat{k}_L(t,z) \in K$*, the following statements are equivalent:*

1). $V(k_f, z, \tau)$ *is bounded as* τ *increases to infinity;*

2). The stability of the system $\Sigma(\mathcal{P}, \hat{k}_L(\tau, z))$ *is unfalsified by* (r,z)*.*

Definition 3.1.6 [SS08] *Stability of the system* $\Sigma : r \mapsto z$ *is said to be falsified by the data* (r,z) *if*

$$\sup_{\tau \in \mathbb{R}_+, \|r\|_\tau \neq 0} \frac{\|z\|_\tau}{\|r\|_\tau} = \infty$$

Otherwise, it is said to be unfalsified.

Chapter 4

Results

4.1 Problem Formulation and Preliminary Results

Consider an unfalsified adaptive control system $\Sigma(\mathcal{P}, \hat{k}_L)$ shown in Fig. 4.1 mapping $r \mapsto (u, y)$, where u and y are the measured plant input and output vector signals respectively, r is reference signal, $\mathcal{P}$ is unknown plant and $\hat{k}_L$ is the controller in the feedback loop. The input signal of supervisor is the measured data

$$z := \begin{bmatrix} u \\ y \end{bmatrix}$$

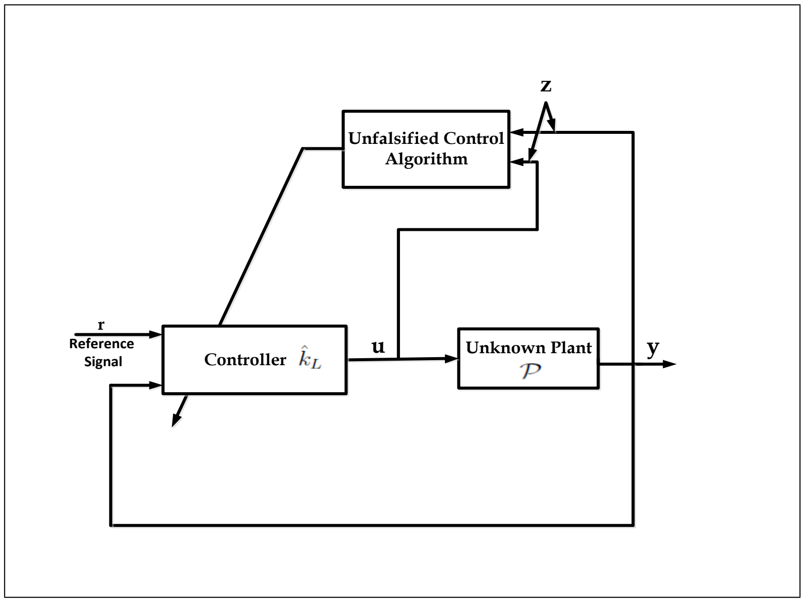

Figure 4.1: Unfalsified adaptive control system $\Sigma(\mathcal{P}, \hat{k}_L)$

and the output of the supervisor is the chosen $\hat{k}_L$ where the adaptive control law has the general form

$$u := \hat{k}_L(t, z) \begin{bmatrix} u \\ y \end{bmatrix}$$

So at each instant of time the supervisor will switch in the loop the best controller among the controller set K based in measured data z and performance criterion.

In this thesis, we call the scalar valued function, $V : K \times \mathbb{Z} \times \mathbb{R}_+ \to \mathbb{R}_+ \cup \{\infty\}$, a performance criterion. It is used to evaluate candidate controllers k based on past data z_τ. The performance criterion $V(k, z, \tau)$ assumed to be causally dependent of z, that is, for all $\tau > 0$ and all $z \in \ell_2$,

$$V(K, z, \tau) = V(K, z_\tau, \tau)$$

where z_τ is the truncated signal z from initial to the current time τ.

Definition 4.1.1 *Consider a continuum controllers set* K *(e.g.* $\mathrm{K} \subset \mathbb{R}^n$*).*

Definition 4.1.2 *The* optimal controller $\hat{k}(\mathrm{t})$*, if exists, at* t *is defined as*

$$\hat{k}(t) = \operatorname*{argmin}_{k \in K} V(K, z, t)$$

Assumption 4.1.1 *The performance criterion $V(k, z, t)$ is continuous in k and t.*

Lemma 4.1.1 [Rud76] *Every nonempty set of real numbers which bounded above has a supremum.*

Assumption 4.1.2 *Adaptive control problem is feasible (Def. 3.1.2).*

Comment 4.1.1 *(i.e. $\exists M \in \mathbb{R}$; s.t. $V(\hat{k}(t), z, t) \leq M < \infty$)*

Let $V_L(z) = \sup_t V(\hat{k}(t), z, t)$

Assumption 4.1.3 *The performance criterion $V(k, z, t)$ is monotonically increasing in t, $V(k, z, t_2) \geq V(k, z, t_1) \;\; \forall t_2 \geq t_1$ and all k.*

Lemma 4.1.2 $V(\hat{k}(t_2), z, t_2) \geq V(\hat{k}(t_1), z, t_1)$ *for all $t_2 \geq t_1$ and all k.*

Proof

(by monotonicity property) $V(k,z,t_2) \geq V(k,z,t_1) \quad \forall t_2 \geq t_1$ (1)

(by definition 4.1.2) $V(\hat{k}(t),z,t) \leq V(k,z,t)$ (2)

(from (1)) $V(\hat{k}(t_2),z,t_2) \geq V(\hat{k}(t_2),z,t_1)$

(from (2)) $\geq V(\hat{k}(t_1),z,t_1)$

$$\Rightarrow \qquad V(\hat{k}(t_2),z,t_2) \geq V(\hat{k}(t_1),z,t_1) \qquad \forall\ t_2 \geq t_1$$

$\Rightarrow$ $V(\hat{k}(t),z,t)$ monotonically increasing sequence. $\Diamond$

Lemma 4.1.3 [Rud76] *If* $V(\hat{k},z,t)$ *is monotonically increasing sequence in* $\mathbb{R}$*. Then* $V(\hat{k},z,t)$ *converges if and only if it is bounded above.*

Assumption 4.1.4 $V(\hat{k}(t),z,t)$ *unique for each* $t \in \mathbb{R}_+$*.*

Lemma 4.1.4 *Let* $V : K \times \mathbb{Z} \times \mathbb{R}_+ \to \mathbb{R}_+ \cup \{\infty\}$ *be a continuous equi-quasi-positive definite function in* k*,* $k \in \mathbb{R}^n$*, and continuous monotonic increasing in*

t. Assume that $V(\hat{k}(t), z, t)$ is bounded above. Then, there exists a time t_M such that $\hat{k}(t)$ lies in a compact subset L, $L \subset K$, for all $t > t_M$.

Proof

Define a family of functionals $f_t(k) = V(k, z_t, t)\ \forall t \geq t_M$. Let $\alpha = \sup_t V(\hat{k}(t), z, t)$ (see lemma 4.1.1). Consider $L(\alpha)$ to be a level set in $\mathbb{R}^n$ (Def. 2.1.7).

