MULTI–CONTROLLER ADAPTIVE CONTROL (MCAC):
COST DETECTABILITY, STABILITY AND SOME APPLICATIONS

by

Ayanendu Paul

A Dissertation Presented to the
FACULTY OF THE GRADUATE SCHOOL
UNIVERSITY OF SOUTHERN CALIFORNIA
In Partial Fulfillment of the
Requirements for the Degree
DOCTOR OF PHILOSOPHY
(ELECTRICAL ENGINEERING)

August 2005

Dedication

To

my parents

Acknowledgements

First and foremost, I would like to express my deepest gratitude to my Ph.D. advisor and committee chairman, Professor Michael G. Safonov, for his guidance, support and inspiration throughout my carrier at USC. I also thank other members of my defense committee: Professor Petros Ioannou and Professor Gary Rosen.

I am grateful to Dr. Mehmet Akar (Previously with USC) for the valuable guidance and insights he provided during a good part of my PhD carrier. I would also like to thank Professor Chaulagi Raghavendra, Professor Viktor K. Prasanna and Professor Urbashi Mitra for helping me diversify my field of research and supporting part of my Ph.D. studies.

I thank fellow graduate students at USC, R. Wang (Crisy) and M. Stefanovic, with whom I shared a lot of interesting technical discussions. I am fortunate to have many good friends at USC, who were and always will be at my side; I thank all of them for making my four years stay at Los Angeles much more exciting.

Lastly and perhaps most importantly, I would like to thank my parents, brother, uncle, and rest of my family for their constant encouragement, confidence, support and friendship; they were always a source of motivation to me.

Contents

List Of Figures

Abstract

In the last two decades, the idea of switching based adaptive control system has gained popularity among control system community. The need for switching based control arises when, with inaccurate assumptions about the unknown and possibly time varying plant, it is difficult to synthesize a single robustly stabilizing controller which would give satisfactory performance under all possible operating environments.

This thesis deals with the use of Multi–Controller Adaptive Control (MCAC) based switching control for imprecisely modeled plants. Here, the potential performance of every candidate controller is evaluated at each time directly from the measured data using some suitably defined, data driven performance index, without trying to estimate or identify the actual process. In this thesis, necessary conditions for cost detectability of the performance index have been developed. Several applications of Multi–Controller Adaptive Control (MCAC) have also been considered in this thesis. The MCAC switching schemes have been successfully applied to the Model Reference Adaptive Control (MRAC) problem. The problem of transmitting optimal power by mobile users in a wireless network has also been studied. An algorithm, employing MCAC technique has been developed for the above problem. The proposed algorithm provides an adaptive technique to tune the gains of a PID–type controller and can adapt to different number of mobile users by employing different candidate gains. This

line of research provides a step towards applying adaptive feedback control systems for

transmitting power by a mobile user in a dynamic wireless environment.

Chapter 1

Introduction

1.1 Motivation

Traditional control theory can be used to design controllers for a nominal process model using available tools. These controllers can be made robust enough to cope with uncertainties, provided these uncertainties are 'sufficiently small'. However, in practice, the actual plant may not match with the assumed plant models; also the actual plant may be complex, unknown and hence difficult to model. External disturbances, sudden or gradual changes in plant dynamics are some other challenges in designing a conventional controller for these processes. When the change in the plant is large or discontinuous, traditional adaptive control may not be able to cope with it due to large transient errors. While gain scheduling in adaptive control is developed to handle such large and rapid change in the plant, the nature of change may not always be known or anticipated. Also, most of the traditional, tuning based adaptive control methods developed prior to 1980s are based on prior assumptions on the plant [NA89,IS95,SB89]. However, such set of assumptions can be often unrealistic in practice. Adaptive control

based on such assumptions can be highly non-robust, as shown by the famous counter–example by Rohrs *et al* in 1985 [RVAS85].

Due to the above difficulties, it may be difficult in some circumstances to construct a single conventional controller which would give satisfactory performance under all possible operating environments. A different approach to solve the above problem is to use multiple candidate controllers, instead of a single controller. Using some logic, one of these candidate controllers, which is thought to be 'best fit' for the unknown process, can be switched in the feedback loop at each time to achieve the specified control objective. Most of the recent switching based algorithms select the candidate controllers intelligently based on some on–line evaluation of the plant input/output data. These algorithms can broadly be divided in two categories: those based on indirect method of process estimation [MGHM88, Bal96, NB94, Nar97, Mor96, KR96, Aka04] (henceforth referred to as Multi–Model Adaptive Control(**MMAC**)); and those based on direct evaluation of the potential performance of candidate controllers [ST97, SF01, PS03, Tsa94, JS99, SWS04, WSS04, WSar, RK01, CZM01, TS01] (henceforth referred to as Multi–Controller Adaptive Control(**MCAC**)).

In Multi–Controller Adaptive Control (MCAC), the potential performance of every candidate controller is evaluated at each time directly from the measured data using some suitably defined, data driven performance index, without trying to estimate or identify the actual process [ST97, SF01, PS03, Tsa94, JS99, SWS04, WSS04, WSar, RK01, CZM01, TS01]. Though prior knowledge about the nominal structure of the plant or its parameters is useful to select the initial set of candidate controllers, this method makes no use, nor tries to identify the plant structure or its parameters while deciding the optimal switching sequence among the candidate controllers. The switching is based

on minimization of some performance index, which is a measure of how closely the output of the closed loop system would have followed some reference input (namely, the *fictitious reference input*), had the candidate controller been in the feedback loop. In some recent papers, stability and convergence of the direct unfalsified switching control were addressed [SWS04,WSS04]. An algorithm was proposed in [SWS04], which guaranteed stability of switching, under minimum assumptions that the performance index chosen is *cost detectable* and that there is at least one stabilizing controller in the candidate controller set. The latter condition is called the *feasibility* condition of the control problem. Without feasibility, there exists no adaptive law that converges to a stabilizing solution in the candidate controller set. The primary topic of this dissertation is to study the MCAC approach. In the next section, the overview of the dissertation and its contributions are discussed.

1.2 Organization and contribution of the dissertation

The dissertation is organized as follows:

Chapter 2 provides a brief survey on existing switching based schemes. A tutorial on the MCAC scheme is also provided.

Chapter 3 introduces necessary conditions for the *cost–detectable* property of the performance index to be used in the MCAC scheme. It has been shown that for a (performance index, candidate controller set) pair to be cost detectable, the candidate controllers have to satisfy some special property (namely the Stably Causally Left Invertability condition) and thus imposes some restrictions on the class of candidate controllers that can be used in the MCAC approach. However, it should be noted

that the restrictions imposed on candidate controllers are still *plant-assumption-free*, i.e. to satisfy the them, one does not have to assume anything about the plant; these restrictions can easily be satisfied by judiciously choosing the candidate controllers. The results of this Chapter are based on [PWSS05, WPSS06].

In Chapter 4, the MCAC technique is applied to solve the Model Reference Adaptive Control (MRAC) problem. Also, a comparison has been made with the Multi–Model Adaptive Control (MMAC) approach developed by Narendra *et al* [NB94, Nar97]. Results of this chapter were published in [PS03].

There are several practical scenarios where it is not easily possible to construct a model for the unknown plant; it might be due to the complex, non–linear structure of the plant. In these situations, it is difficult to implement the MMAC based switchig schemes. However, if the controller to be used has a simple structure, the more direct MCAC switching scheme might be easy to implement. One such scenario arises while controlling transmitted powers of several mobile users in a wireless cellular environment. Due to the complex interconnections between powers of various mobile users through a non–linear interference term, it is difficult to model the unknown and time varying process. In Chapter 5, the direct MCAC based switching scheme has been used to control transmitted power of several mobile users in an wireless environment. The results presented are based on [PASM04b, PASM05, PAMS04].

Each of the chapters 3, 4 and 5 is self–contained and may be read in any order. Conclusion and future research directions is presented in Chapter 6.

Chapter 2

Multiple Controller Adaptive Control (MCAC) : Basic Concepts

2.1 Background

Most of the traditional, tuning based adaptive control methods developed prior to 1980s are based on prior assumptions on the plant [NA89, IS95, SB89], like

1. The sign of high frequency gain is known,

2. The order of the plant, n, is known,

3. Its relative degree, n^*, is known,

4. The plant is minimum phase.

However, such set of assumptions can be often unrealistic in practice. Adaptive control based on such assumptions can be highly non-robust, as shown by the famous counter–example by Rohrs *et al* in 1985 [RVAS85].

A lot of research has been done so far on the relaxation of these assumptions. Nussbaum first showed that assumption (1) is not necessary [Nus83]. The assumption

(2) was relaxed by Willems *et al* in [WB84] for plant relative degree $n^\star = 1, 2$. Tao and Ioannou [TI93] developed a MRAC scheme which required knowledge of only an upper bound of the relative degree and not the actual relative degree of the plant, thereby partially relaxing the assumption (3).

A different approach for relaxing the assumptions was used by Martensson in [Mar86], who showed that the order of a stabilizing controller was sufficient information for adaptive stabilization. He proposed a pre–routed search among a finite set of the candidate controllers, *i.e.* the the controllers were switched in the feedback loop in a pre–determined sequence until one controller was found which could achieve the control objective. However, such switching schemes gave poor transient response; and switching to a stabilizing controller took long time, specifically when the number of candidate controllers were large. These kinds of switching scheme had little practical applications. However, the pioneer work by Martenson led several adaptive control researchers explore logic based switching schemes. Some other pre–routed based switching scheme can be found in [FB86, MD91, Fu96].

Most of the recent switching based algorithms no longer follow the pre–routed search technique; they select the candidate controllers intelligently based on some on–line evaluation of the plant input/output data. These switching schemes are more efficient and result in better performance than the blind pre–routed search. These algorithms can broadly be divided in two categories: those based on indirect method of process estimation [MGHM88, Bal96, NB94, Nar97, Mor96, KR96, Aka04]; and those based on direct evaluation of the potential performance of candidate controllers [ST97, Tsa94, JS99, PS03, JZM01].

2.1.1 Estimator based Indirect Multi–Model Adaptive Control (MMAC)

The indirect estimator based switching [MGHM88, Bal96, NB94, Nar97, Mor96, KR96, Aka04] relies on system identification. Here, the actual plant behavior is constantly compared with that of several candidate plant models and the model that best approximates the actual plant is identified at each time. Then, based on certainty equivalence principle, a controller suitable for the identified candidate plant model is switched in the loop. The basic assumption here is that there is at least one plant model in the candidate model set which is sufficiently close to the actual process to be controlled.

2.1.2 Controller Performance based Direct Multi–Controller Adaptive Control (MCAC)

An alternative to the above method is the more direct controller performance based switching [ST97, JS99, PS03, JZM01]. In this case, the potential performance of every candidate controller is evaluated directly from the measured data using some suitably defined performance index, without trying to estimate or identify the actual process. This index is a measure of how closely the output of the closed loop system would have followed some reference input, had the candidate controller been in the feedback loop. Note that the performance index of all the candidate controllers can be evaluated without actually inserting all of them in the feedback loop. Depending on this index, a suitable controller is selected and switched in the feedback loop to control the system. The only assumption for this kind of switching based scheme to be stable is that there is at least one stabilizing controller in the set of candidate controllers, provided one judiciously chooses the performance index [SWS04].

Fig. 2.1 represents a general architecture for a switched system. A key component of multi–controller based switching scheme is the supervisor unit, which decides on the switching logic based on the plant input/output data. As stated before, this supervisory unit can be estimator based or the controller performance based.
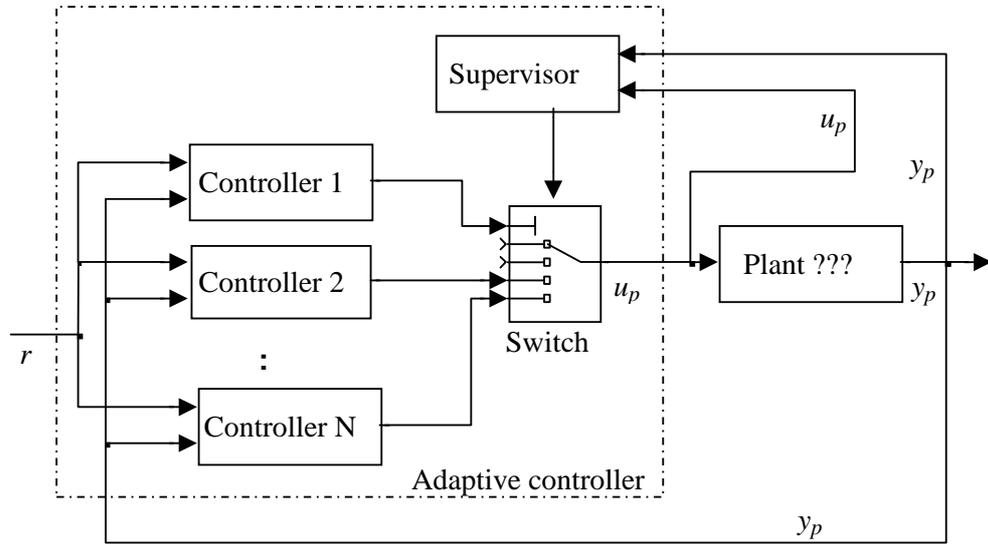
Figure 2.1: A general architecture for switching based control

Chapter 3

Cost Detectability of Performance Index for Unfalsified Multi–Controller Adaptive Control

Unfalsified direct adaptive control using multiple controllers and switching has been examined in past. Conditions for stability of this approach has been proved in the literature, under the minimum assumption of presence of at least one stabilizing controller in the pool of available candidate controllers, provided one judiciously chooses a cost detectible performance index. In this chapter, we show that for a certain narrow class of candidate controllers, the cost detectability property of the performance index cannot be fulfilled. We provide a plant-assumption-free necessary condition on the structure of the candidate controllers, namely the Stably Causally Left Invertability (SCLI) condition, to overcome this problem. The results of this Chapter are based on [PWSS05, WPSS06].

3.1   Introduction

Multiple model adaptive switching control has gained popularity among the adaptive control community in the past few decades. Early work in this field was done in mid

80s by Martensson [Mar86] , who proposed a pre–routed search among the candidate controllers until one controller was found which could achieve the control objective. Most of the recent switching based algorithms select the candidate controllers more intelligently, based upon on–line evaluation of the plant input/output data. These algorithms can broadly be divided in two categories: those based on indirect method of process estimation [NB94, Bal96, Mor96, KR96]; and those based on direct evaluation of the potential performance of candidate controllers [ST97, SF01, PS03, Tsa94, JS99, SWS04, WSS04, WSar, CZM01, RK01, TS01].

The indirect estimator based switching [NB94, Bal96, Mor96, KR96] relies on system identification. Here, the actual plant behavior is constantly compared with that of several candidate plant models and the model that best approximates the actual plant is identified at each time. The basic assumption here is that there is at least one plant model in the candidate model set which is *sufficiently* close to the actual process to be controlled.