Since $L(\alpha) \subset \mathbb{R}^n$ it is sufficient to show that $L(\alpha)$ is bounded and closed. Define $s(t)$ to be

$$s(t) = \begin{cases} \min\limits_{\rho\ \to\infty} \sup\limits_{\|k\|\geq\rho} V(k, z, t), & if \quad exists \\ \infty, & otherwise. \end{cases}$$

Since $V(k, z, t)$ is continuous equi-quasi-positive definite function in k and has a unique minimum $\hat{k}(t)$ at each t (assumption 4.1.4), then $s(t) > V(\hat{k}(t), z, t)$. Let $\epsilon(t) = s(t) - V(\hat{k}(t), z, t) > 0$. By lemma 4.1.2, $V(\hat{k}(t), z, t)$ monotonic increasing sequence. Since $V(\hat{k}(t), z, t)$ is monotonic increasing sequence bounded above by α, then for every $\epsilon(t) > 0$ there exists t_M such that $\alpha - V(\hat{k}(t), z, t) < \epsilon(t)$ for all $t \geq t_M$ (this is true by lemma 4.1.3).

We know that for all $t \geq t_M$ the condition $\alpha - V(\hat{k}(t), z, t) < \epsilon(t)$ is satisfied.

$$\Rightarrow \alpha - V(\hat{k}(t), z, t) < s(t) - V(\hat{k}(t), z, t) \;\; \forall t \geq t_M$$

$$\Rightarrow \alpha < s(t) \;\; \forall t \geq t_M$$

In what follows, we will show that $L(\alpha)$ is bounded and closed for all $t \geq t_M$ (i.e., $f_t(k) = V(k, z_t, t)$).

First, we show $L(\alpha)$ is bounded for $t \geq t_M$. Suppose to the contrary that $L(\alpha)$ is not bounded. Then there exists a sequence $\{k_m\} \subseteq L(\alpha)$ such that $\lim\limits_{m\to\infty} \|k_m\| = \infty$. Since $f_t(k)$ is equi-quasi-positive definite function, $\lim\limits_{m\to\infty} f_t(k_m)$ $\geq s > \alpha$ ($i.e.$ $\exists N \in \mathbb{N}$ such that $\forall \ell \geq N$ $f_t(k_\ell) > \alpha$). Then $\{k_m\} \not\subset L(\alpha)$, which contradicts the above assumption. Hence, $L(\alpha)$ is bounded.

Next, we show that $L(\alpha)$ is closed for $t \geq t_M$: Let $\{k_m\} \subseteq L(\alpha)$ be a convergent sequence, and $k_f = \lim\limits_{m\to\infty} k_m$. Since f_t is continuous, $f_t(k_f) = \lim\limits_{m\to\infty} f_t(k_m)$. Also, $f_t(k_m) \leq \alpha \; \forall t$. Then, $f_t(k_f) = \lim\limits_{m\to\infty} k_m \leq \lim\limits_{m\to\infty} \alpha = \alpha$, so $k_f \in L(\alpha)$. Thus $L(\alpha)$ is closed and bounded, therefore compact. $\Diamond$

Lemma 4.1.5 [Ber99]*(Weierstrass theorem) Let K be a non empty subset of $\mathbb{R}^n$ and let $V : K \mapsto \mathbb{R}$ be lower semicontinuous at all points of K. If K is compact, then $\hat{k}(t) = \underset{k \in K}{\operatorname{argmin}} V(K, z, t)$ exists.*

Lemma 4.1.6 *If $V(k, z, t) - V(\hat{k}(t), z, t) \geq \phi(\|k - \hat{k}(t)\|)$ then, $V(\hat{k}(t_2), z, t_2) - V(\hat{k}(t_1), z, t_1) \geq \phi(\|\hat{k}(t_2) - \hat{k}(t_1)\|) \quad \forall\, t_2 \geq t_1$.*

Proof

Since, $V(k, z, t) - V(\hat{k}(t), z, t) \geq \phi(\|k - \hat{k}(t)\|)$

$$\Rightarrow \quad V(\hat{k}(t_2), z, t_1) - V(\hat{k}(t_1), z, t_1) \geq \phi(\|\hat{k}(t_2) - \hat{k}(t_1)\|)$$

(by monotonicity property) $V(\hat{k}(t_2), z, t_2) \geq V(\hat{k}(t_2), z, t_1) \quad \forall\, t_2 \geq t_1$

$\Rightarrow V(\hat{k}(t_2), z, t_2) - V(\hat{k}(t_1), z, t_1) \geq V(\hat{k}(t_2), z, t_1) - V(\hat{k}(t_1), z, t_1) \geq \phi(\|\hat{k}(t_2) - \hat{k}(t_1)\|) \quad \forall\, t_2 \geq t_1$

$$\Rightarrow \quad \mathrm{V}((\mathrm{t}_2), z, t_2) - V(\hat{k}(t_1), z, t_1) \geq \phi(\|\hat{k}(t_2) - \hat{k}(t_1)\|) \quad \forall\, t_2 \geq t_1. \qquad \Diamond$$

Lemma 4.1.7 [Rud76] *In $\mathbb{R}^n$, every Cauchy sequence converges.*

4.2 Main Result

Theorem *(Main Result) Consider the feedback adaptive control system $\Sigma(\mathcal{P}, \hat{k}_L)$ in Fig. 2.1. Assume that the adaptive control problem is feasible, and the associated performance criterion $V(K, z, t)$ is monotone increasing in t and continuous in k. Assume further that $V(K, z, t)$ is equi-quasi-positive definite function in k (Def. 2.1.8) and $V(\hat{k}(t_i), z, t_i)$ unique for each t_i. Then, the adaptive control system converges to a unique controller as time proceeds.*

Proof

(from lemma 4.1) $\quad V(\hat{k}(t_2), z, t_2) - V(\hat{k}(t_1), z, t_1) \geq \phi(\|\hat{k}(t_2) - \hat{k}(t_1)\|)$ $\forall\ t_2 \geq t_1$

Since, $\quad V_L(z) \geq V(\hat{k}(t_2), z, t_2)$

$$\Rightarrow \mathrm{V}_L(z) - V(\hat{k}(t_1), z, t_1) \geq V(\hat{k}(t_2), z, t_2) - V(\hat{k}(t_1), z, t_1)$$

$$\geq \phi(\|\hat{k}(t_2) - \hat{k}(t_1)\|)$$

(from lemma 4.1.3)   for each $\epsilon > 0$ there exists t_N such that

$$\epsilon \geq |V(\hat{k}(t_2), z, t_2) - V(\hat{k}(t_1), z, t_1)| \geq \phi(\|\hat{k}(t_2) - \hat{k}(t_1)\|) \;\; \forall \; \mathrm{t}_1, t_2 \geq t_N$$

Since, ϕ is nondecreasing continuous function and satisfies $\phi(0) = 0$. Then $\forall \delta > 0$ $\exists \; t_N$ such that

$\epsilon(\delta) \geq \phi(\|\hat{k}(t_2) - \hat{k}(t_1)\|)$. Therefore, $\delta \geq \|\hat{k}(t_2) - \hat{k}(t_1)\| \;\; \forall \; \mathrm{t}_1, t_2 \geq t_N$.