An alternative to the above method is the more direct controller performance index based switching, popularly referred to as the unfalsification approach in some recent publications [ST97, SF01, PS03, Tsa94, JS99, SWS04, WSS04, WSar] and it was successfully applied to some practical problems [RK01, CZM01, TS01]. Here, the potential performance of every candidate controller is evaluated at each time directly from the measured data using some suitably defined, data driven, performance index, without trying to estimate or identify the actual process. The performance index is chosen to be a measure of how closely the output of the closed loop system would have followed some reference input (namely, the *fictitious reference input*), had the candidate controller been in the feedback loop. In some recent papers, stability and convergence of

the direct unfalsified switching control were addressed [SWS04, WSS04]. An algorithm was proposed in [SWS04], which guaranteed stability of switching, under minimum assumptions that the performance index chosen is *cost detectable* and that there is at least one stabilizing controller in the candidate controller set. The latter condition is called *feasibility* of the control problem. Without feasibility, there exists no adaptive law that converges to a stabilizing solution in the candidate controller set.
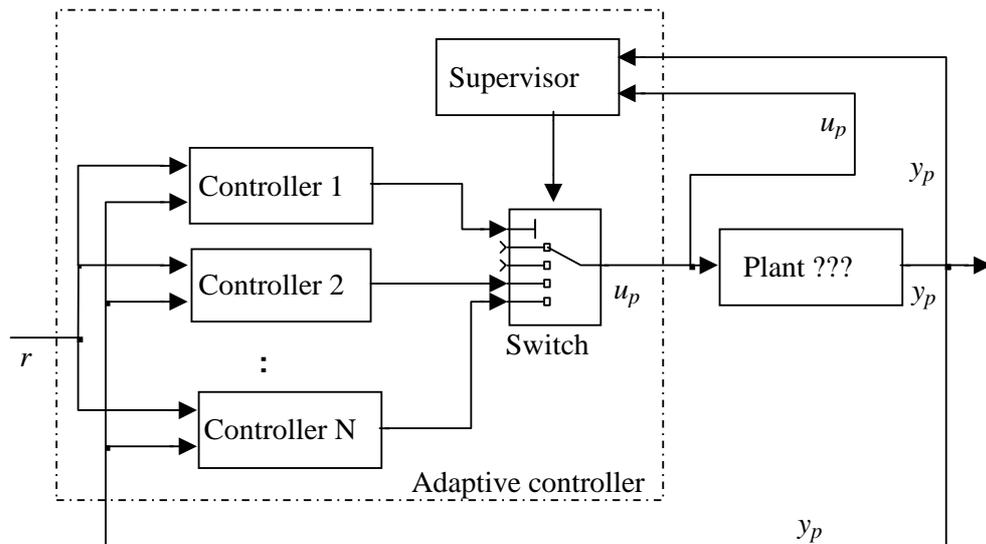
Figure 3.1: A general architecture for an adaptive controller using switching based logic

A main component behind any multiple controller based adaptive control scheme is to associate candidate controllers with a data driven performance index. The controllers are ordered and chosen based on minimizing the performance index; the performance index is a reflection of the ability of a candidate controller to give good performance when placed in loop with the unknown plant. In this chapter, the cost detectability property of the performance index is studied. It is shown that for a certain narrow class of candidate controllers, this property cannot be fulfilled, irrespective of the structure of the performance index used. Hence, for this class of candidate controller, as of now,

it is not possible to apply the multi–controller based direct switching scheme using un-falsification concept. However, the author wants to stress that this class of candidate controllers is quite restrictive and for most practical applications, unfalsification approach can be used. Also, a necessary condition on the controller structure is proposed in the chapter for the performance index to be cost detectible.

In the next section, we give the detailed description of the direct multi–controller based adaptive unfalsification approach. Section 3.3 contains the main results of the chapter, where the necessary condition for cost detectability of performance index is detailed. The condition is elucidated using several examples. We conclude in the last section.

3.2 Problem Formulation

Consider a unknown plant, with (u_p, y_p) as the measurable plant Input/Output (I/O) signals and r as the reference signal. Given a finite set of candidate controllers, repre-sented by

$$\mathbb{C} = \{C_i\},\ i = 1, \ldots, N.$$

Unfalsification approach is a *plant-assumption-free* approach, i.e. we do not assume anything about the unknown plant. However, we have certain minimum assumption on the candidate controller set for stabilization of the plant, which is referred to as the *feasibility* assumption [SWS04].

Assumption 1. *The candidate controller set contains at least one robustly stabilizing and performing candidate controller.* ◇

We now briefly introduce the following notations and definitions.

Let $\overline{\mathbf{Z}}$ be the set of all possible plant Input/Output(I/O) data. Let $\mathbf{Z}$ be the set of experimental data, measured from time o to time ∞. Also, let $\mathbf{Z}_t$ be the set of measured plant I/O data, measured from time 0 to current time t. So, $\mathbf{Z}_t$ is the truncated space of $\mathbf{Z}$, measured till current time t and represents the plant I/O data (u_p, y_p) from initial to the current time.

Definition 1 *(L_{2e} signals). We define the truncation of signal x over a time interval $[0, \tau]$ as*

$$
x_\tau(t) = \begin{cases} x(t), & \text{if } t \in [0, \tau] \\ \\ 0, & \text{otherwise.} \end{cases}
$$

We say $x \in L_{2e}$ if $\| x_\tau \|$ exists for any finite time τ where

$$
\| x_\tau \| = \sqrt{\int_0^\infty \| x(t) \|^2 \, dt}
$$

$\diamond$

Definition 2 *(Stability) A system with input w and output v is stable if for any $w \in L_{2e}$, $w \neq 0$,*

$$
\sup_{\tau \in (0, \infty]} \frac{\| v_\tau \|}{\| w_\tau \|} < \infty.
$$

$\diamond$

Definition 3 *(Bi-proper transfer function [SP00]) A SISO system $G(s)$ is bi-proper if $G(s) \to D \neq 0$ as $s \to \infty$. For a transfer function to be bi-proper, the order of*

numerator and denominator polynomial has to be same, i.e. relative degree has to be

0. ◇

Definition 4 *(Fictitious Reference Signal) [JS99] Given a set of past measured plant*

I/O data (u_p, y_p), and a candidate controller $C_i \in \mathbb{C}$, a fictitious reference signal for this

candidate controller is a hypothetical reference signal that would have produced exactly

the measured data (u_p, y_p) had the candidate controller C_i been in the feedback loop with

the unknown plant during the entire time period over which the measured data (u_p, y_p)

were collected. ◇

If the ith controller is actually in the loop during the entire period over which plant

I/O data (u_p, y_p) were collected, then one i^{th} fictitious reference signal would be the

actual reference signal; else it would in general be different from the actual reference

signal (hence the name *fictitious*).

We denote the fictitious signals by $\widetilde{r}(C_i, u_p, y_p)$ and for notational convenience, also

by $\widetilde{r}_i$. A fictitious reference generator for a general setup is shown in Fig. 3.2.

Definition 5 *(Unfalsified stability) [SWS04] Given a candidate controller C_i and cor-*

responding fictitious reference signal $\widetilde{r}(C_i, u_p, y_p)$, we say the stability of the system with

the ith candidate controller is falsified *by measurement data (u_p, y_p) if $\exists \; \widetilde{r}(C_i, u_p, y_p)$*

such that $\lim\limits_{t \to \infty} \dfrac{\|y_p, u_p\|}{\|\widetilde{r}(C_i, u_p, y_p)\|} = \infty$. ◇

Definition 6 *(Fictitious error signal) For each candidate controller, a fictitious error*

signal can be defined as the error between its fictitious reference signal and the actual

plant output. Hence, this would have been the error signal, had that candidate controller

been in the feedback loop with (u_p, y_p) as the measured plant data and $\widetilde{r}(C_i, u_p, y_p)$ as

the reference signal. ◇
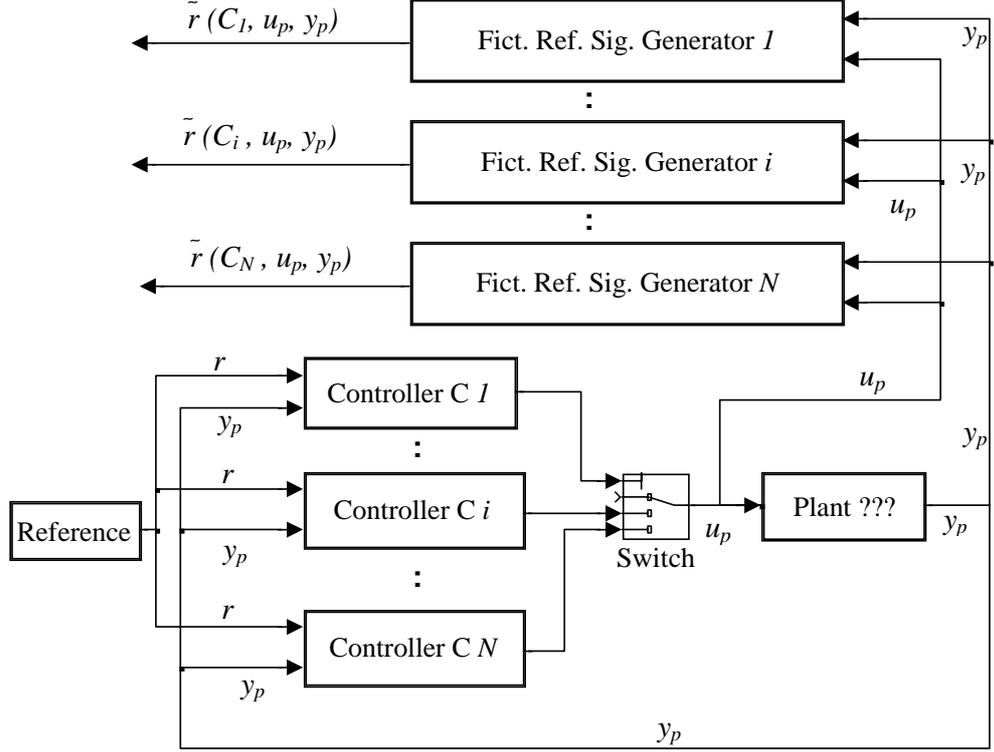
Figure 3.2: General setup of filters for generating Fictitious reference signals

The fictitious error signal for the candidate controller C_i is represented by $\widetilde{e}(C_i, u_p, y_p)$ and for notational convenience, would also be denoted by $\widetilde{e}_i$ and is given by

$$\widetilde{e}_i(C_i, u_p, y_p) \triangleq \widetilde{r}_i(C_i, u_p, y_p) - y_p. \tag{3.1}$$

Now, y_p is the output of the actual process with C_i controller in the loop and with the ith fictitious reference signal $\widetilde{r}(C_i, u_p, y_p)$ as the reference signal (follows from the definition of fictitious reference signal). Hence, the error $\widetilde{e}(C_i, u_p, y_p)$ would have been the control error, had the ith candidate controller C_i been in the feedback loop during the entire time period over which the measured data (u_p, y_p) were collected, with plant input/output data as (u_p, y_p) and fictitious reference signal $\widetilde{r}(C_i, u_p, y_p)$ as

the reference signal. Thus, the error $\widetilde{e}(C_i, u_p, y_p)$ is a is used in evaluating how effective the ith candidate controller might be if it were placed in the loop.

Definition 7 *(Performance index) For a candidate controller $C_i \in \mathbb{C}$, the performance index $J(C_i, u_p, y_p, \tau)$ is a mapping: $J : \mathbf{Z}_t \times C_i \times \mathbb{R}_+ \rightarrow \mathbb{R}_+$, where $\mathbf{Z}_t$ is the truncated space of the measured signals (u_p, y_p) from initial to the current time. This is a measure of performance of the closed loop system, had the candidate controller C_i been in the feedback loop with the unknown plant, with the corresponding fictitious reference signal as the reference signal and (u_p, y_p) as the measured plant I/O data.* $\diamond$

For notational convenience, the performance index for i^{th} candidate controller would also be denoted by J_i. The ordering of candidate controllers and switching is based on the performance index. Several performance index has been suggested in literature; a good overview can be found in [SWS04]. An example of a performance index is given from [SWS04] as

$$J(C_i, u_p, y_p, \tau) = \max_{\tau \in (0,t)} \frac{\widetilde{e}_i^2(\tau) + \int_0^\tau \exp^{(-\lambda(\tau-\sigma))} \widetilde{e}_i^2(\sigma) d\sigma}{\int_0^\tau \exp^{(-\lambda(\tau-\sigma))} \widetilde{r}_i^2(\sigma) d\sigma}, \tag{3.2}$$

where $\widetilde{e}_i$ and $\widetilde{r}_i$ are the fictitious error and fictitious reference signal of the ith candidate controller respectively; λ is a non–negative exponential forgetting factor that deemphasizes the importance of distance past signals and places more emphasis on the current value of the signal. The max operator ensures monotonically increasing cost. Detailed discussion on the choice of this performance index and the rationale behind can be found in [SWS04].

Definition 8 *(Cost Detectability of performance index) In switching adaptive control, given cost function J and candidate controller set $\mathbb{C}$. Suppose for an input r, the time varying online controller is $\widehat{C}(\tau) \in \mathbb{C}, \forall \tau$, and $\widehat{C}(\tau) = C^f, C^f \in \mathbb{C}$, for $\tau > t_f$. $\mathbf{Z}_t$ is the plant data collected during $\widehat{C}$ in the close loop. The pair $(J, \mathbb{C})$ is said to have* cost-detectability *property for this input r, if the following statements are equivalent:*

1). $J(C^f, \mathbf{Z}_t, \tau)$ is bounded during $\mathbb{R}_+$;

2). The stability of the system with $\widehat{C}(\tau)$ is unfalsified by $(r, \mathbf{Z}_t)$. ◇

When we say a system has the cost-detectability property, we mean the pair of $(J, \mathbb{C})$ of that system is cost-detectable for any input r. This is a plant-independent notion. This definition implies that system with cost-detectability can detect instability when an unstable controller finally stays in the feedback loop.

For the sake of completeness and to fully describe the unfalsification approach, we would now present an algorithm from [SWS04]. This algorithm describes a switching scheme under which the stability of the adaptive system and convergence of the adaptive law are guaranteed. The switching algorithm described below is a ε - performance minimization algorithm [SWS04]; however there are several different ways of choosing a controller [ST97, Tsa94]. The algorithm outputs, at each instant τ, a controller $\widehat{C}_\tau$ which is placed in the feedback loop.

Algorithm 1 [SWS04]

1. Initialization: Let $t = 0, \tau = 0$; choose $\varepsilon > 0$. Let $\widehat{C}_t \in \mathbb{C}$ be the first controller in the loop.

2. $\tau \leftarrow \tau + 1$.

   If $J(\widehat{C}_t, u_p, y_p, \tau) > \min_{C \in \mathbb{C}} J(C, u_p, y_p, \tau) + \varepsilon$,

   then $t \leftarrow \tau$ and $\widehat{C}_t \leftarrow \arg\min_{C \in \mathbb{C}} J(C, u_p, y_p, \tau)$

3. $\widehat{C}_\tau \leftarrow \widehat{C}_t$; $return\ \widehat{C}_\tau$.

4. Go to 2.

The switching occurs only when the current performance index related to the current active controller exceeds the minimum performance index of other candidate controllers at current time by at least ε. This non-zero design constant prevents arbitrary fast switching and ensures a non-zero dwell time between switches. A detailed description of the algorithm and proof of convergence of the algorithm can be found in [SWS04]. However, as the focus of this chapter is on cost detectability property of the performance index, details are avoided here.

3.3 Results

In this section, we impose certain restriction on the candidate controller set for the unfalsification approach to work. First we define a key property associated with the candidate controllers.

Definition 9 (*S*tably *C*ausally *L*eft *I*nvertable Controller (*SCLI*)) *A candidate controller is said to be Stably Causally Left Invertible (SCLI), if the mapping from the collected measurement data (u_p, y_p) to the corresponding fictitious reference signal is causal and stable.* ◇

Note: The above definition implies that for a controller to be SCLI, the corresponding fictitious reference signal generator has to be causal and stable (See Fig. 3.2).

Using the following situations, we now demonstrate the restrictions imposed on the controller structure for the SCLI property to be satisfied.
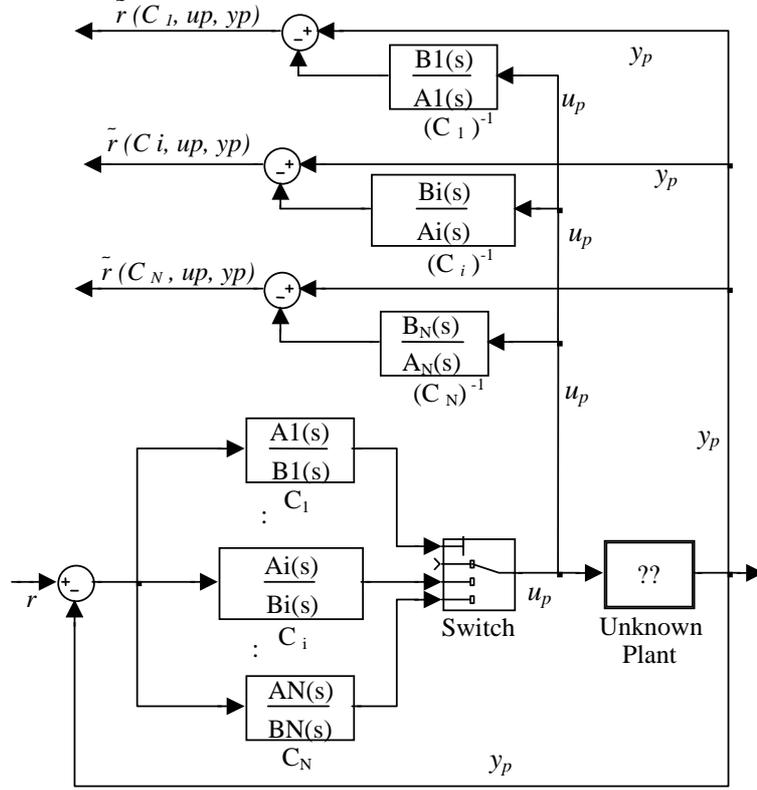
Figure 3.3: Fictitious reference signal generator for Situation 1

Situation 1: Consider a class of candidate controllers, given by (Fig 3.3)

$$C_i(s) = \frac{A_i(s)}{B_i(s)}, i = 1, \ldots, N,$$

and the control input given by

$$u_p = \widehat{C}(s)(r - y_p), \tag{3.3}$$

19

where $\widehat{C}(s)$ is the candidate controller currently in the loop, determined by any suitable switching algorithm (for example, Algorithm 1 or any other suitable algorithm described in [ST97, Tsa94] or references therein). As per definition given earlier, the fictitious reference signal for controller C_i would then be given by

$$\widetilde{r}(C_i, u_p, y_p) = y_p + C_i(s)^{-1} u_p = y_p + \frac{B_i(s)}{A_i(s)} u_p. \tag{3.4}$$

The mapping from (u_p, y_p) to $\widetilde{r}(C_i, u_p, y_p)$ is causal and stable if $C_i(s)^{-1}$ is causal and has no RHP poles. Hence, the candidate controllers, when placed in the forward path of the feedback loop, would be SCLI if they are minimum phase and bi-proper; i.e. $A_i(s), i = 1, \ldots, N$ should be stable polynomial and order of $A_i(s)$ and $B_i(s), i = 1, \ldots, N$ should be same for the SCLI property to be fulfilled. $\diamond$

Situation 2: Now consider a class of candidate controller, given by (Fig 3.4)

$$C_i(s) = \frac{E_i(s)}{F_i(s)}, i = 1, \ldots, N,$$

and the control input given by

$$u_p = r + \widehat{C}(s) y_p, \tag{3.5}$$

where $\widehat{C}$ is the candidate controller currently in the loop, determined by any suitable switching algorithm (for example, Algorithm 1 or any other algorithm described
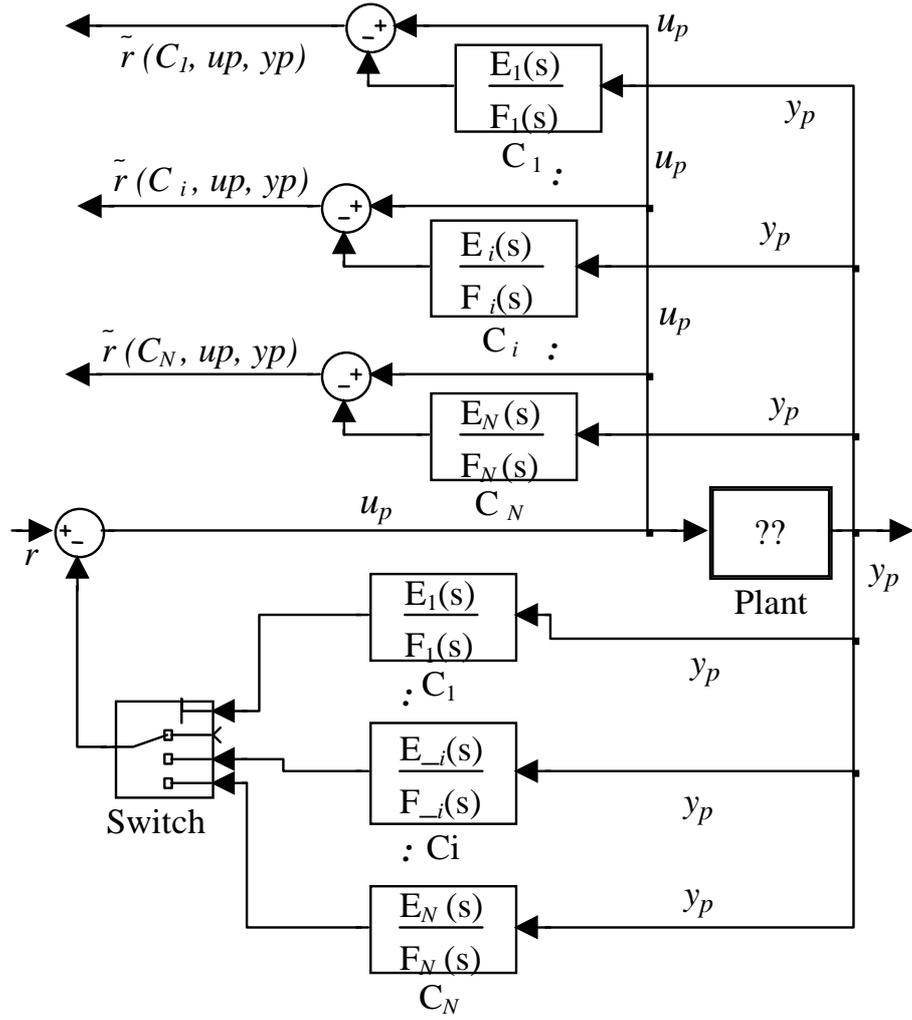
Figure 3.4: Fictitious reference signal generator for Situation 2

in [ST97, Tsa94] or references therein). Note that, in this case, the candidate controllers are placed in the backward path of the feedback loop. As per the definition given earlier, the fictitious reference signal for controller C_i would be given by

$$\widetilde{r}(C_i, u_p, y_p) = u_p - C_i(s)y_p = u_p - \frac{E_i(s)}{F_i(s)}y_p. \tag{3.6}$$

Note that, when the candidate controllers are in the backward path of the feedback loop, to calculate the fictitious reference signal, the candidate controller transfer function is

needed (and not the inverse of the candidate controller transfer function, which was needed for Situation 1, where in controllers were placed in the forward path of the feedback loop). Thus, for this situation, for fictitious reference signal to exists and be stable, the candidate controllers should have no RHP poles, which is also a common requirement. ◇

So, from the above two situations, we see that when the candidate controllers are in the feed forward loop, for the fictitious reference signal generator to be causal and stable, the corresponding candidate controller has to be minimum phase and bi-proper. However, if the candidate controllers are to be placed in the feedback loop, candidate controllers with no RHP poles is enough for the fictitious reference signal generator to be causal and stable.

Situation 3: Now, using a third situation, we illustrate the importance of the SCLI property of the controllers. Consider just a single candidate controller, C_1, to be used to control the plant and no switching involved. Let C_1 have structure similar to that in Situation 1, i.e.

$$C_1(s) = \frac{A_1(s)}{B_1(s)},$$

and the control input be given by

$$u_p = C_1(s)(r - y_p). \tag{3.7}$$

Let the performance index be chosen same as (3.2). Let C_1 be chosen such that it is non minimum phase ($A_1(s)$ having RHP poles), biproper and destabilizes the unknown plant. The fictitious reference signal corresponding to C_1 can be calculated as

$$\widetilde{r}(C_1, u_p, y_p) = y_p + C_1(s)^{-1}u_p = y_p + \frac{B_1(s)}{A_1(s)}u_p. \tag{3.8}$$

Note that in this situation, C_1 is the only candidate controller and hence it is switched in the loop throughout. Now, from the definition of the fictitious reference signal, ideally, as C_1 is in the loop the entire time during which the plant data (u_p, y_p) were collected, the fictitious reference signal should be same as the actual reference signal; however, due to the RHP pole in the $A_1(s)$ polynomial in the fictitious reference signal generator, slight noise or disturbance would make the fictitious reference signal unstable. Hence, in practice, the fictitious reference signal would always be unstable and $\widetilde{r}_1 \to \infty$ as $t \to \infty$. As C_1 is non-minimum phase, it does not satisfy the SCLI property and although the fictitious reference signal exists, it grows unbounded with time irrespective of the collected measurement data (u_p, y_p). Now, examine the performance index corresponding to C_1, given in (3.2). The denominator grows unbounded as $t \to \infty$, irrespective of the plant data (u_p, y_p). Now, even if C_1 was chosen to be a destabilizing controller, with y_p growing unbounded with time, but $\widetilde{r}_1$ can grow at a rate faster then y_p, thereby resulting in a finite performance index; hence although the closed loop system can be unstable with C_1 in the feedback loop, it might not be reflected in the performance index $J(C_1, u_p, y_p, \tau)$. $\diamond$

Example 3.1. Consider a case, where the unknown plant is given by (Same as in Situation 3)

$$G(s) = \frac{10}{s + 10}.$$ (3.9)

Consider a controller, given by

$$C(s) = \frac{s - 4}{s},$$ (3.10)

which is obviously not SCLI, and the control law given by

$$u = C(r - y).$$ (3.11)

Also, let the controller $C(s)$ be in the loop, without any switching involved.

Then, the closed loop system is

$$T(s) = \frac{10s - 40}{s^2 + 20s - 40},$$ (3.12)

with closed loop poles lying at -20.1980 and 0.1980. Clearly the closed loop system is unstable.

The fictitious reference signal can be calculated as

$$\widetilde{r} = \frac{C^{-1}C}{1 + CG}r + \frac{CG}{1 + CG}r,$$

$$\text{or} \quad \widetilde{r} = \frac{s(s - 4)}{s(s - 4)} \frac{1}{1 + CG}r + \frac{CG}{1 + CG}r.$$ (3.13)

Now, since the fictitious reference signal generator involves a cancellation of a right half pole–zero, the fictitious reference signal may go to infinity due to disturbance and noise. We implement the simple version of cost (3.2),

$$J(C, (u, y), \tau) = \max_{t \leq \tau} \frac{\|\widetilde{e}\|}{\|\widetilde{r}\|} \tag{3.14}$$

and simulate the plant using a step input. We see that the cost is bounded while the system is unstable(Fig. 3.5), which means the cost function does not capture the instability of the system and thus violates the definition of cost-detectability. $\qquad\square$
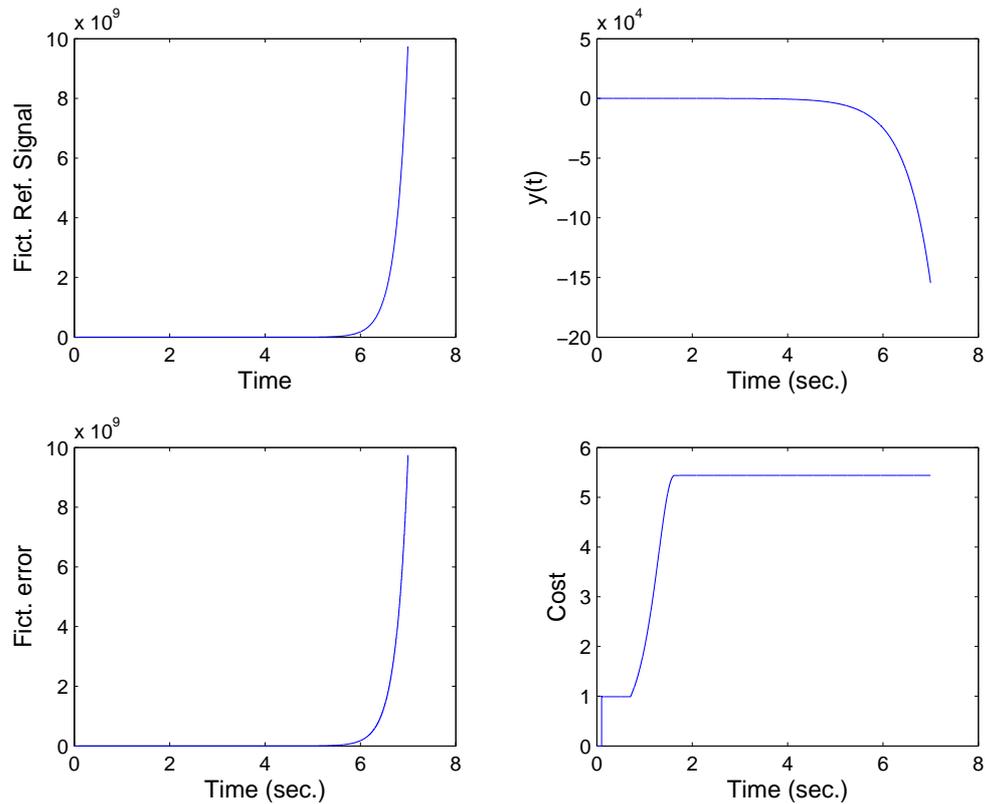
Figure 3.5: Signals for the Example.

From the above, we conclude that if a candidate controller does not satisfy the SCLI property, the corresponding performance index may lose the cost detectability property.

Theorem 1 : *A necessary condition for a performance index of form (3.2) to be cost detectible is that the candidate controllers are Stably Causally Left Invertible (SCLI).*

Proof: Suppose cost detectability holds and that C_1 is a candidate controller which does not satisfy the SCLI property. Suppose that C_1 is switched into the loop at some time t_1 and remains in the loop thereafter and that the true input is r and $\|r\| < \infty$. Since controller C_1 is not SCLI, we may assume that the fictitious reference signal $\tilde{r}_1(t)$ grows unboundedly. Since cost-detectability is by definition a plant-independent property, it holds for any plant. Choose the plant so that C_1 destabilizes it, but only just barely so that the plant output $y_p(t)$ grows less rapidly than $\tilde{r}_1(t)$ (cf. Situation 3 and Example 3.1). Clearly, the data (u_p, y_p) falsifies stability. However, since $y(t)$ grows less rapidly than $\tilde{r}_1(t)$, it follows that the cost $J_1(t)$ given by (3.2) remains finite as $t \to \infty$, contradicting cost detectability. $\diamond$

3.4   Discussion and Conclusion

As noted in several previous publications, the main motivation behind unfalsification theory is to develop a *plant-assumption-free* method that would ensure stability of a system, under minimum assumptions of presence of at least one stabilizing controller in the candidate controller set and a cost detectable performance index. Now, Proposition 1 says that for a performance index to be cost detectable, the candidate controllers have to satisfy the SCLI property and thus imposes some restrictions on the class of

candidate controllers that can be used in the unfalsification approach. However, it should be noted that the restrictions imposed on candidate controllers are still *plant-assumption-free*, i.e. to satisfy the them, one does not have to assume anything about the plant; these restrictions can easily be satisfied by judiciously choosing the candidate controllers.

Also, as noted in the Theorem and Situations, the restrictions on the controller structure apply when the controllers are in the forward path of the feedback loop. In that case, the candidate controllers are required to be minimum phase; in practice, most controllers used are minimum phase and hence unfalsification approach can still be applied to most practical cases.
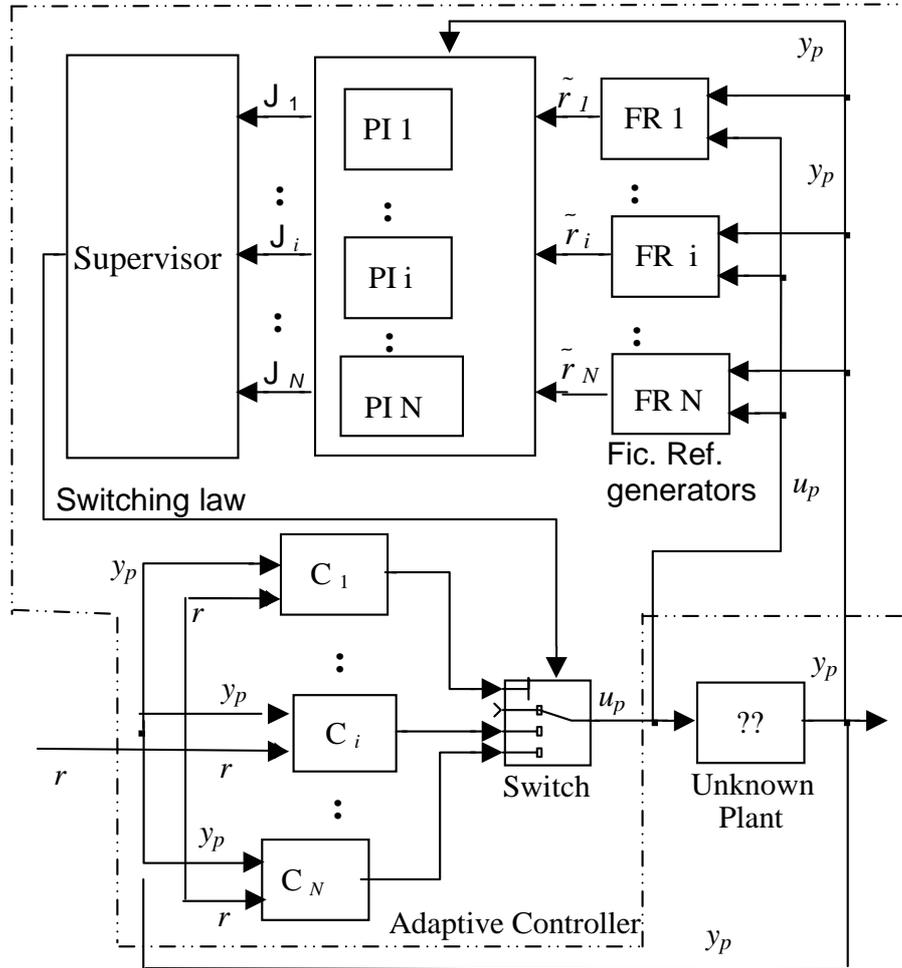
Figure 3.6: General setup for switching based control. $\widetilde{r}_i, i = 1, \ldots, N$ represents the fictitious reference signal $\widetilde{r}(C_i, u_p, y_p)$ and FR 1,..., FR N represent filters used to generate these signals. Similarly, $J_i, i = 1, \ldots, N$ represents performance index $J(C_i, u_p, y_p, \tau)$ and PI 1,...,PI N represent filters used to generate them. The dotted line represents the adaptive controller.