By Cauchy criterion (lemma 4.1.7) $\hat{k}(t)$ converges. Since $\hat{k}(t)$ is unique (assumption 4.1.4), Hence the adaptive control system converges to a unique controller. $\Diamond$

Chapter 5

Performance Criterion

5.1 Performance Criterion Example

Consider an unfalsified adaptive control system $\Sigma(\mathcal{P}, \Theta_L)$ shown in Fig. 5.1 mapping $r \mapsto (u, y)$, where u and y are the measured plant input and output vector signals respectively, r is reference signal, $\mathcal{P}$ is unknown plant and Θ_L is the controller in the feedback loop.

Consider a controller structure in Fig. 5.2, where θ_{ii} is a constant parameter, $\theta_{ii} \in \Theta$, Θ is a parameter vectors, $\Theta \in \mathbb{R}^{2n}$ and $\bar{\Theta}$ is a set of parameter vectors. Then the control low has the form

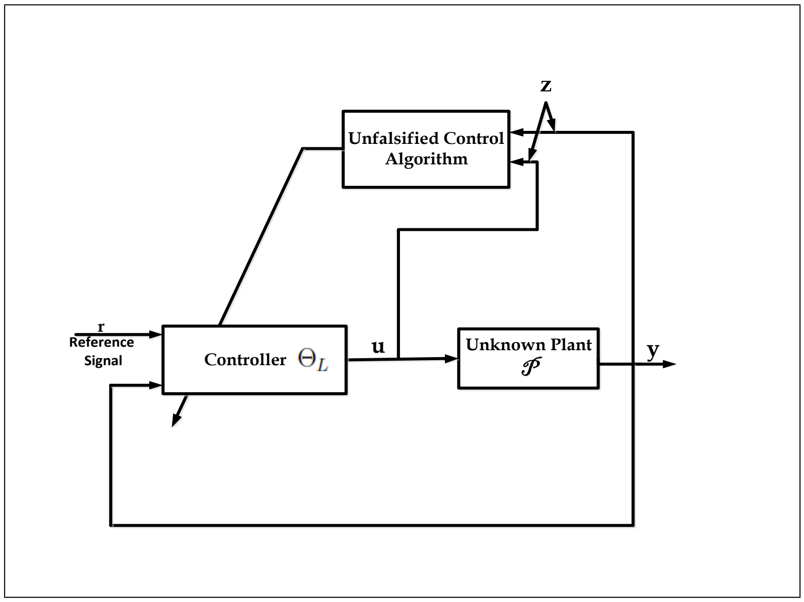

Figure 5.1: Unfalsified adaptive control system $\Sigma(\mathcal{P}, \Theta_L)$

$$u(t) = r(t) + \theta_{1i} f_{i1} u(t) + \theta_{2i} f_{i2} y(t)$$

where $f_{i1} = \mathcal{L}^{-1}(F_{i1}(s))$ and $f_{i2} = \mathcal{L}^{-1}(F_{i2}(s))$,

θ'_{1i} and θ'_{2i} are $n \times 1$ vectors,

$$\theta'_{1i} = \begin{pmatrix} \theta_{11} \\ \theta_{12} \\ \vdots \\ \theta_{1n} \end{pmatrix}, \qquad \theta'_{2i} = \begin{pmatrix} \theta_{21} \\ \theta_{22} \\ \vdots \\ \theta_{2n} \end{pmatrix}$$

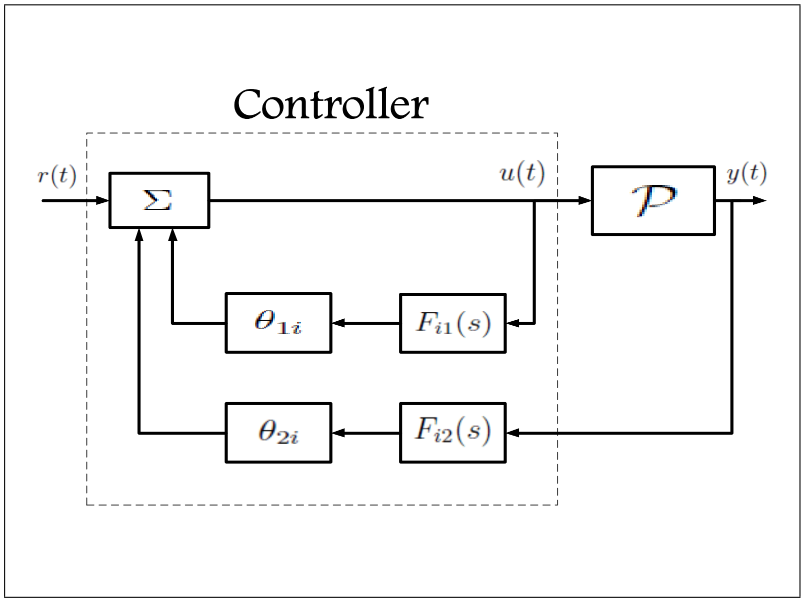

Figure 5.2: Control configuration

and Θ is $1 \times 2n$ vector,

$$\Theta = (\theta_{11} \quad \theta_{12} \quad \cdots \quad \theta_{1n} \quad \theta_{21} \quad \theta_{22} \quad \cdots \quad \theta_{2n})$$

and $F_{i1}(s)$ and $F_{i2}(s)$ are $n \times 1$ vectors of stable filters as shown below

$$F_{i1} = \begin{pmatrix} F_{11} \\ F_{21} \\ \vdots \\ F_{n1} \end{pmatrix}, \qquad F_{i2} = \begin{pmatrix} F_{12} \\ F_{22} \\ \vdots \\ F_{n2} \end{pmatrix}$$

Consider the performance criterion $V(\Theta, z, t)$. The optimal controller parameters $\Theta^*(t)$ at each instant of time is defined as

$$\Theta^*(t) = \underset{\Theta \in \bar{\Theta}}{\text{argmin}} \; V(\bar{\Theta}, z, t)$$

Fictitious reference signal $\tilde{r}$ is not the true signal (Def. 3.1.1). For each Θ_i there is a fictitious reference signal $\tilde{r}_i$ that would have produced exactly the same measurements data (u, y) had the candidate controller Θ_i been in the feedback loop with the unknown plant during the entire time period over which the measurements data (u, y) were collected. Given data $z = (u, y)$ and controller Θ with the structure in Fig. 5.2. Its fictitious reference signal $\tilde{r}(\Theta, z)$ would be

$$\begin{aligned}\tilde{r}(\Theta, z) &= T(\Theta)z \\ &= u - \theta_{1i}F_{i1}(s)u - \theta_{2i}F_{i2}(s)y\end{aligned}$$

where $T(\Theta)$ is a fictitious reference generator of the controller configuration in Fig. 5.2. The fictitious reference generator $T(\Theta)$ structure is illustrated in Fig. 5.3.