Chapter 4

Model Reference Adaptive Control using Multiple Controllers and Switching

In this chapter, we develop an adaptive multi–controller system to solve the MRAC problem using unfalsified control theory. Switching is done among the candidate controllers based on some suitably defined performance index, which is obtained without actually inserting the candidate controllers in the feedback loop. Though prior knowledge about the nominal plant structure or its parameters is useful to select the initial set of candidate controllers, this method makes no use nor tries to identify the plant structure or its parameters while deciding the optimal switching sequence among the candidate controllers. Simulation results are presented to indicate successful switching strategy. We also compare this approach with the switching scheme developed by Narendra for the MRAC problem in [NB94, NS97].

4.1   Introduction

In the last few years, the idea of switching between various plant models has developed rapidly: Narendra et al [NB94, NS97] and Morse, Hespanha et al [Hes01, MMG92].

The main idea behind many of these approaches, also known as indirect adaptive control approach, is as follows: There are multiple candidate models of the plant to be controlled and there is a corresponding candidate controller for each candidate plant model. The correct controller identification and switching is done in two stages: first, the plant model that best represents the actual plant behaviour is identified from the set of candidate plant models and then, based on certainty equivalence principle, the corresponding controller is switched in the feedback loop.

An alternative to the above approach is a more direct adaptive control approach, also referred to as unfalsification method. Using unfalsification concept, we dispense any assumption on the plant structure and directly identify and switch to the optimal controller among a set of candidate controllers. This approach can be used to evaluate performance of all candidate controllers directly at every time instant without actually inserting them in the feedback loop; by performance of a candidate controller we mean how closely the closed loop plant would have followed the given reference model, had that candidate controller been in the feedback loop. Thus, switching between candidate controllers can be based directly and only on the performance of candidate controllers. The concept was proposed by Safonov *et al* [CS96, ST97, JS99, SF01]. In this Chapter, we apply it to the Model Reference Adaptive Control (MRAC) problem. At the end, we briefly discuss the indirect adaptive control approach developed by Narendra in [NB94, NS97] and compare it with the method developed in this Chapter.

4.2 Statement of MRAC problem

The statement of our problem is similar to the general MRAC problem and we present
it from [NB94]. The plant to be controlled is linear, time invariant, with input u_p and
output y_p related by

$$y_p = W_p(s)u_p, \tag{4.1}$$

where

$$W_p(s) = K_p \frac{Z_p(s)}{R_p(s))}$$

is the transfer function of the plant. Here $K_p \in \Re \setminus \{0\}$, and $R_p(s)$ and $Z_p(s)$ are
monic coprime polynomials of degree n and m respectively, with $m < n$, and unknown
coefficients and $Z_p(s)$ is Hurwitz.

The reference model to be followed is linear, time invariant with input r (which is
piece continuous, uniformly bounded), output y_m related by

$$y_m = W_m(s)r, \tag{4.2}$$

where

$$W_m(s) = K_m \frac{Zm(s)}{Rm(s)}$$

is the transfer function of the reference model. Here $K_m \in \Re \setminus \{0\}$, and $R_m(s)$ and
$Z_m(s)$ are monic coprime Hurwitz polynomials of degree n and m.

Let the control error e_c be the difference between output of the plant and the
reference model, given by

$$e_c = y_p - y_m. \tag{4.3}$$

The objective of MRAC problem is to select a differentiator free control input such that $e_c(t)$ converges to zero asymptotically. However, we modify this objective in view of our fixed (no tuning involved) multiple controller based switching scheme. We aim to select a controller from the candidate controller set which would minimize a suitably defined norm of this error.

4.3   Overview of switched system

In this section, a brief description of the direct multi–controller based switching scheme and unfalsification approach is presented. Consider an unknown plant P for which we need to determine a control law K so that the closed loop system response, say T, satisfies a specification requiring that for all command inputs $r \in \Re$, the triple (r, y, u) be in the given specification set $\mathbb{T}_{spec}$.

Definition [Safonov and Tsao, [ST97]]: A controller $K \in \mathbb{K}$ is said to be *falsified* by measurement information if this information is sufficient to deduce that the performance specification $(r, y, u) \in \mathbb{T}_{spec} \; \forall \; r \in \Re$ would be violated if the controller were in the feedback loop. Otherwise the control law K is said to be unfalsified.

Let $\mathbb{K}$ be a given class of admissible control laws; $\mathbb{P}_{data}$ be the set of triples (r, y, u) consistent with past measurement of (u, y). For each $K \in \mathbb{K}$, let $K \triangleq graph(K)$, *i.e.* , $\mathbb{K}$ is the set of triples (r, y, u) that satisfy input–output relation of K.

Theorem 2 *[ST97]: A control law $K \in \mathbb{K}$ is unfalsified by measurement information $\mathbb{P}_{data}$ if, and only if, for each triple $(r_0, y_0, u_0) \in \mathbb{P}_{data} \cap K$, there exists at least one pair (u_1, y_1) such that*

$$(r_0, y_1, u_1) \in \mathbb{P}_{data} \cap \mathbb{K} \cap \mathbb{T}_{spec}. \tag{4.4}$$

General Methodology: The proposed system has a finite set of N candidate controllers, denoted by K_i, $i \in \mathcal{I} = 1, \ldots, N$. These candidate controller models can be selected or designed off–line before the process starts, based on some nominal model of the process. Then at every instant, one of the candidate controllers is selected from this set, based on some suitably defined performance criterion, and switched into the feedback loop.

Candidate Controllers: The unknown plant is assumed to belong to a compact set $\mathcal{S}$. First, N nominal plant models, evenly distributed in the set S, are chosen. When less prior knowledge of the plant is available, this set can be made larger to include all possibilities. Corresponding to each plant model, we choose a candidate controller that can meet performance requirements of that plant. The actual plant can take any of the infinite number of values within the compact set S and not necessarily match exactly with the initially chosen N number of plant models for which the candidate controllers were designed. The objective is to select the optimal controller among the set of candidate controllers, which would minimize a given performance index.

Fictitious Reference Signal: Given a set of past plant input/output data $u_p(t), y_p(t)$, we now define fictitious reference input $\widetilde{r}(K_i, y_p, u_p)$ for the candidate controller K_i. This is a hypothetical command signal that would have produced exactly the measured data (u_p, y_p) had the candidate controller K_i been in the feedback loop with the unknown plant during the entire time period over which the measured data (u_p, y_p) were collected [JS99]. Note that this signal is not the actual reference signal, hence the name

fictitious. For example, for the system in Fig. 4.1, the fictitious reference signal for the ith candidate controller with controller parameter as K_{ai} and K_{bi} would be

$$\widetilde{r}(K_i, u_p, y_p) \triangleq (u_p - K_{ai}y_p)K_{bi}^{-1}Z. \tag{4.5}$$

If the ith controller is actually in the loop during which plant I/O data (u_p, y_p) were collected, then the ith fictitious reference signal would be same as the actual reference signal r; else it would be different from the actual reference signal.
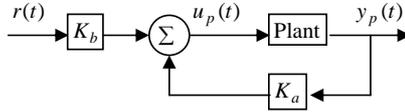
Figure 4.1: Example controller structure

Remark: To determine the fictitious reference signal, the controller has to be causally-left-invertible, that is, from the past and present output of the controller, one should be able to uniquely determine its present input [JS99]. Corresponding to each candidate controller, a fictitious output $\widetilde{y}(K_i, u_p, y_p)$ can be defined as

$$\widetilde{y}(K_i, u_p, y_p) \triangleq W_m \widetilde{r}(K_i, u_p, y_p). \tag{4.6}$$

For each candidate controller, a fictitious error signal can be defined as the error between its fictitious output and the actual plant output. Hence, i^{th} fictitious error signal would be

$$\widetilde{e}(K_i, u_p, y_p) \triangleq y_p - \widetilde{y}(K_i, u_p, y_p). \tag{4.7}$$

To keep notation simple, we represent signals as

$$\widetilde{r}_i = \widetilde{r}(K_i, u_p, y_p),$$

$$\widetilde{y}_i = \widetilde{y}(K_i, u_p, y_p)$$

and

$$\widetilde{e}_i = \widetilde{e}(K_i, u_p, y_p).$$

Performance index: The fictitious output $\widetilde{y}_i$ is the output of the reference model with the ith fictitious reference signal $\widetilde{r}_i$ as the command signal (4.6). And y_p is the output of the actual process with K_i controller in the loop and with the ith fictitious reference signal $\widetilde{r}_i$ as the command signal (follows from the definition of fictitious reference signal). Hence, the error $\widetilde{e}_i$ would have been the control error (4.3), had the ith candidate controller K_i been in the feedback loop during the entire time period over which the measured data (u_p, y_p) were collected, with plant input/output data as (u_p, y_p) and fictitious reference signal $\widetilde{r}_i(t)$ as the reference signal. Thus, the error $\widetilde{e}_i$ is a measure of effectiveness of the ith candidate controller if it is placed in the loop, i.e. how closely will the feedback system with i^{th} controller in the loop follow the given reference model. So, the switching among the candidate controllers should be based on some norm of this error.

We consider a γ dependent performance specification[1]

$$\mathbb{T}_{spec}(\gamma) = \{(r, y, u) \mid J(r, u, y, t) \leq \gamma \ \forall t\}$$

$$J(r, y, u, t) \triangleq \alpha e_c^2(t) + \beta \int_0^t \exp^{\lambda(t-\tau)} e_c^2(\tau) d\tau, \qquad (4.8)$$

and $e_c = y_p - Wm(s)r$. Parameter γ is the performance level and parameters $\alpha, \beta, \lambda \geq 0$ are design parameters. The parameter a penalizes instantaneous errors while β penalizes accumulated past error. These design parameters can be adjusted depending on the given problem. For example, for a rapidly time varying plant, more weighting can be given to instantaneous error with a small time windowing; for a time invariant or slowly varying plant equal weighting can be given to the two error terms. The non-negative forgetting factor λ determines the weight of past error; it also ensures the boundedness of the integral terms. At time t, the performance index for i^{th} candidate controller is represented by

$$\widetilde{J}_i(t) \triangleq J(\widetilde{r}, u_p, y_p, t) \qquad (4.9)$$

The controllers are naturally ordered on the basis of their unfalsified performance level g; the lower is this performance level for a candidate controller, the more is the likelihood of this controller giving better performance. By Theorem 1, K_i is unfalsified at time t by plant input/output data (u_p, y_p), if and only if the performance index for the ith controller satisfies

$$\max_{\tau \leq t} \widetilde{J}_i(\tau) < \gamma, \qquad (4.10)$$

---

[1] A recent publication [SWS04] shows that the switching scheme studied in this dissertation (and in [PS03, Bal96, NB94, Nar97]) might fail to choose a stabilizing controller over a destabilizing one if the performance index is not judiciously chosen; the problem, as pointed out in [SWS04], is that the performance index (4.8) is not cost detectable.

where γ is a positive threshold and design parameter; if for a candidate controller, the performance index exceeds this threshold at any time, it is not suitable for the actual unknown plant and hence is falsified and taken out of the candidate controller set. Switching is done among as yet unfalsified candidate controllers only.

The left-hand side of (4.10) is the unfalsified performance level associated with controller K_i at time t. Since it is non-decreasing in time t, it follows that for each specified performance level γ the unfalsified controller set shrinks as time increases; controllers which are not suitable for the actual plant gradually get falsified and thus eliminated from the candidate controller set. The spectral richness of reference signal (somewhat analogous to persistent excitation in system identification) and the threshold γ will determine how fast the bad controllers are eliminated from the unfalsified candidate controller set. A falsification approach based on a monotone increasing unfalsified performance level $\max_{\tau \leq t} \widetilde{J}_i(\tau)$ ensures that with accumulating plant data, the unfalsification process converges in finite time to a stabilizing controller satisfying (4.10) provided that the candidate controller set contains at least one stabilizing controller; convergence is assured without the usual standard assumptions [NA89] of MRAC on P, e.g., known sign of high frequency gain, known order, etc.

Fading Memory: For time varying plants, candidate controller not suitable for current plant may work in future. In the remainder of this proposal, we use in place of (4.10), the *fading memory unfalsification criterion*, given by

$$\widetilde{J}_i(t) \leq \gamma. \tag{4.11}$$

This time-varying criterion allows old data to be forgotten so that a controller falsified long ago can be reconsidered (i.e., become unfalsified again). However, in this case the candidate controller set no longer shrinks with time, thereby sacrificing some of the advantages of unfalsification. For example, convergence to a stabilizing controller may no longer be guaranteed without assumptions on P.

Switching Strategy: At every instant, a candidate controller among the candidate controller set with minimum unfalsified performance level is switched in the feedback loop. Given a set of candidate controllers K_i, $i \in I = 1, \ldots, N$, the problem is to identify and switch to the optimal controller $K^\star(t)$, that would minimize the unfalsified performance level γ at each instant:

$$K^\star(t) = \arg \min_{K_i, i \in \{1, \ldots, N\}} \max_{\tau \leq t} \widetilde{J}_i(\tau) \tag{4.12}$$

Some other robust switching strategies, like hysteresis and dwell time switching, can be found in [Hes01, MMG92] and references therein.

4.4   Controller structure in MRAC

We use the same structure of the controller in MRAC problem as in [NB94]. However, we would not use the plant parameterization given there as unlike [NB94], our scheme is based on multi models of controller and not of the plant. Hence, we just show that the controller structure used has enough degree of freedom to satisfy the requirements of MRAC problem [NA89]. Construct $w_1, w_2 : \Re \to \Re^{n-1}$ by

$$\dot{w}_1 = \wedge w_1 + l u_p, \dot{w}_2 = \wedge w_2 + l y_p, \tag{4.13}$$

where $(\wedge, l)$ is an asymptotically stable system in controllable canonical form, with

$$\lambda(s) \triangleq \det(sI - \wedge) = \lambda_1(s)Z_m(s) \tag{4.14}$$

for some monic Hurwitz polynomial $\lambda_i(s)$ of degree $n - m - 1$.

The control input is given by

$$u_p(t) = \theta^T(t)\underline{w}(t), \tag{4.15}$$

where

$$\underline{w} \triangleq (r, w_1^T, y_p, w_2^T)^T \in \Re^{2n}, \tag{4.16}$$

and

$$\theta \triangleq (K_c, \theta_1^T, \theta_0, \theta_2^T)^T \in \Re^{2n}, \; K_c, \theta_0 \in \Re, \; \theta_1, \theta_2 \in \Re^{n-1} \tag{4.17}$$

are the controller parameters. Define

$$\frac{C(s)}{\lambda(s)} = \theta_1^T(sI - \wedge)^{-1}l$$

and

$$\frac{D(s)}{\lambda(s)} = \theta_0 + \theta_2^T(sI - \wedge)^{-1}l. \tag{4.18}$$

Then overall transfer function of the feedback system can be obtained as

$$W_o(s) = \frac{K_c K_p Z_p(s)\lambda_1(s)Z_m(s)}{R_p(s) \; (\lambda(s) - C(s)) - K_p Z_p(s)D(s)} \tag{4.19}$$
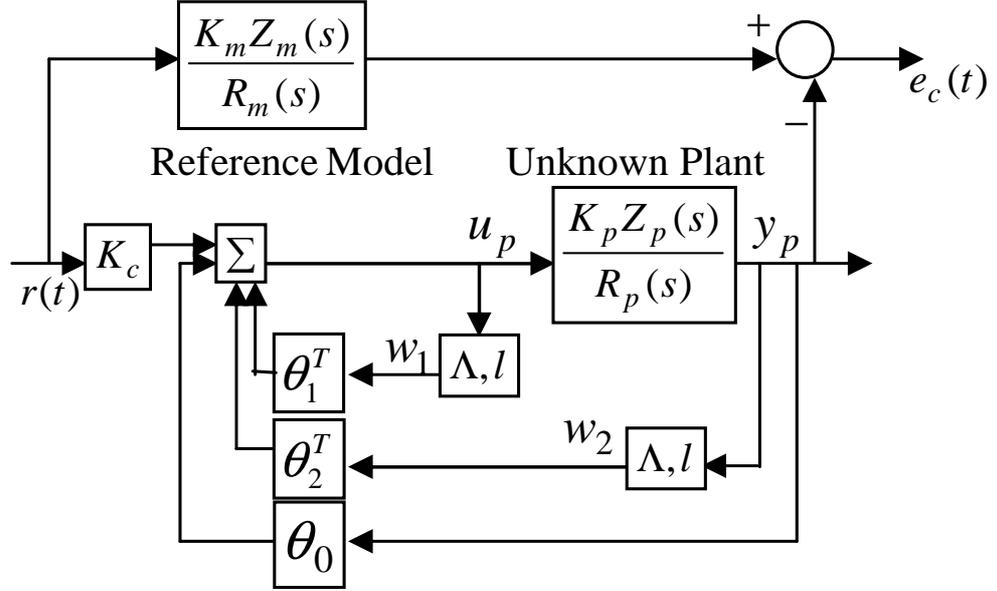
Figure 4.2: Controller realization in MRAC problem

Since $Z_p(s)$ and $R_p(s)$ are coprime, using Bezout Identity, unique polynomials $C^\star(s)$ and $D^\star(s)$ can be found such that the denominator of (4.19) equals to $Z_p(s)\lambda_1(s)R_m(s)$. If $K_c^\star = \frac{K_m}{K_p}$, then the over all transfer function of (4.19) becomes equal to the transfer function of the given reference model. Hence,

$$R_p(s)\left(\lambda(s) - C^\star(s)\right) - K_p Z_p(s) D^\star(s) = Z_p(s)\lambda_1(s)R_m(s)$$

and

$$K_c^\star = \frac{K_m}{K_p}. \tag{4.20}$$

So, the problem is to identify this $C^\star(s), D^\star(s)$ and $K_c^\star$ which are related to the optimal controller parameter $\theta^\star$ by (4.18) and (4.17), such that the closed loop plant

transfer function given by (4.19) equals the given reference model. The N candidate controller are given by $\theta_i, i = 1..N$, where

$$\theta_i = (k_{ci}, \theta_{1i}^T, \theta_{0i}, \theta_{2i}^T)^T \in \Re^{2n}. \tag{4.21}$$

It is assumed that there is at least one controller in the candidate controller set which is close to $\theta^\star$ (so that it can fulfill the MRAC objectives); the task of the switching system is to identify the controller nearest or equal to this $\theta^\star$. The control input $u(t)$, if i^{th} candidate controller θ_i is in the loop, is given by

$$u(t) = \theta_i^T \underline{w}(t). \tag{4.22}$$

4.5   Proposed switching scheme

From the control law given in (4.15), the fictitious reference signal for i^{th} candidate controller θ_i is given by

$$\widetilde{r}_i(t) = \frac{1}{K_{ci}}(u_p(t) - \theta_{1i}^T w_1 - \theta_{0i} y_p - \theta_{2i}^T w_2),$$

which can be written as

$$\widetilde{r}_i(t) = \underline{\theta}_i^T w, \tag{4.23}$$

where

$$\underline{\theta}_i^T \triangleq (\frac{1}{K_{ci}}, -\frac{\theta_{1i}^T}{K_{ci}}, -\frac{\theta_{0i}}{K_{ci}}, -\frac{\theta_{2i}^T}{K_{ci}}) \in \Re^{2n} \tag{4.24}$$

and

$$w \triangleq (u_p(t), w_1(t), y_p(t), w_2(t)). \tag{4.25}$$

It is seen that all candidate controllers share the same regression vector w for generating the fictitious reference signal. The fictitious output corresponding to i^{th} candidate controller is given by

$$\widetilde{y}_i(t) = W_m \widetilde{r}_i(t). \tag{4.26}$$

The error signal corresponding to the ith candidate controller would be

$$\widetilde{e}_i(t) = y_p(t) - \widetilde{y}_i(t). \tag{4.27}$$

As per justification given in section 4.3, this error is an indication of performance of ith controller.

Implementation Issues: The above control law can be implemented in both continuous and discrete time. However, as we monitor all the above signals in continuous fashion, we choose to implement it in a continuous fashion. The performance index of (4.9) needs some modification. The state space of (4.9) can be achieved easily as

$$\dot{x}_i(t) = \lambda x_i(t) + \beta \widetilde{e}_i^2(t)$$

$$\widetilde{J}_i(t) = \dot{x}_i(t) + \alpha \widetilde{e}_i^2(t) \tag{4.28}$$

Algorithm 1 *(Proposed switching strategy)*

INITIAL SETTING: A finite set of N number of candidate controllers, given by $\theta_i, i \in$

$\mathcal{I} = 1, \ldots, N$ *where* θ *is as in (4.21) Initial performance index* $\widetilde{J}_i(0) = 0$, *initial state of filter in (4.28) be* $x_i(t) = 0; i = 1, \ldots, N$

Let the resulting switching index of candidate controllers in the loop be given by $\widetilde{i}^\star(t)$. *Set* $\widetilde{i}^\star(0) = N$, *that is, place the* N^{th} *controller in the loop at* $t = 0$.

PROCEDURE: (1) For each $i \in 1, \ldots, N$: *Continuously update the following signals: fictitious reference signal* $\widetilde{r}_i(t)$, *fictitious output* $\widetilde{y}_i(t)$, *fictitious error signal* $\widetilde{e}_i(t)$ *and* i^{th} *performance index* $\widetilde{J}_i(t)$ *using (4.23), (4.26), (4.26) and (4.28).*

(2) Continuously identify the current optimal controller index by $\widetilde{i}^\star(t) = \arg\min_{i \in \mathcal{I}} \widetilde{J}_i(t)$ *and switch the controller* $\widetilde{\theta}^\star(t) = \theta_{\widetilde{i}^\star}$ *in the loop.*

Remark:

(a) $\widetilde{\theta}^\star(t)$ is the resulting switching sequence of Algorithm 1.

(b) We could have falsified any unsuitable candidate controllers without fading memory using (4.10) and incorporated it after step (1) of the Algorithm as follows:

1a *If* i^{th} *controller does not satisfy (4.10), then it is falsified by evolving plant I/O data* (u_p, y_p). *Delete the controller index element i. from candidate controller set* $\mathcal{I}$. *If the set is empty, then all the controllers have been falsified, hence terminate the algorithm; else continue.*

However, later in the chapter, we compared our algorithm with a switched system developed by Narendra who hasn't used the falsification concept; hence to make the comparison, we didn't incorporate the falsification concept in the above algorithm.

4.6 Narendra's plant model based switched system

The fictitious output of the ith candidate controller in (4.26) can be written as

$$\widetilde{y}_i(t) = W_m \frac{1}{K_c} \left(u_p(t) - \theta_{1i}^T w_1 - \theta_{0i} y_p - \theta_{2i}^T w_2 \right) = \underline{\theta}_i^T \overline{w}, \qquad (4.29)$$

where

$$\overline{w} \triangleq W_m(s) I_{2n} w$$

and

$$\underline{\theta}_i^T \triangleq \frac{1}{K_{ci}} (1, -\theta_{1i}^T, -\theta_{0i}, -\theta_{2i}^T) \in \Re^{2n}. \qquad (4.30)$$

When $\underline{\theta}_i = \underline{\theta}^\star$, the actual plant output can be represented by

$$y_p = \underline{\theta}^{\star T} \overline{w}. \qquad (4.31)$$

Hence, the actual plant can be parameterized by this $\underline{\theta}^\star$, with $\overline{w}$ forming the input and $\theta^\star$ forming the plant parameters. In [NS97, NB94], Narendra had used a group of N candidate plant models $P_i, i \in \{1, ... N\}$, each with corresponding controller C_i. The candidate controllers were designed so as to meet control objective of the corresponding candidate plant models. The candidate plant models were either fixed or tuned using some suitable tuning algorithm. Here, we consider the case where Narendra had considered fixed plant models (No tuning of candidate plant models involved). The candidate plant model, which best represents the actual plant, was identified at each instant and the corresponding controller was switched in the loop. To identify the correct plant model, N identifiers, with output $\widehat{y}_i$, were proposed. Equations similar to

(4.29) was used in [NA89, NS97, NB94] to parameterize the plant and construct the N identifiers, with elements of $\underline{\theta}_i, i, i \in 1, ... N$ forming the set of N candidate plant model parameters and the signal vector $\overline{w}$ forming the input to the identifier; if $\underline{\theta}(t) = \underline{\theta}^\star$, the output of the identifier would then be same as the plant output. The ith identifier output was given by

$$\widehat{y}_i = \underline{\theta}_i^T \overline{w}. \tag{4.32}$$

Each candidate plant model with parameters $\underline{\theta}_i$ had θ_i as the corresponding controller parameters, so that the ith candidate plant model, with corresponding controller in the loop, produces zero control error. Each identifier had a corresponding identification error

$$\widehat{e}_i = y_i - y_p. \tag{4.33}$$

This error was an indication of how close the ith plant model was to the actual plant and the switching was based on performance index $\widehat{J}_i(t)$, given by

$$\widehat{J}_i(t) \triangleq \alpha \, \widehat{e}_i{}^2(t) + \beta \int_0^T \exp^{-\lambda(t-\tau)} \widehat{e}_i{}^2(\tau) \, d\tau. \tag{4.34}$$

The problem is then to identify from the candidate plant set, at each instant, the optimal plant model $P^\star(t)$ that is closest to the actual plant and switch the corresponding candidate controller set in the feedback loop.

Algorithm 2 *(Narendra's method [NB94, NS97])*

INITIAL SETTING:

- *Given N fixed candidate plant model (No tuning involved), parameterized by $\underline{\theta}_i, i \in 1, ...N$, as in (4.30). Also given N identifiers corresponding to each plant model, each having structure as in (4.32).*

- *Given N candidate controller models, parameterized by $\theta_i, i = 1, \ldots, N$, as in (4.21) such that each θ_i is able to meet the performance specification for the corresponding plant model $\underline{\theta}_i$, so that when θ_i is placed in feedback loop with plant $\underline{\theta}_i$, it results in zero control error.*

- *Initial performance index $\widehat{J}_i(0) = 0, i = 1, \ldots, N$, that is, initial state of state space representation of (4.34) is 0.*

- *The resulting index of candidate controllers in the loop be given by $\widehat{i}^\star(t)$. Set $\widehat{i}^\star(0) = N$, that is, place the Nth candidate controller in the loop at $t = 0$.*

PROCEDURE:

(1) For each $i \in \mathcal{I} = 1, \ldots, N$: Continuously update the following signals: output of ith identifier $\widehat{y}_i(t)$, identification error $\widehat{e}_i(t)$ and finally i^{th} performance index $\widehat{J}_i(t)$ using (4.32),(4.33) and (4.34).

(2) Continuously compute the index of the plant model that is closest to the actual plant by $\widehat{i}^\star(t) = \arg\min_{i \in \mathcal{I}} \widehat{J}_i(t)$. Switch the corresponding candidate controller, $\widehat{\theta}^\star(t) = \theta_{\widehat{i}^\star}$ in the loop.

Remark: Let $\widehat{\theta}^\star(t)$ is the resulting switching sequence of Algorithm 2.

4.7 Comparison

Assumptions: (a) Let the candidate controller set in Algorithm 1 be selected such that they meet the control objectives for the corresponding candidate plant models selected in algorithm 2, that is, let the candidate controller set and their index in the above two algorithm be the same. (b) Let the initial candidate controller (at $t = 0$), the reference signal applied and the initial conditions of the plant in the above two algorithms be same. The above assumptions are essential to compare the performance of the two algorithms.

Theorem 3

(a) The fictitious error $\widetilde{e}_i(t)$ of ith candidate controller in Algorithm 1 and the identification error $\widehat{e}_i(t)$ of the ith candidate plant/controller pair in Algorithm 2 are equal, i.e., $\widehat{e}_i(t) = \widetilde{e}_i(t)$ for all t and $i = 1, \ldots, N$.

(b) The switching sequences of the two algorithms are equal, i.e., $\widetilde{\theta}^\star(t) = \widehat{\theta}^\star(t)$ for all t.

Proof: The ith fictitious output in Algorithm 1 is $\widetilde{y}_i(t)$ and the output of ith identifier in algorithm 2 is $\widehat{y}_i$. As per justification given in the beginning of section 4.6, we have $\widehat{y}_i(t) = \widetilde{y}_i$. Hence, the fictitious error of Algorithm 1 and the identification error of Algorithm 2 for corresponding models are equal, *i.e.,* $\widetilde{e}_i(t) = \widehat{e}_i(t)$. Hence, the performance index of the ith candidate controller in algorithm 1 would be identical to the index of ith candidate plant/controller pair in Algorithm 2. Since switching is based on this performance index, the switching sequences for the two algorithms are equal. □

So, in this proposal, we arrive to the same results as Narendra's, but using a completely different approach. Narendra has used his method (Algorithm 2) to identify

the candidate plant model, which is closest to the actual plant, and then switch to the corresponding controller. And in this proposal, we directly identify and switch to the candidate controller based on the fictitious error concept. So, we can say that in [NB94,NS97], in the process of selecting the best plant model representing the actual plant, Narendra has also selected the controller, which would have given best performance, if placed in the loop with the unknown plant. This similarity is due to the special parameterization of the plant used by Narendra to construct the identifiers.

Narendra proved the validity of the special parameterization of the plant and structure of the identifier [NB94]; here, using a different approach, we give a different motivation behind using such parameterization and structure.

However, parameterizing the plant in some other way and constructing some other observers may not result in this similarity between two conceptually different approaches. The beauty of our method is it is a plant model free approach; we do not try to identify the plant. The switching is based only and directly on the performance of the candidate controllers. Hence, it can be applied to a broader class of problems, where estimating a complex plant might prove difficult; though prior plant knowledge is useful to build the initial set of candidate controllers.

4.8 Simulations

The actual plant to be controlled has a transfer function of the form

$$W_p(s) = \frac{k_p}{s^2 + as + b}.$$

The sign of K_p is unknown. The parameters K_p, a, b are known to lie in the compact set given by

$$\mathcal{S} = \{0.1 \leq |K_p| \leq 1, \ -.5 \leq a \leq 2, \ -1.5 \leq b \leq 1.5\}.$$

Here, for simplicity, we impose these assumptions on the plant. When less prior information about the plant is available, the candidate controller set has be made larger to accommodate all possible plant structures; but this set can be reduced monotonically with the unfalsification concept in (4.10).

The reference model is given by (same as in [NB94])

$$W_m(s) = \frac{1}{s^2 + 1.4s + 1}.$$

The reference input $r(t)$ is given by a square wave, with unit amplitude and period 10 units. The candidate controllers are chosen for plants models, evenly distributed in the set $\mathcal{S}$, given by

$$K_p \in \{-1, -0.7, -0.4, -0.1, 0.1, 0.4, 0.7, 1\},$$

$$a \in \{-0.5, 0, 0.5, 1, 1.5, 2\}$$

and

$$b \in \{-1.5, -1, -0.5, 0, 0.5, 1, 1.5\}.$$

The parameter of the performance index in (4.11) is taken as

$$\lambda = 0.05, \alpha = \beta = 1.$$

The simulations were carried out with a 0.1 mean, 2 variance disturbances in the input of plant, applied after 40 seconds of starting of the simulation. So,

$$d(t) = N(0.1, 2)u(t - 40) \qquad (4.35)$$

Figure 4.3: Case 1: Plant response $y_p(t)$ (solid line), reference output $y_m(t)$ (dotted line), reference input $r(t)$ (dotted line).