A fictitious error signal $\tilde{e}$ is the error between the fictitious reference signal and the actual plant output y, which can be written as

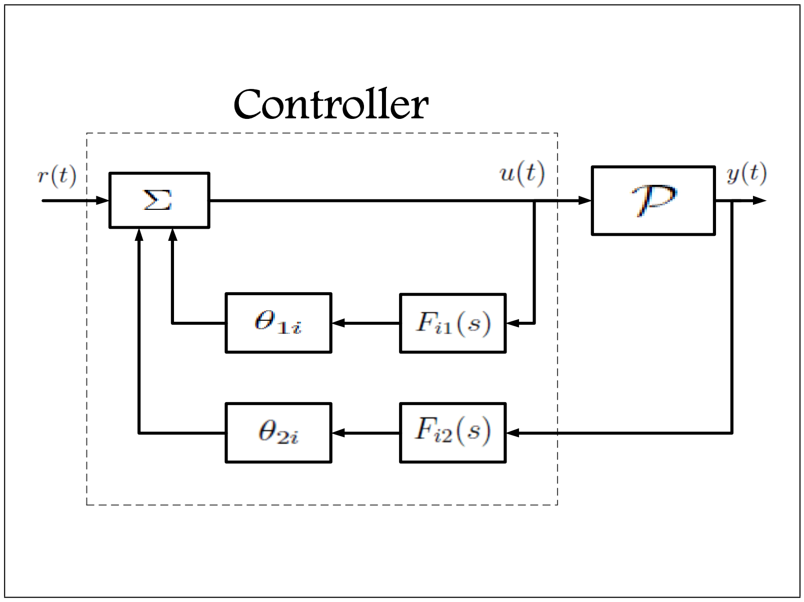

Figure 5.3: Fictitious reference generator

$$\tilde{e}_i = \tilde{r}_i - y$$

The fictitious error signal for the data $z = (u, y)$ and controller Θ with the structure in Fig. 5.2 is

$$\begin{aligned}\tilde{e}(\Theta, z) &= \tilde{r}(\Theta, z) - y \\ &= u - \theta_{1i}F_{i1}(s)u - \theta_{2i}F_{i2}(s)y - y\end{aligned}$$

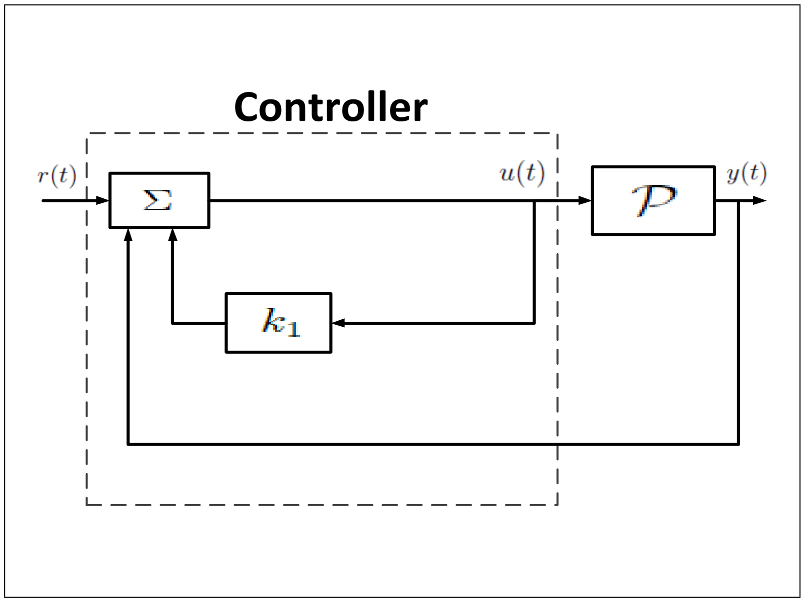

Figure 5.4: Control configuration example

An example of the performance criterion and the conditions under which it ensures convergence according to the previous theorem may be constructed as follows. Consider a controller structure in Fig. 5.4 Its fictitious reference signal would be

$$\tilde{r}(\Theta, z) = u - k_1 u - y$$

where θ_{1i}, θ_{2i}, f_{i1}, and f_{i2} in this example are

$$\theta'_{1i} = \begin{pmatrix} k_1 \\ 0 \\ \vdots \\ 0 \end{pmatrix}, \qquad \theta'_{2i} = \begin{pmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix}$$

and

$$f_{i1} = \begin{pmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix}, \qquad f_{i2} = \begin{pmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix}$$

And the associated fictitious error signal is

$$\begin{aligned} \tilde{e}(\Theta, z) &= u - k_1 u - y - y \\ &= u(1 - k_1) - 2y \end{aligned}$$

Consider the well-known performance criterion integral norms of estimation errors

$$V(\Theta, z(t), \tau) = \int_0^\tau \|e(t)\|^2 dt$$

In the unfalsified adaptive algorithm, the only available information about the actual plant is the data (u, y), so we will use the fictitious error signal instead of the true error. This fictitious error signal should converge to the true signal if the fictitious reference generator has a stable structure (See Reference [ST97]).

$$V(\Theta_i, z(t), \tau) = \int_0^\tau \|\tilde{e}_i(t)\|^2 dt = \int_0^\tau \|u(t)(1-k_1) - 2y(t)\|^2 dt$$

Then,

$$\nabla_{k_1} V(\Theta_i, z(t), \tau) = -2 \int_0^\tau u(t)(u(t)(1-k_1) - 2y(t))dt$$

and

$$\nabla_{k_1}^2 V(\Theta_i, z(t), \tau) = 2 \int_0^\tau u(t)^2 dt$$

Definition 5.1.1 *We say that the system is* persistently excited *if the hessian is strictly positive definite for all t sufficiently large.*

Under the persistent excitation assumption, the function $V(\Theta_i, z(t), t)$ is uniformly convex function in k for sufficiently large time t.

For this example, we can derive explicit conditions on $\tilde{e}_i$ that guarantee parameter convergence by considering u^2 does not tend to zero. Therefore, whenever the systems is persistently excited, this performance criterion has the the uniform convexity property. The persistent excitation (PE) property defined by us is crucial in many adaptive schemes where parameter convergence is one of the objectives and is closely related to the persistent excitation of [NA87, Eyk74, BS86, Bit84, AB66, And77].

The main idea of this thesis is to introduce a new algorithm for adaptive controller convergence without using any constraint on the switching scheme (removing the constraints on the switching scheme, "e.g. dwell-time, average dwell-time, hysteresis switching"). The idea introduces in this thesis is investigated in the context of the unfalsified adaptive control algorithm. We believe that the unfalsified adaptive control algorithm is one of the best algorithms in adaptive control theory since it requires the minimum number of assumptions (i.e., at least one controller in the controller set has the ability to satisfy the adaptive control performance) about the plant to ensure convergence and stability.

Such a contribution could also be used under a different adaptive control algorithm (e.g., multiple model adaptive control) to enhance the performance, as we will show in the next chapter.