Case 1: An unstable plant, given by

$$W_p(s) = \frac{0.4}{s^2 + s - 1}.$$

The plant parameters match exactly with one of the nominal plant model for which there exists a matching candidate controller. The correct controller, as expected, is determined almost immediately and the switching stops thereafter.
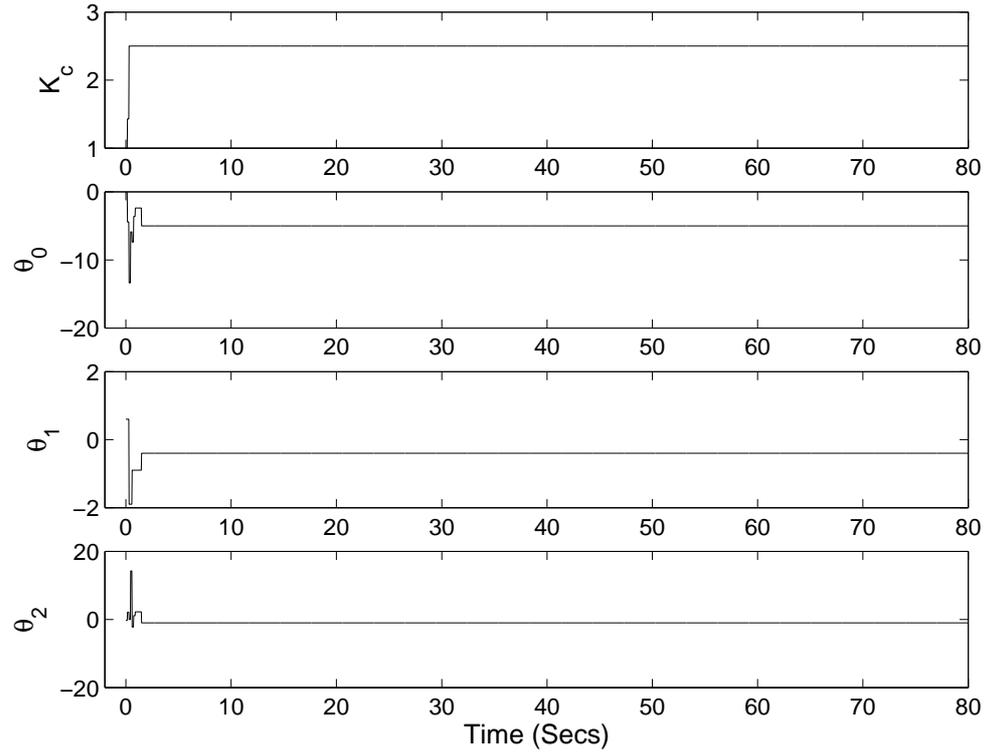
Figure 4.4: Case 1: Switching of the controller parameters.

Case 2: The plant is given by

$$W_p(s) = -\frac{0.5}{s^2 + 1.1s + 1.2}.$$

The candidate controller set does not have a perfect controller for this plant; however, it selects the best available controller among the set for the unknown plant to minimize the control error. The system reacts to the disturbance (4.35) in this case by switching between two candidate controllers.
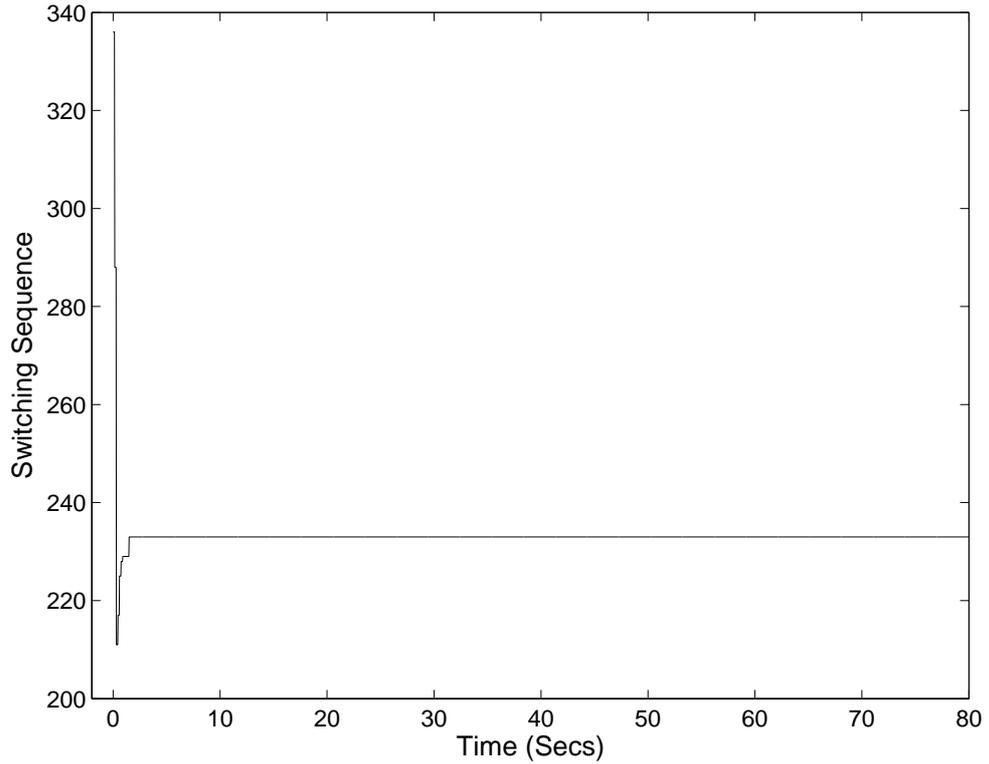
Figure 4.5: Case 1: Switching sequence among the candidate controllers.

4.9 Discussion

In this chapter, we have described a way to apply the direct controller model based approach to build a switched system for the MRAC problem. This is a plant model free approach; we do not try to estimate the unknown plant while deciding the optimal switching sequence, though prior knowledge about the plant is always useful to construct the candidate controller set. Like all other fixed model based switching system, the success of this approach depends on the selection of candidate controller set. Making this set large assures better result (less control error), but increases the computational burden. However, the unfalsification criterion in (4.10) ensures a monotonically shrinking set of candidate controllers. We have also pointed out the duality
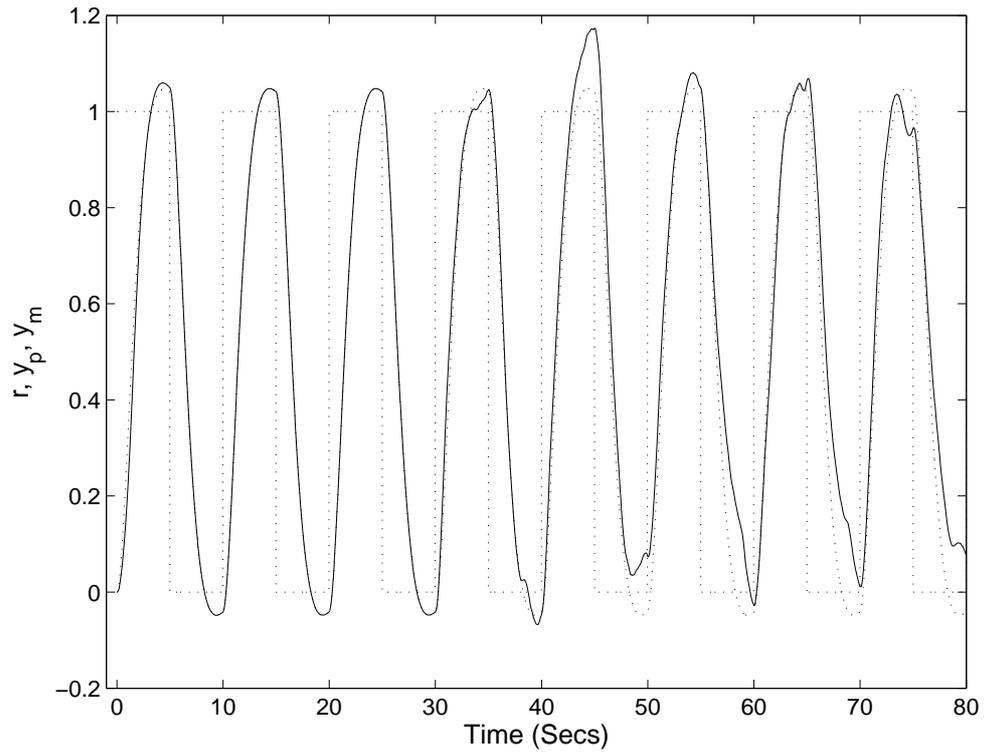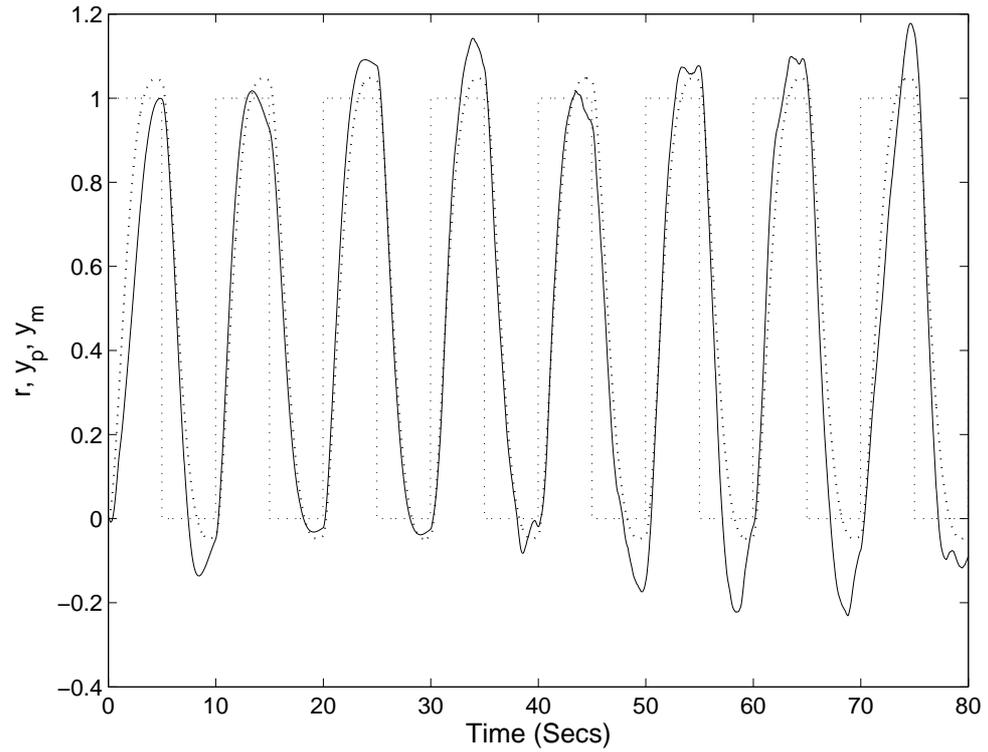
Figure 4.6: Case 2: Plant response $y_p(t)$ (solid line), reference output $y_m(t)$ (dotted line), reference input $r(t)$ (dotted line).

between our switching strategy and that of Narendra's. These two conceptually differ-ent approaches produce same result when the unfalsification criterion (4.10) is relaxed to the fading memory criterion (11); the similarity owes to the special parameterization used by Narendra to construct the identifiers.
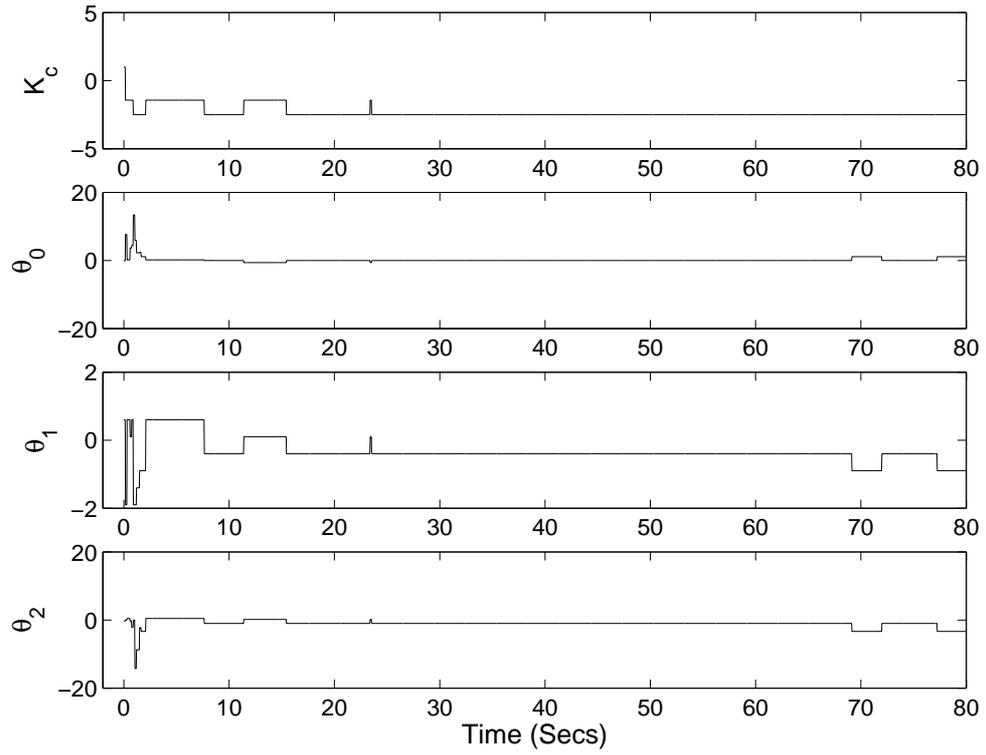
Figure 4.7: Case 2: Switching of the controller parameters.
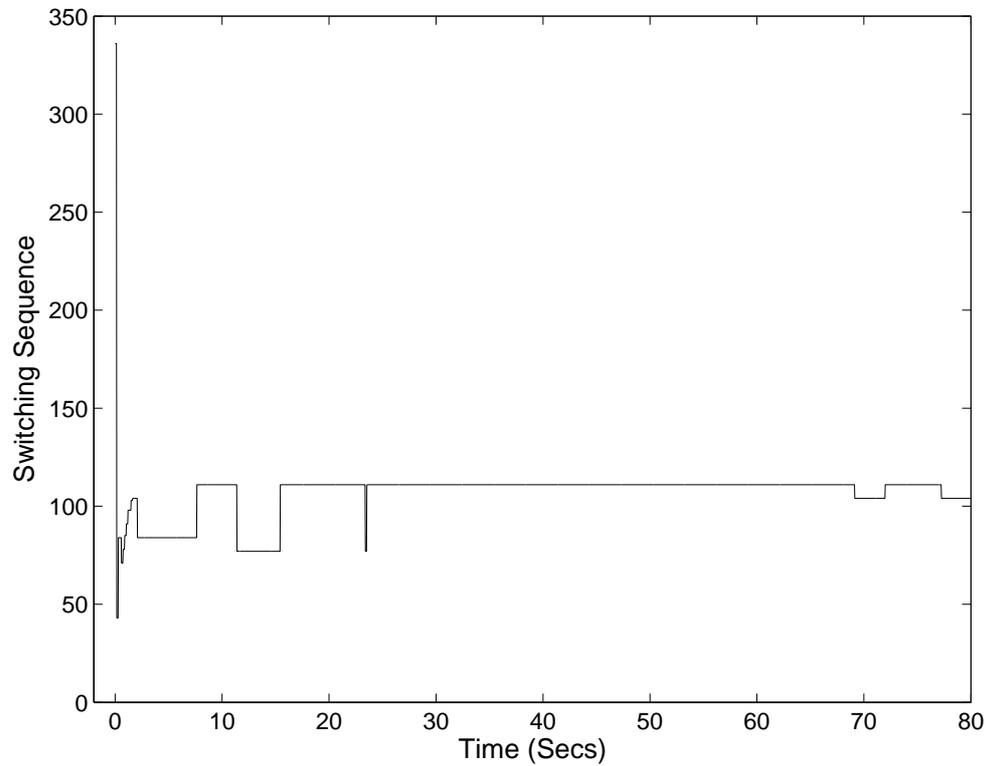
Figure 4.8: Case 2: Switching sequence among the candidate controllers.
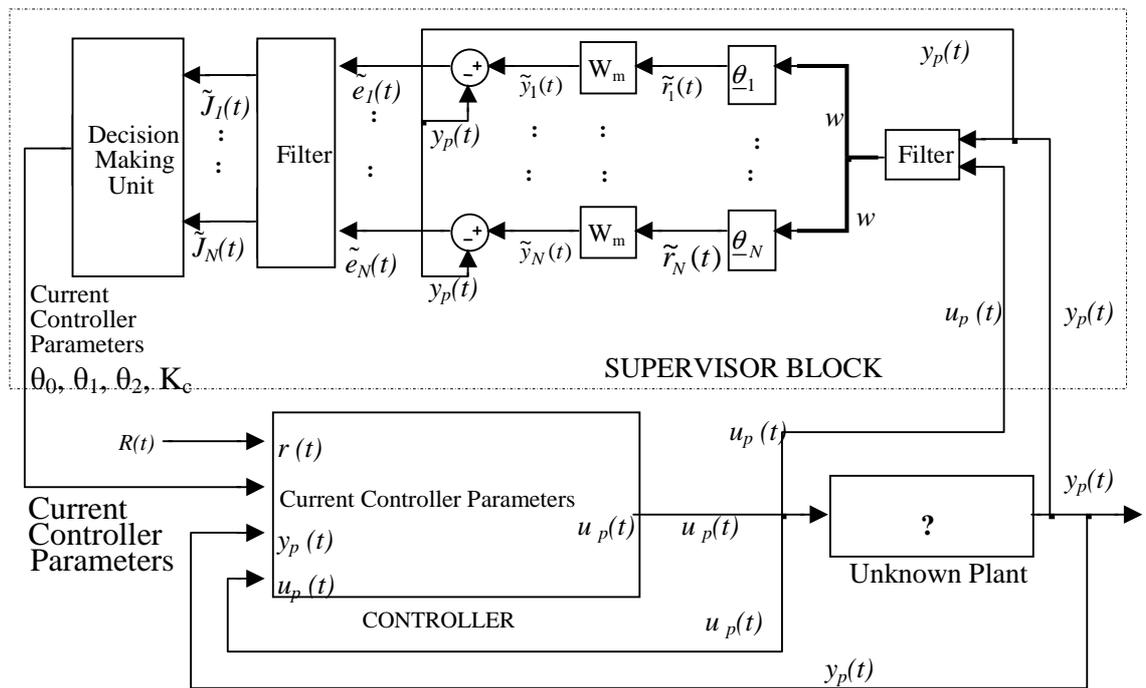
Figure 4.9: Architecture for control using N candidate controllers for MRAC problem

Chapter 5

Power Control for Wireless Networks Using Multiple Controllers and Switching

In previous chapters, a comparison between Multi–Model Adaptive Control (MMAC) and Multi–Controller Adaptive Control (MCAC) was made. The MMAC [Bal96, NB94, Nar97, Mor96, KR96, Aka04] relies on system identification, where the actual plant behavior is constantly compared with that of several candidate plant models and the model that best approximates the actual plant is identified at each time. Then, based on certainty equivalence principle, a controller suitable for the identified candidate plant model is switched in the loop. On the other hand, in MCAC, [ST97, JS99, PS03, JZM01] the potential performance of every candidate controller is evaluated directly from the measured data using some suitably defined performance index, without trying to estimate or identify the actual process. Depending on this index, a suitable controller is selected and switched in the feedback loop to control the system.