Chapter 6

Comparison

Adaptive control using a continuum set of candidate controllers has recently received considerable attention, with several successful applications being reported (e.g., [HLM$^+$01, HLM03, SS08]). Some of these applications have been a source of inspiration for the idea introduced in this thesis. Our main goal in this thesis is to establish the conditions for performance criterion under which the convergence constraint on the switching schemes (i.e., strictly positive hysteresis constant) has been relaxed.

The aim of this chapter is to show, by a literature review, how this new idea could be useful in relaxing some of the unnecessary assumptions. In [HLM$^+$01, HLM03], Hespanha and his coworkers introduced new modifications to a hysteresis switching technique that have the ability to deal with an infinite set of candidate controllers (typically, a continuum of controllers) and ensure adaptive control convergence. The two switching logics are called hierarchical hysteresis switching and local priority hysteresis switching logic, and were reported in [HLM03] and [HLM+01], respectively. The primary idea of the first switching logic relies on a partition of the continuum set into a finite number of subsets. The switching strategy in this logic is based on two stages. The first step is to choose controllers "system's parameters" that satisfy the minimum value for the performance criterion "monitoring signal" in each subset and then compare the signal values produced by these controllers to select the one that satisfies the overall minimum. For further details on hierarchical hysteresis switching, we refer the reader to [HLM03, LHM00].

The main idea of the second switching logic, local priority hysteresis switching logic, relies on giving priority to the neighborhood's parameters before switching to another one, as we will present in more detail in the sequel.

The concepts of these switching logics are almost the same and the switching between controllers "system's parameters" occurs in a discrete switching form even though we use a continuum set of candidate controllers. Combining discrete switching with continuous dynamical systems will drive the designer to deal with hybrid dynamical systems instead of dealing with continuous systems. Since this will make the systems more complicated to deal with, we try to avoid this situation in this thesis by introducing continuous adaption. In this chapter, we will choose the second switching logic, local priority hysteresis switching, for our case study. We perform a comparison and try to show how the idea presented in this thesis could contribute in this context and relax some assumptions.

We now present some concepts and equations about local priority hysteresis switching logic. For more details, the reader can refer to [HLM$^+$01]. The inputs of the local priority hysteresis switching logic are continuous signals, μ_p, $p \in \mathbb{P}$, where μ_p assumed to be strictly positive and monotone increasing in t and $\mathbb{P}$ is a compact set. Define a set D_γ

$$D_\gamma(q) := \{p \in \mathbb{P} : |q - p| \leq \gamma\}$$

where γ is a proper positive constant and $|\cdot|$ is a norm function in $\mathbb{P}$. The output of the switching logic, at each instant of time, is a switching signal, $\sigma(t)$. Pick a hysteresis constant $h > 0$ and set $\sigma(0) = \underset{p \in \mathbb{P}}{\operatorname{argmin}} \{\mu_p(0)\}$. Suppose that at time t_i, σ has just switched to some $q \in \mathbb{P}$ and kept fixed until a time $t_{i+1} > t_i$ such that the following inequality is satisfied:

$$(1 + h) \min_{p \in \mathbb{P}} \{\mu_p(t_{i+1})\} \leq \min_{p \in D_\gamma(q)} \{\mu_p(t_{i+1})\}$$

At this time, we set $\sigma(t_{i+1}) = \underset{p\in\mathbb{P}}{\operatorname{argmin}} \{\mu_p(t_i + 1)\}$. By repeating these steps we can generate a sequence of switching signal which will converge as time increase.

In this study, the authors stated that the constant γ should be sufficiently small to ensure the tractability property for subset D_γ. It is reasonable to ask about the kind of upper bound needed for the constant γ to ensure this property? Furthermore, if there is one, does this bound work for all possible performance criteria? The answers to these questions can be found in [SS08]. In [SS08], Stefanovic succeeded to avoid these difficulties by using the uniform continuity property for the performance criterion. In this study, the constant γ is adjusted by choosing the hysteresis constant, h, using the continuity property of the function. The uniform continuity property of the performance criterion helps to ensure that the adaptive control system does not switch to another controller outside the neighborhood of radius γ, until all of the controllers in this neighborhood have been falsified. More details can be found in [SS08].

In all of above studies, the only way to ensure adaptive control convergence for the case of a continuum set of candidate controllers is by adding constraints to the switching logics through using strictly positive constants. These constraints may prevent the adaptive control system from reaching optimality, as we will show in Section 6.2.

6.1 Compactness Property

In [HLM+01], the authors used the "well-known" integral norms of the estimation errors performance criterion and the compactness property of the parameter set, $\mathbb{P}$. In this part of the study we will show that the idea of this thesis can relax the compactness assumption by using the same performance criterion used in [HLM+01].

Suppose $e_p = (p^T A + b)x_{\mathbb{E}} - y$ and consider the integral norms of estimation errors performance criterion [HLM+01]

$$\mu_p(\tau) = \int_0^{\tau} \|e_p(t)\|^2 dt = \int_0^{\tau} ((p^T A+b)x_{\mathbb{E}}(t)-y(t))^2 dt \tag{I}$$

Then,

$$\nabla_p(\mu_p(\tau)) = 2\int_0^{\tau} ((p^T A + b)x_{\mathbb{E}}(t) - y(t))x_{\mathbb{E}}(t)^T A^T dt$$

and

$$\nabla_p^2(\mu_p(\tau)) = 2\int_0^{\tau} A x_{\mathbb{E}}(t) x_{\mathbb{E}}^T(t) A^T dt$$

Definition 6.1.1 *We say that the system is* persistently excitated *if the hessian is strictly positive definite for all t sufficiently large.*

Under the persistent excitation assumption, the function $\mu_p(t)$ is uniformly convex function in p for sufficiently large time t.

Let $\hat{p}(t) = \underset{p}{\operatorname{arginf}}\ \mu_p(t), \ \ t \in \mathbb{R}_+$.

Lemma 6.1.1 *Let $\mu_p : \mathbb{R} \times \mathbb{R}_+ \to \mathbb{R}_+ \cup \{\infty\}$ be a continuous function in p, and continuous monotonic increasing in t. Suppose that the system is persistently excited and that $\mu_{\hat{p}(t)}(t)$ is bounded above. Then, there exists a time t_C such that $\hat{p}(t)$ lies in a compact subset L, $L \subset \mathbb{R}$, for all $t > t_C$.*

Proof

Since $\hat{p}(t)$ minimizes $\mu_p(t)$, we have $\nabla_p(\mu_{\hat{p}(t)}(t)) = 0$ (1)

Since $\mu_p(t)$ is uniformly convex in p then $\nabla_p^2(\mu_p(t)) \geq \alpha > 0$ (2)

By Definition 2.1.5, equations (1) and (2), $\mu_p(t)$ can be written as

$$\mu_p(t) \geq \mu_{\hat{p}(t)}(t) + \frac{\alpha}{2}\|p - \hat{p}(t)\|$$

Hence, $\mu_p(t)$ is equi-quasi-positive definite function (Def. 2.1.8) with a unique minimum and the proof proceeds like the proof of lemma 4.1.4. $\Diamond$

6.2 Optimality

The difficulty of using the hysteresis switching algorithm [MMG92] and its modifications [HLM+01, HLM03, SS08] is that when using the usual requirement that the hysteresis constant is strictly positive, this constraint may prevent the adaptive control system from achieving optimality. For this reason, our aim in this thesis is to reexamine the adaptive control convergence in the context of the well-known hysteresis switching algorithm by setting the hysteresis constant to zero (relaxing the switching scheme constraint). Relaxing this constraint allows the adaptive controller to converge to a unique optimum in the case of an infinite (continuum) candidate controller set as $t \to \infty$.