There are several practical scenarios where it is not easily possible to construct a model for the unknown plant; it might be due to the complex, non–linear structure of the plant. In these situations, it is difficult to implement the MMAC based switchig schemes. However, if the controller to be used has a simple structure, the more direct

MCAC switching scheme might be easy to implement. One such scenario arises while controlling transmitted powers of several mobile users in a wireless cellular environment. Due to the complex interconnections between powers of various mobile users through a non–linear interference term, it is difficult to model the unknown process. We apply the direct MCAC based switching scheme for this problem and adaptively select a PID controller from a set of candidate controllers.

Controlling transmitted power in a wireless network is critical for maintaining quality of service, maximizing channel utilization and minimizing near-far effect for suboptimal receivers. In this Chapter, a general PID (Proportional–Integral–Derivative) type algorithm for controlling transmitted powers in wireless networks is studied and a systematic way to adapt or tune the parameters of the controller in a distributed fashion, using MCAC based switching schemes, is suggested. The proposed algorithm utilizes multiple candidate PID gains. Depending on the prevailing channel conditions, it selects an optimal PID gain from the candidate gain set at each instant and places it in the feedback loop. Simulation results indicate that the proposed scheme performs better than several candidate controllers, including a well known distributed power control algorithm. The results of this chapter have been published in [PASM04b, PASM05].

5.1 Introduction

Power control in wireless networks is important to maintain reliable communication links between base stations and mobile users, and to maximize the battery life. This objective can be met by using a centralized algorithm [Aei73, Zan92, GRVZ93], which also minimizes the total transmitted power and interference. However, centralized

algorithms are not practical as they require complete information on the link gains. This difficulty can be avoided by using the distributed power control algorithm proposed by Foschini and Miljanic [FM93]. Their algorithm, widely known as Distributed Power Control (DPC) in the power control literature, converges to the optimal solution of the centralized case provided that the channel gains are fixed and the system is feasible.

Adjusting transmitted powers has also been addressed using control–theoretic methods including PID (Proportional+Integral+Derivative) controllers [GGB99, Gun00] and game theoretic methods [ABD04, JH98], in order to improve the convergence rate. In [GGB99], Gunnarsson *et al* provide specific parameter values for their PI controller in log–linear scale, albeit it leads to a suboptimal system performance in general because the design ignores the cross coupling between powers of various users through the interference term. Later, they extended their work by computing a range of stabilizing PID controller parameters [Gun00]; however it uses linearization of the non–linear interference term and the small gain theorem, which is known to produce conservative results.

Although the algorithms in [GGB99, Gun00] with appropriately chosen parameters lead to better convergence rates than DPC, there is not a systematic procedure that would update the controller parameters based on the dynamic wireless environment which could improve performance. Furthermore, a single set of controller parameters might not be suitable for all users in the network, *i.e.*, the optimal controller for an individual user might be a function of its location within the cell, the current channel conditions, the total number of active users in the cell, and the interference it is experiencing. For instance, a mobile user close to the base station may find it more suitable

to use a low gain controller whereas other users far from the base station may have to use some other controller to achieve faster convergence.

In this chapter, we propose an adaptive distributed scheme which can tune the controller parameters of individual users in order to improve the overall system performance. The proposed system utilizes a set of candidate PID controllers. The idea of using multiple controllers has become popular over the last decade [NB94, Nar97, Mor96, ST97, JS99, AM04, PS03, RK01, CZM01, TS01]. The basic problem here is to control a complex, unknown, possibly time–varying system for which there is no way to construct a traditional controller that would give satisfactory results; the limitation in constructing such a controller might be due to lack of sufficient knowledge about the system or the complexity of the system. Hence, instead of using a single controller to control the system, a set of candidate controllers is used. Based on measurement data collected from the system, a supervisory unit adaptively switches among these candidate controllers and tries to place the best available controller in the feedback loop.

Most switching based methods belong to two broad categories: Indirect methods based on system identification [NB94, Nar97, Mor96] and methods that directly identify the controller [ST97, JS99, AM04, PS03, RK01, CZM01, TS01]. The indirect estimator based switching [NB94, Nar97, Mor96] relies on system identification. Here, the actual plant behavior is constantly compared with that of several candidate plant models and the model that best approximates the actual plant is identified at each time; then, based on certainty equivalence principal, an appropriate controller is switched in the loop. The basic assumption here is that there is at least one plant model in the candidate model set which is *sufficiently* close to the actual process to be controlled. However,

as it would be evident later in the chapter, the unknown plant here consists of the unknown link gains between the transmitters and the receivers; without a centralized algorithm, it would be difficult to continuously estimate these gains and hence the indirect switching scheme cannot be applied for decentralized power control applications.

An alternative to the above method is the more direct controller performance index based switching, popularly referred to as the unfalsification approach in some recent publications [ST97, JS99, AM04, PS03]; it has also been successfully applied to some practical problems [RK01, CZM01, TS01]. Here, the potential performance of every candidate controller is evaluated at each time directly from the measured data using some suitably defined data driven performance index, without trying to estimate or identify the actual process. The performance index is chosen to be a measure of how closely the output of the closed loop system would have followed some reference input (namely, the *fictitious reference input*), had the candidate controller been in the feedback loop. In some recent papers, stability and convergence of the direct unfalsified switching control were addressed [SWS04, WSS04]. An algorithm was proposed in [SWS04], which guarantees stability of switching, under minimum assumptions that the performance index chosen is *cost detectable* and that there is at least one stabilizing controller in the candidate controller set. The latter condition is called the *feasibility* of the control problem. Without feasibility, there exists no adaptive law that would converge to a stabilizing solution. The underlying theory is quite general and can be applied to many power control algorithms that satisfy some mild conditions on invertibility of the controller, stated in Section 5.3. In this chapter, we use the direct method to choose the most suitable PID gains for each user from a candidate set.

The rest of the chapter is organized as follows. In Section 5.2, we present the power control problem and a simple PID–type distributed power control algorithm. In Section 5.3, we describe the proposed multiple controller based switching scheme. In Section 5.4, we apply this technique to tune the parameters of the PID power control algorithm discussed in Section 5.2. Finally, we present the simulation results and the conclusions in Sections 5.5 and 5.6, respectively.

5.2 Distributed power control

Consider a wireless system, with N mobile users sharing the same channel. To simplify the analysis, we assume a mobile is connected to a single base station at any given time. We assume that the ith, $i = 1, \ldots, N$, mobile user is connected to the ith base station. If two mobile users, say i and j are assigned to the same base station, then the base station indices i and j refer to the same physical base station. All values are in linear scale unless otherwise mentioned. We study only the uplink, although the results are also applicable to the downlink.
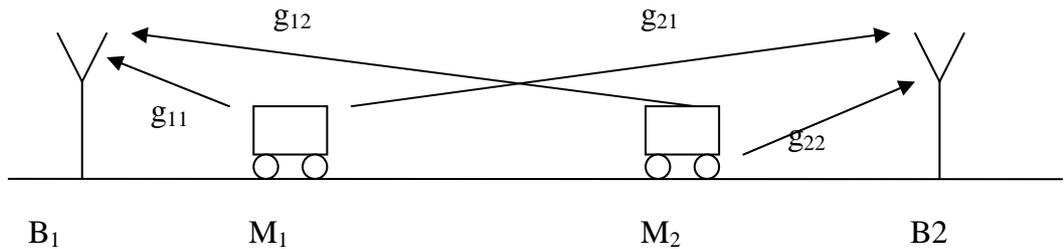

Figure 5.1: An example situation with 2 mobiles and 2 base stations. M_i, $i = 1, 2$ represents the ith mobile user; B_i, $i = 1, 2$ represents the ith base station.

Let g_{ij} represent the channel gain between the jth mobile transmitter and the ith base station receiver and the transmitted power vector be given by

$$P(t) = [p_1(t), \ p_2(t), \ldots, p_N(t)]^T,$$

with

$$p_i(t) \geq 0$$

denoting the power from the ith transmitter. The achieved Signal to Interference plus Noise Ratio (SINR) for the ith user can be expressed as

$$\gamma_i(t) = \frac{g_{ii}p_i(t)}{\sum_{j=1, \ j \neq i}^{N} g_{ij}p_j(t) + \nu_i} \ , \ i = 1, \ldots, N, \tag{5.1}$$

where ν_i is the thermal noise at the ith receiver. The objective of the power control algorithm is to update the power levels $p_i(t)$ such that the achieved SINR satisfies

$$\gamma_i(t) \geq \Gamma, \ \ i = 1, \ldots, N, \tag{5.2}$$

where Γ is the target SINR that is determined from the Quality of Service constraints. This goal can be achieved using several distributed power control algorithms [FM93, GGB99, Gun00]. In this chapter, we employ a general linear PID–type controller structure and propose a method to adaptively tune the PID gains in real time. However, the theory behind the proposed tuning scheme is quite general in nature and can be easily applied to the afore–mentioned power control algorithms of [FM93, UJK00, GGB99, Gun00].

Our starting point is the DPC algorithm, which is an Integral (I)–type algorithm proposed in [FM93]; we extend it by adding the proportional and derivative terms as well. Since practical power control algorithms are usually implemented in discrete–time, we directly present the discrete–time version of the proposed PID algorithm. The power update equation for the ith user is given by:

$$p_i(k) = \alpha e_i(k) + \beta x_i(k) + \theta[e_i(k) - e_i(k-1)], \qquad (5.3)$$

where

$$e_i(k) = \left[1 - \frac{\Gamma}{\gamma_i(k-1)}\right] p_i(k-1), \qquad (5.4)$$

and

$$x_i(k) = x_i(k-1) + e_i(k), \qquad (5.5)$$

with α, β, θ forming the controller parameters. This is a typical PID–controller, with the proportional, integral and derivative gains represented by α, β and θ, respectively.

In this chapter, we apply the proposed multiple–controller based approach to tune the PID–parameters α, β and θ. The general idea is as follows: There are multiple candidate controllers, having different sets of gains for the controller parameters α, β and θ. The proposed scheme monitors the data and associates a performance index with each candidate controller. Based on this index, the best available controller is selected from the candidate set and is used in the feedback loop.
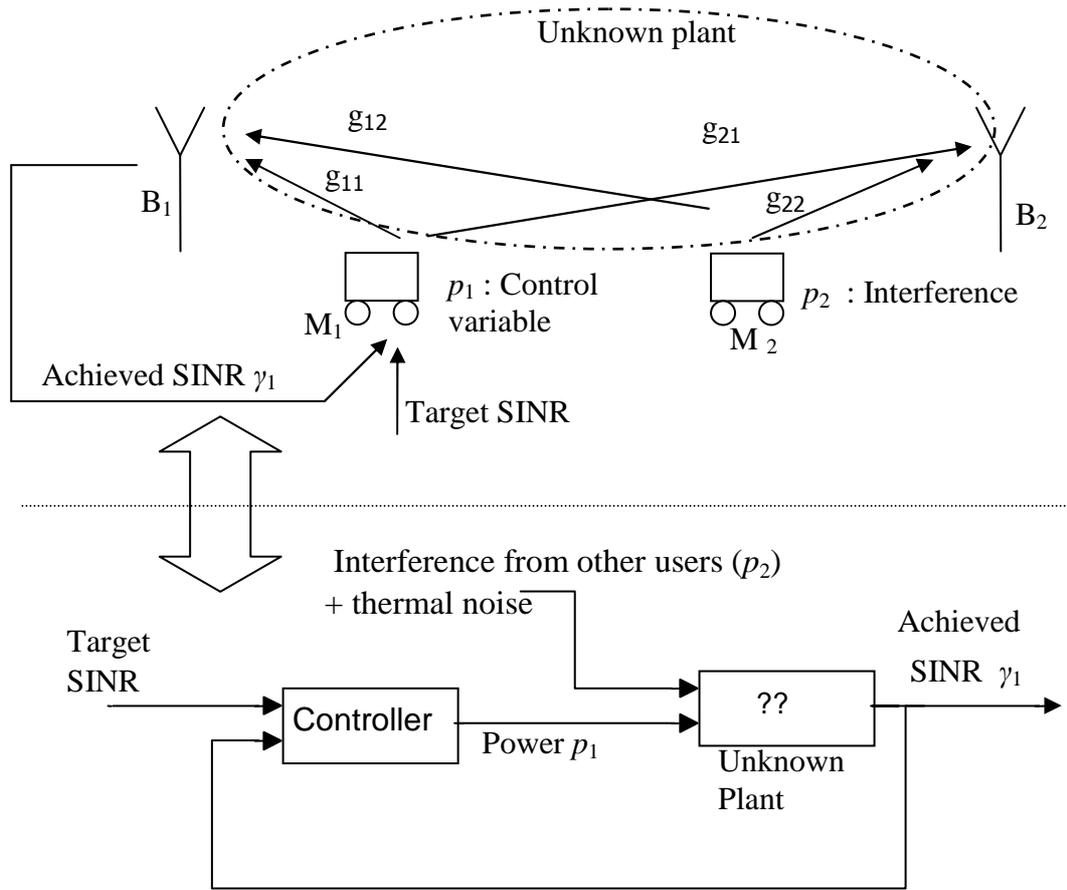
Figure 5.2: The closed loop plant for a 2 mobile, 2 base station scenario.

5.3 Multiple Controller System

In this section, we present a general overview of a direct multi–controller based switching system which we will use in the next section to tune the gains of the PID controller parameters. Consider an unknown plant $\mathcal{P}$, with $(p(t), \gamma(t))$ being the plant input/output (I/O) data that can be measured (see Fig. 1). The plant output $\gamma(t)$ is required to track a reference input, $\Gamma(t)$. The proposed scheme utilizes a set of M candidate controllers, denoted by

$$C_j, j \in \mathcal{M} \triangleq \{1, ..., M\},$$

where each candidate controller is characterized by the triplet $(\alpha_j, \beta_j, \theta_j)$. The objective is to select at each instant the best controller among the set of available controllers using some performance index and place it in the feedback loop. Construction of this set of candidate controllers is crucial for the success of the algorithm. The assumption is that the set $\mathcal{M}$ contains at least one controller which can stabilize the plant. For the power control problem, this assumption is easily satisfied by including the controller with the parameters,

$$\alpha = \theta = 0, \ \beta = 1,$$

in the candidate controller set, as this controller corresponds to the DPC algorithm in [FM93] and is known to produce a stable system under static channel conditions. The candidate controller set should be constructed such that there are several stabilizing controllers in the set, including some potentially good ones. When the ranges of stabilizing candidate controller parameters are partially or completely unknown, this set has to be made large by including several candidate controllers.