Another noticeable difficulty with the local priority hysteresis switching logic is that other factors could prevent the adaptive control system from reaching optimality besides the hysteresis constant, including the choice of the other constant γ and the performance criterion "monitoring signal", μ_p. Since there is no upper bound for choosing the constant γ and no clear rules for choosing the performance criterion, this drawback could be worse with bad choice for constant γ and the performance criterion.

Lemma 6.2.1 *Local priority hysteresis switching logic may stop switching (become prematurely stuck with one controller in the feedback loop) even though there are controllers in the controller set that satisfy the condition*

$$(1+h)\mu_{p_{other}}(t_i) < \mu_{p_{fl}}(t_i)$$

where $\mu_{p_{fl}}$ is the monitoring signal associated with controller in the feedback loop and $\mu_{p_{other}}$ is the monitoring signal associated with other controllers in the controller set.

Proof

Let's start with with $\sigma(0) = \underset{p\in\mathbb{P}}{\text{argmin}} \{\mu_p(0)\}$ and at certain time t_i, σ has just switched to some $q \in \mathbb{P}$. Suppose that at time $t_{i+1} > t_i$ there exists a globally minimizing p_m such that, $p_m = \underset{p\in\mathbb{P}}{\text{argmin}} \{\mu_p(t_{i+1})\} = \underset{p\in D_\gamma(q)}{\text{argmin}} \{\mu_p(t_{i+1})\}$ and $p_m \in D_\gamma(q)$. Then equation

$$(1+h)\min_{p\in\mathbb{P}} \{\mu_p(t_{i+1})\} \leq \min_{p\in D_\gamma(q)} \{\mu_p(t_{i+1})\}$$

becomes

$$h * \min_{p \in \mathbb{P}} \{\mu_p(t_{i+1})\} \leq 0$$

Since $\mu(t)$ is positive function, this condition cannot be satisfied (i.e., if $\mu_{p_{p_m}}$ is the monitoring signal associated with the controller p_m and μ_{p_q} is the monitoring signal associated with the controller q the system will not switch to the controller p_m that satisfies the global minimum "$p_m = \operatorname*{argmin}_{p \in \mathbb{P}}\{\mu_p(t_{i+1})\}$" whatever the difference between $\mu_{p_{p_m}}$ and μ_{p_q}). ◊

6.3 Convergence

The primary goal of this thesis is to establish conditions for the performance criterion under which the convergence constraint on the switching schemes (i.e., strictly positive hysteresis constant) may be relaxed. In this section, we shall get to the main point of this chapter by showing that using the same performance criterion that was used in our case study [HLM+01], it is possible to prove adaptive control convergence without a strictly positive hysteresis constant (e.g., without $h > 0$ and $\gamma > 0$), which allows the adaptive controller to converge to a unique optimal solution where the "optimal performance has been satisfied".

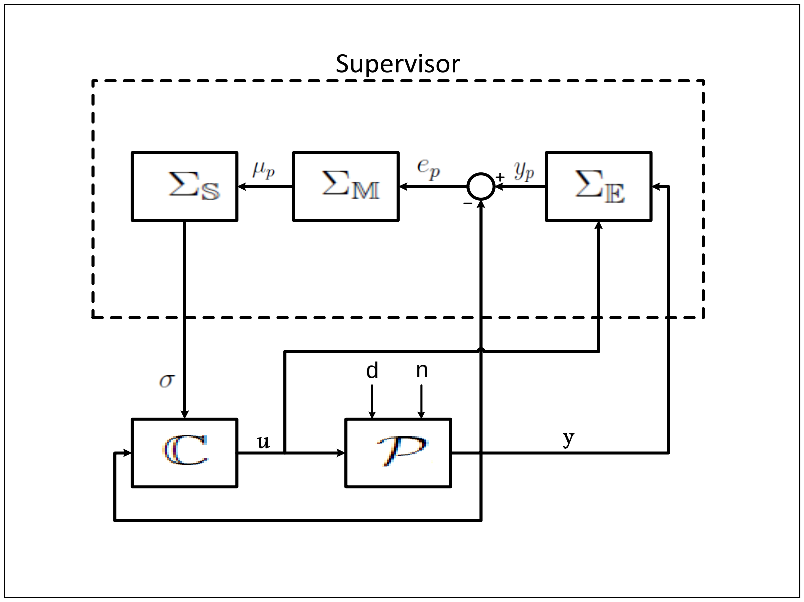

Figure 6.1: Supervisory control block diagram

To make this comparison we need to recall some required and necessary notations and definitions from [HLM+01].

The switching process in the local priority hysteresis switching logic is orchestrated by a supervisory unit, which is responsible for switching into the feedback loop, at each instant of time, the best controller from the controller set $\Im$ based on the measured data and performance criterion. This supervisor consists of three subsections, as shown in Fig. 6.1.

1) Multi-estimator $\Sigma_{\mathbb{E}}$ — a dynamical system whose inputs are the output y

and the input u of the process $\mathcal{P}$ and whose outputs are the signals y_p, $p \in \mathbb{P}$.

2) Monitoring signal generator $\Sigma_{\mathbb{M}}$ — a dynamical system whose inputs are the estimation errors $e_p = y_p - y$, $p \in \mathbb{P}$ and whose outputs μ_p, $p \in \mathbb{P}$ are suitably defined integral norms of the estimation errors, called monitoring signals.

3) Switching logic $\Sigma_{\mathbb{S}}$ — a switched system whose inputs are the monitoring signals μ_p, $p \in \mathbb{P}$ and whose output is a switching signal σ taking values in $\mathbb{P}$, which is used to define the control law u.

State-space equations for the supervisory system is described in detail in [Mor96], recall the state-space equations for the three subsystems. As p ranges over $\mathbb{P}$, let realizations of the transfer functions of the candidate controllers be:

$$\dot{x}_{\mathbb{C}} = A_p x_{\mathbb{C}} + b_p y$$
$$u = k_p x_{\mathbb{C}} + r_p y$$

where $x_{\mathbb{C}}$ is controller state and $\mathbb{C}_q$ is one controller parameter in the candidate controller set $\Im$ (i.e. $\{\mathbb{C}_q : q \in \Im\}$). It have been assumed that there is a controller in the candidate controller set that able to solve the tracking error and regulation problems for each unknown process $\mathcal{P}$.

The state space realization of multi-controller $\mathbb{C}$ can be define as:

$$\dot{x}_{\mathbb{C}} = A_\sigma x_{\mathbb{C}} + b_\sigma y$$
$$u = k_\sigma x_{\mathbb{C}} + r_\sigma y$$

and multi-estimator $\Sigma_{\mathbb{E}}$ has the following realization:

$$\dot{x}_{\mathbb{E}} = A_{\mathbb{E}} x_{\mathbb{E}} + b_{\mathbb{E}} y + d_{\mathbb{E}} u$$
$$y_p = c_p x_{\mathbb{E}}, \qquad p \in \mathbb{P}$$

where $x_{\mathbb{E}}$ is estimated state and its assumed to be available for the controller in all time, $A_{\mathbb{E}}$ is a stable matrix and d is a process disturbance.

The matrices c_p, $p \in \mathbb{P}$ is design in such way for each $p \in \mathbb{P}$, c_p exists and unique (See Reference [Mor96] Section IV). Moreover, for the case of $\mathbb{P}$ to be continuum c_p, $p \in \mathbb{P}$ assumed to depend linearly on p to ensure the tractability property (See Reference [Mor96] Section XI). So the matrix c_p can be represented in the form:

$$c_p = p^T A + b$$

For SISO system, A is $n \times n$ nonzero matrix, p is $n \times 1$ unknown process parameters and b is $1 \times n$ vector.

In our case study [HLM$^+$01], authors used the "well-known" performance criterion integral norms of estimation errors:

$$\mu_p(\tau) = \int_0^\tau \|e_p(t)\|^2 dt$$

where $e_p = y_p - y$ and $y_p = c_p x_{\mathbb{E}}$ so, μ_p can be written as

$$\mu_p(\tau) = \int_0^\tau \|(p^T A + b)x_{\mathbb{E}}(t) - y(t)\|^2 dt$$

or

$$\mu_p(\tau) = \int_0^\tau ((p^T A + b)x_{\mathbb{E}}(t) - y(t))^T((p^T A + b)x_{\mathbb{E}}(t) - y(t))dt$$

Then,

$$\nabla_p(\mu_p(\tau)) = 2\int_0^\tau ((p^T A + b)x_{\mathbb{E}}(t) - y(t))x_{\mathbb{E}}(t)^T A^T dt$$

and

$$\nabla_p^2(\mu_p(\tau)) = 2\int_0^\tau A x_{\mathbb{E}}(t) x_{\mathbb{E}}^T(t) A^T dt$$

Then μ_p is uniformly convex in p when the hessian satisfies $\nabla_p^2(\mu_p(t)) > \epsilon > 0$, which holds since the systems is persistently excited.

For this example, we can derive explicit conditions on $e_p(t)$ that guarantee parameter convergence by considering $\|x_{\mathbb{E}}\|^2$ does not tend to zero. Therefore, whenever the systems is persistently excited, this performance criterion has the the uniform convexity property. The persistent excitation (PE) property defined by us is crucial in many adaptive schemes where parameter convergence is one of the objectives and is closely related to the persistent excitation of [NA87, Eyk74, BS86, Bit84, AB66, And77].

Under the persistent excitation assumption, the function $\mu_p(t)$ has a unique minimum parameter $\hat{p}(t)$ "controller" for sufficiently large time t, let the optimal parameter $\hat{p}(t)$ at each t is defined as

$$\hat{p}(t) = \operatorname*{argmin}_{p \in \mathbb{P}} \mu_p(t)$$

In the sequel, we will use the same assumptions that have been used in [HLM+01], except:

1) Compactness property: we showed that by using the same performance criterion that has been used in [HLM+01] we were able to relax the compactness property for the parameter set $\mathbb{P}$ (See Section 6.1).

2) Strictly positive constants (i.e., $h > 0$ and $\gamma > 0$): these two constants have

been used in local priority hysteresis switching logic to ensure adaptive control convergence for the case of a continuum set of candidate controllers. In this section, we provide the main result for this chapter, which relies on proving that the adaptive controller convergence for the case of a continuum set of candidate controllers without using any constraints on the switching logic (i.e., $h = 0$, $\gamma = 0$). Relaxing these constraints allows the adaptive control system to overcome these limitations and ensure the optimal performance.

Theorem (Main Result) Consider the feedback adaptive control system in Fig. 6.1. Assume that the adaptive control problem is feasible (Def. 3.1.2), and that the associated performance criterion "$\mu_p(t)$" is suitably defined integral norms of the estimation errors. Assume further that $\mu_p(t)$ is monotone increasing in t and continuous in t and p. Then, the adaptive control system converges to a unique optimal controller as time proceeds.

Proof

By using Taylor's theorem the performance criterion $\mu_p(t)$ can be written as:

$$\mu_p(t) = \mu_{\hat{p}(t)}(t) + (p-\hat{p}(t))^T \nabla_p(\mu_{\hat{p}(t)}(t)) + \tfrac{1}{2}(p-\hat{p}(t))^T \nabla_p^2(\mu_{\xi(t)}(t))\,(p-\hat{p}(t))$$

Where $\xi(t)$ can be written as $\quad \alpha p + (1-\alpha)\hat{p}(t)\ ; \quad \alpha \in [0,1]$

Since $\hat{p}(t)$ minimizes $\mu_p(t)$, we have $\nabla_p(\mu_{\hat{p}(t)}(t)) = 0$ (1)

Since $\mu_p(t)$ is uniformly convex in p then $\nabla_p^2(\mu_{\xi(t)}(t) \geq c > 0$ (2)

By definition 2.1.5, equations (1) and (2), $\mu_p(t)$ can be written as

$\mu_p(t)$ - $\mu_{\hat{p}(t)}(t) \ \geq \frac{c}{2}\|p - \hat{p}(t)\|^2$

or, equivalently,

$\mu_{\hat{p}(t_m)}(t_n)$ - $\mu_{\hat{p}(t_n)}(t_n) \ \geq \frac{c}{2}\|\hat{p}(t_m) - \hat{p}(t_n)\|^2$ $(\star)$

From monotonicity $\Rightarrow \mu_{\hat{p}(t_m)}(t_m) \geq \mu_{\hat{p}(t_m)}(t_n) \quad \forall t_m \geq t_n$

Then $(\star)$ can be written as

$\mu_{\hat{p}(t_m)}(t_m)$ - $\mu_{\hat{p}(t_n)}(t_n) \ \geq \frac{c}{2}\|\hat{p}(t_m) - \hat{p}(t_n)\|^2 \ \ \forall t_m \geq t_n$ $(\star\star)$

(from lemma 4.1.3) for each $\epsilon > 0$ there exists t_N such that

$\mu_{\hat{p}(t_m)}(t_m)$ - $\mu_{\hat{p}(t_n)}(t_n) \ \leq \epsilon \quad \forall t_m, t_n \geq t_N$

Then, $(\star\star)$ can be written as

$$\tfrac{c}{2}\|\hat{p}(t_m) - \hat{p}(t_n)\|^2 \leq \epsilon \quad \Rightarrow \quad \|\hat{p}(t_m) - \hat{p}(t_n)\| \leq \underbrace{\sqrt{\frac{2\epsilon}{c}}}_{\delta} \quad \forall \ \mathrm{t}_m, t_n \geq t_N$$

It is clear that as $\epsilon \to 0$ implies that $\delta \to 0$

By Cauchy criterion (lemma 4.1.7) $\hat{p}(t_n)$ converges. Since $\hat{p}(t_n)$ is unique by uniformly convexity property and since for each parameter p there is a unique controller that satisfy the adaptive control performance then, the adaptive control system converges to a unique optimal controller. $\Diamond$

6.4 Performance Improvement

The main reason for introducing the supervisory control approach [Mor96, Mor97] is to ensure a satisfactory performance (e.g., regulation and tracking problem) of a closed-loop system by switching among a given set of candidate controllers. The basic idea behind the controller selection strategy is to determine which nominal process model is associated with the smallest monitoring signals, and then select the corresponding candidate controller.

According to the certainty equivalence concept [Mor92]:

> *The nominal process model with the smallest performance criterion signal "best" approximates the actual process, and therefore the candidate controller associated with that model can be expected to do the best job of controlling the process.*

The contribution of the local priority hysteresis switching logic in the context of supervisory control is to introduce a new switching logic that has the ability to deal with the case where the unknown parameters belong to a continuum set.

Now, suppose the unknown process $\mathcal{P}$ shown in Fig. 6.1 whose input and output signals u and y are the input of multi-estimator $\Sigma_{\mathbb{E}}$ where the output of $\Sigma_{\mathbb{E}}$ is y_p, $p \in \mathbb{P}$. Each y_p would converge to y if the transfer function of $\mathcal{P}$ was equal to the nominal process model transfer function ϑ_p in the absence of disturbances, unmodeled dynamics and noises. Disturbance input and noise signal are represented by d and n respectively. Assumed that the transfer function of $\mathcal{P}$ from u to y belongs to a family of admissible process model transfer functions

$$F = \bigcup_{p \in \mathbb{P}} F(p) \qquad (\star\star\star)$$

for each p, $F(p)$ denotes a family of transfer functions 'centered' around some known nominal process model transfer function ϑ_p where p is a parameter taking values in some index set $\mathbb{P}$. In the absence of noises, unmodeled dynamics and disturbances equation $(\star\star\star)$ will be equivalent to

$$V = \bigcup_{p \in \mathbb{P}} \vartheta_p$$

In our case study [HLM+01], the authors assumed that a candidate controllers set $\Im = \{\mathbb{C}_p : p \in \mathbb{P}\}$ is chosen in such a way that for each $p \in \mathbb{P}$; $\mathbb{C}_p$ will satisfy the adaptive control performance, where $\mathcal{P}$ is any element of F.

In the sequel we assume that all assumption in [HLM+01] are hold except the compactness property of $\mathbb{P}$ that has been relaxed in Section 6.1.

6.4.1 Performance In the Presence of Disturbances

In the Multiple model adaptive control, the adaptive control problem is placed in a setting of a standard optimization problem and the nominal process model with the smallest performance criterion signal is the model that best fits the available data ('certainty equivalence'). Therefore the candidate controller associated with this model can be expected to do the best job of controlling the process.

As shown in Section 6.2, local priority hysteresis switching logic may fail to optimize the performance criterion (I) while the new idea introduced in this thesis (which relies on relaxing the local priority hysteresis switching logic constraints) ensure the optimal signal for the performance criterion (I) as shown in Section 6.3. By certainty equivalence concept [Mor92], our idea improves the adaptive control performance.

6.4.2 Performance In the Absence of Disturbances

For the case of free of disturbance, unmodeled dynamics and noise the transfer function $F(p)$ is equal to ϑ_p, $p \in \mathbb{P}$. In Section 6.3, we showed that our idea succeeded to ensure convergence to optimal solution "$y_p \rightarrow y$" therefore the exact match between the actual process $\mathcal{P}$ and nominal process model ϑ_p is achieved, since there is a controller parameter that has the ability to achieve adaptive control performance for each ϑ_p, $p \in \mathbb{P}$, then targeting performance is satisfied as $t \rightarrow \infty$.

6.5 Summary

The idea introduces in this thesis is investigated in the context of the unfalsified adaptive control algorithm. We believe that the unfalsified adaptive control algorithm is one of the best algorithms in adaptive control theory since it requires the minimum number of assumptions "feasibility" about the plant to ensure convergence and stability.

The aim of this chapter is to show that such contribution could also be used under different adaptive control algorithm like multiple model adaptive control in order to enhance the system performance. Common goal for the different adaptive control algorithms is to satisfy the best performance for the system under minimum assumptions about the plant and its structure. Therefore combining this contribution with the unfalsified adaptive control may help to achieve this goal.

Chapter 7

Conclusion and Future Direction

7.1 Summary

In this thesis we discussed recent progress in the design and analysis of the hysteresis switching algorithm for the case of a continuum set of candidate controllers. The main contribution of this thesis is to study the Morse-Mayne-Goodwin hysteresis switching algorithm for continuous adaptive control and establish condition on performance criterion under which the hysteresis constant may be set to zero. It has been shown that using an equi-quasi-positive definite performance criterion is sufficient to ensure adaptive control convergence. The primary focus of this dissertation is to relax the usual requirement that the hysteresis constant is strictly positive. Relaxing this constraint allows the adaptive controller to converge to a unique optimum, yielding an improved performance (regulation and tracking), as shown in Chapter 6.

7.2 Future Directions

The method of controlling a system using adaptive control is not new. The idea was discovered more than a half century ago. It seemed natural to switch between different controllers when no single controller was capable of achieving the performance goal. At that time, the stability and convergence proofs of adaptive control were based on several plant assumptions, which could cause limited practical applications of this method. Since then, a fair amount of research has been done to relax these assumptions. It has been found that some of these assumptions are not crucial and can be relaxed.

In this thesis, we examined continuously switched adaptive control systems in the context of unfalsified adaptive control, without using any of the usual constraints on the switching process (e.g., hysteresis switching, dwell-time, average dwell-time), and were able to theoretically prove the system convergence to a unique "optimum" controller based on the feasibility and some assumptions on the performance criterion used for controller selection.

The main contribution of the unfalsified adaptive control algorithm is that it does not require any assumption about the plant (i.e., plant-assumption-free method) in order to ensure the stability of the system, given the feasibility of the adaptive control problem and a cost detectable performance criterion.

• The cost-detectability property is a condition of the performance criterion that ensures closed-loop stability for the switched multi-controller adaptive control (MCAC) system whenever stabilization is feasible. For this reason, an adaptive control system that employs cost-detectability has been called a "safe adaptive control system" [WPSS05]. Unfortunately, the performance criterion introduced in this thesis does not have this property. The possibility of achieving cost-detectable safe adaptive control with continuous switching is a topic for future research.

• Unfalsified adaptive control approach assumes that, there is at least one controller in a candidate controller set has the ability to satisfy the adaptive control performance (feasibility assumption). Feasibility is the weakest assumption under which adaptive asymptotic system stability and performance can be guaranteed. The question of the relation of feasibility assumption to other assumptions commonly used in adaptive control will be examined.

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