Let us define the control error

$$e_c(t) \triangleq \Gamma(t) - \gamma(t) \ . \tag{5.6}$$

Given a set of past plant I/O data $(p(t), \gamma(t))$, we now define the fictitious reference signal $\widetilde{\Gamma}(C_j, p, \gamma)$ for the jth candidate controller C_j, $j \in \mathcal{M}$.

Definition 10 *(Fictitious Reference Signal) [JS99] Given a set of past measured plant I/O data $(p(t), \gamma(t))$, and a candidate controller C_j, $j \in \mathcal{M}$, a fictitious reference signal for this candidate controller is a hypothetical reference signal that would have*
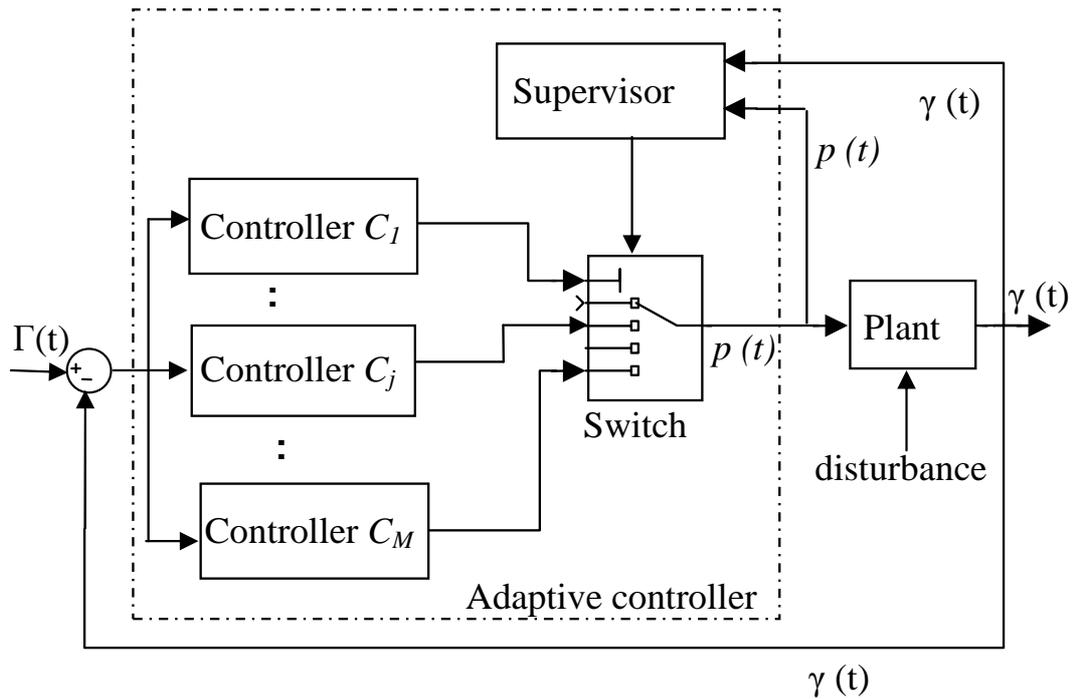
Figure 5.3: General plant–controller configuration for switching based control

produced exactly the measured data $(p(t), \gamma(t))$, had the candidate controller C_j been in the feedback loop with the unknown plant during the entire time period over which the measured data $(p(t), \gamma(t))$ were collected. ◇

If the jth candidate controller is actually in the loop during the entire period over which plant I/O data were collected, then the jth fictitious reference signal would be the actual reference signal; else it would in general be different from the actual reference signal (hence the name *fictitious*).

For example, for the system in Fig. 1, let C_j be the jth candidate controller. Note that the controller used can be in the form of a gain or a transfer function; in that case, we will have to use the inverse of the transfer function to obtain the fictitious reference

signal. Then, the fictitious reference signal for the jth candidate controller C_j is given as

$$\widetilde{\Gamma}(C_j, p, \gamma) = C_j^{-1} p + \gamma \;, \tag{5.7}$$

where C_j^{-1} is the inverse of the jth candidate controller transfer function.

Remark: To uniquely determine the fictitious reference signal, the controller has to be causally-left-invertible [JS99], that is, from the past and present output of the controller, one should be able to uniquely determine its present input and hence the inverse of the controller should also be a proper transfer function. Controllers with bi–proper transfer function, including the controller studied in this chapter have this property. ◇

Definition 11 *(Fictitious error signal) For each candidate controller, a fictitious error signal can be defined as the error between its fictitious reference signal and the actual plant output. Hence, this would have been the error signal, had that candidate controller been in the feedback loop with $(p(t), \gamma(t))$, as the measured plant data and $\widetilde{\Gamma}(C_j, p, \gamma)$ as the reference signal.* ◇

The fictitious error signal for the candidate controller C_j is represented as $\widetilde{e}(C_j, p, \gamma)$ and is given by

$$\widetilde{e}(C_j, p, \gamma) \triangleq \widetilde{\Gamma}(C_j, p, \gamma) - \gamma. \tag{5.8}$$

For convenience, in the sequel, we will use the following notations for the fictitious reference signal and fictitious error signal of the jth candidate controller:

$$\widetilde{\Gamma}_j(t) \triangleq \widetilde{\Gamma}(C_j, p, \gamma, t),$$

67

$$\widetilde{e}_j(t) \triangleq \widetilde{e}(C_j, p, \gamma, t).$$

From the definition of the fictitious reference signal given above, it is clear that the error $\widetilde{e}_j$ would have been the control error of (5.6) with (p, γ) as the plant I/O data, had the jth candidate controller C_j been in the feedback loop. So, $\widetilde{e}_j$ is a measure of the effectiveness of the jth candidate controller, if it were used to control the plant. Hence, switching among candidate controllers will be based on this error. We can construct some suitably defined performance indices, $\widetilde{J}(j, t), j \in \mathcal{M}$, which will be discussed explicitly in the next section. Given a set of candidate controllers $C_j, j \in \mathcal{M}$, the problem then is to identify and switch to the best available controller C^* that would minimize the performance index at each instant, that is:

$$C^*(t) = C_{j^*(t)}, \tag{5.9}$$

where

$$j^*(t) = \arg \min_{j \in \mathcal{M}} \widetilde{J}(j, t). \tag{5.10}$$

5.4 Switching Based Approach to Power Control Problem

In this section, we apply the switching based approach to the power control algorithm described earlier. We first construct a bank of PID controllers, $C_j, \ j \in \mathcal{M}$, each with different values of α, β and θ, i.e., the jth controller has α_j, β_j and θ_j as its parameters. Given this set, we now proceed to construct the fictitious reference signal, as described in the previous section.

As we consider the local loops, that is, the signals corresponding to individual users are dealt with separately, we henceforth discard the index i. Thus, in the sequel, $p(k)$ will indicate the power $p_i(k)$ of the ith user; similarly $\gamma(k)$ will denote the achieved SINR $\gamma_i(k)$ for the ith user. Using the delay operator q, the PID algorithm in (5.3) can be written as

$$p(k) = \frac{(\alpha + \beta)q^2 - \alpha q + \theta(q^2 - 2q + 1)}{q(q-1)}$$
$$\cdot \left[p(k-1) - \frac{\Gamma}{\gamma(k-1)} p(k-1) \right], \qquad (5.11)$$

or

$$\Gamma = \gamma(k-1)$$
$$- \frac{q^2 - q}{(\alpha + \beta + \theta)q^2 - (\alpha + 2\theta)q + \theta} \frac{p(k)\gamma(k-1)}{p(k-1)}.$$

Hence, the fictitious reference signal $\widetilde{\Gamma}_j(t)$ for the jth candidate controller C_j, with parameters as α_j, β_j and θ_j can be expressed as

$$\widetilde{\Gamma}_j(k) = \gamma(k-1)-$$

$$\frac{q^2 - q}{(\alpha_j + \beta_j + \theta_j)q^2 - (\alpha_j + 2\theta_j)q + \theta_j} \frac{p(k)\gamma(k-1)}{p(k-1)}. \qquad (5.12)$$

Note that here j is the index of the candidate controller and all signals correspond to the ith user. This would have been the reference SINR, had the jth controller C_j been in the loop, with $p(k)$ and $\gamma(k)$ being the actual transmitted power and the actual achieved SINR. The fictitious error signal $\widetilde{e}_j$ for the jth controller is then given by (5.8),

which is an indication of how closely the achieved SINR would have followed the target SINR, had C_j been used to control the plant.

5.4.1 Choice of Performance Index

An appropriate choice of the index is crucial for the performance of the switched system. The performance index proposed here is motivated by the loop shaping techniques, a well known tool used in robust control for H_∞ design [SP00, JS99]. Other choices of performance index can be found in [SWS04] and references therein.

The objective in any power control algorithm is that the achieved SINR satisfies the requirement in (5.2) using minimum transmitted power. The control error for the power control algorithm is, $e_c(t) = \Gamma - \gamma(t)$, and the reference input is the target SINR (Γ). Hence, the sensitivity function for the power control problem, which is defined as the transfer function from the reference signal to the control error of the system, is given by [SP00]

$$S = \frac{e_c}{\Gamma}.$$

The sensitivity function S assumes an important role in robust performance of any system and is widely used in H_∞ control system design [SP00]. It can be shown that it is also the transfer function from external disturbances to the output of the system [SP00]. Thus to minimize the effect of disturbance on the output of the system and minimize the error, it is desirable that the magnitude of the sensitivity function

be low at all frequencies. However, for a stable plant and controller, the sensitivity

function has to satisfy the Bode sensitivity integral equality [SP00, Thm. 5.1]

$$\int_0^\infty \ln(|S(j\omega)|)d\omega = 0 \tag{5.13}$$

and hence, if the sensitivity is low at certain frequencies, it has to be high at some

other frequency (known as the "waterbed effect" in robust control theory). In the low

frequency region (inside the system bandwidth), it is desirable that the magnitude of

the sensitivity be small, and is allowed to be large in the high frequency region to

compensate for small values in the low frequency region.

In robust control theory and $\mathcal{H}_\infty$ design, to ensure that the sensitivity function of a

system meets the above specification, a controller is designed such that the sensitivity

function lies below the frequency response of the reciprocal of some known filter W_1 at

all frequencies, that is [SP00],

$$|S(j\omega)| \le 1/|W_1(j\omega)|, \quad \forall \omega , \tag{5.14}$$

which can be written as

$$\|W_1(j\omega)S(j\omega)\|_\infty \le 1, \tag{5.15}$$

where $\|.\|_\infty$ denotes the $\mathcal{H}_\infty$ norm. W_1 is a filter to be specified by the designer – for

details, refer to [SP00, § 2.7]. The filter $1/W_1$ has low gain at low frequency and is

allowed to have high gain at high frequency.

Let the system be required to satisfy the integral performance specification

$$J(t) \triangleq \zeta\, T_{spec}(\Gamma(t), p(t), \gamma(t)) +$$

$$\delta\, \int_0^t \exp^{-\phi(t-\tau)} T_{spec}(\Gamma(\tau), p(\tau), \gamma(\tau)) d\tau \leq \mu, \qquad (5.16)$$

where the objective is to minimize the parameter μ by optimally choosing the controller; $T_{spec}(\cdot)$ is the performance specification to be chosen later and ζ, δ, ϕ are non-negative design constants. The performance index for the jth candidate controller can then be given by

$$\widetilde{J}(j, t) = \zeta\, T_{spec}(\widetilde{\Gamma}_j(t), p(t), \gamma(t)) +$$

$$\delta\, \int_0^t \exp^{-\phi(t-\tau)} T_{spec}(\widetilde{\Gamma}_j(\tau), p(\tau), \gamma(\tau)) d\tau. \qquad (5.17)$$

The parameters ζ and δ are weighing factors for the instantaneous and accumulated performance specifications, respectively, and ϕ is the forgetting factor that determines the weight of past data and ensures the boundedness of the integral term. Making ζ large results in very fast switching whereas large δ results in comparatively slow switching. The non-negative forgetting factor ϕ de–emphasizes the importance of old data set so that the performance index can adopt to the dynamic channel. These design parameters can be chosen based on the application domain. When the environment is dynamic and changes rapidly (i.e. when the plant parameters change rapidly), the forgetting factor can be made large to rapidly de–emphasis the past data and the weight

of instantaneous term, ζ, can be made higher. On the other hand, for a static or very slowly varying environment (i.e. when the plant does not change or very slowly changes with time), knowledge acquired from past data is still relevant and a small forgetting factor may be used, with a small ζ. For power control applications in wireless network, the environment is dynamic and the network conditions change rapidly; hence we can choose the parameters accordingly. A qualitative discussion on how to choose these constants and their effect on switching can be found in [NB94].

For convenience, we use the following notation in the sequel to represent performance specification of the jth candidate controller:

$$\widetilde{T}_{spec}(j,t) \triangleq T_{spec}(\widetilde{\Gamma}_j(t), p(t), \gamma(t)).$$

Let us define a signal $f(j,t)$ corresponding to the jth candidate controller as

$$f(j,t) = w_1(t) * (\widetilde{\Gamma}_j(t) - \gamma(t)), \tag{5.18}$$

where $*$ is the convolution operator and $w_1(t)$ is the time response of filter W_1 described before. The performance specification is taken as

$$\widetilde{T}_{spec}(j,t) = \frac{|f(j,t)|^2}{|\widetilde{\Gamma}_j(t)|^2} \ . \tag{5.19}$$

Note that
$$\max_{\Gamma(t)} \frac{||w_1(t) * (\Gamma(t) - \gamma(t))||^2}{||\Gamma(t)||^2} = ||W_1(j\omega)S(j\omega)||_\infty$$

forms the left hand side of (5.15), where $||\cdot||$ is the 2-norm of the signal and $||\cdot||_\infty$ is the $\mathcal{H}_\infty$ norm of transfer function $W_1 S$. The continuous–time state space representation of the performance index of (5.17) can be obtained as

$$\dot{x}(t) = -\phi \, x(t) + \delta \, \widetilde{T}_{spec}(j, t),$$

$$\widetilde{J}(j, t) = x(t) + \zeta \, \widetilde{T}_{spec}(j, t), \tag{5.20}$$

where $x(t)$ is the state of the filter generating the performance specification. For implementation, a discrete–time version of the above can be easily obtained. The following algorithm summarizes the multiple controller based switching scheme.

Algorithm 1:

INITIALIZATION

For each user $i = 1, ..., N$:

- Define a set of candidate controllers given by C_j, $j \in \mathcal{M}$. Choose an initial controller to be used in the loop at time $\tau = 0$.

- Set initial performance specification $\widetilde{J}(j, 0) = 0$, $j \in \mathcal{M}$.

PROCEDURE (at each time $\tau = k\Delta t$)

For each user $i = 1, ..., N$, repeat the following so long as the user is connected to the network:

(1) Measure $(p(k\Delta t), \gamma(k\Delta t))$.

(2) Calculate $\widetilde{J}(j, k\Delta t), \; j \in \mathcal{M}$, using discrete-time version of (5.17) and (5.19).

(3) Determine the best available controller C^* using (5.9) and update the controller parameters of the ith user.

(4) Update the transmitted power using (5.3). $\hfill \diamond$

The success of any switching scheme depends on judicious choice of the candidate controller set, as well as on the choice of cost function. Considerations in choosing the cost function are ease of evaluation and minimization, as well as stability related properties like 'cost detectability' [SWS04, WSS04].The computational complexity of the algorithm increases linearly with the increase in the number of candidate controllers. However, a large controller set increases the chances of having *good* controllers in the set. So, the set of candidate controllers has to be chosen carefully, keeping in mind all these constraints. When a range of stabilizing controller parameters is known, the candidate controllers can be selected such that their parameters lie evenly in that range. In absence of such knowledge, if a stabilizing controller is known, it can be used as a nominal controller and several other candidate controllers can be constructed with parameters lying around those of the nominal controller. From [FM93], we know that the controller corresponding to the DPC algorithm ($\alpha = 0, \; \beta = -1, \; \theta = 0$) produces a stable system. Hence, DPC can be set as a nominal controller and the candidate controllers can be chosen from all possible combinations of the parameters lying around the parameters of the DPC. This nominal controller can also be used as the initial controller for the process while executing Algorithm 1.

Also, the proposed scheme does not necessarily have to increase the computational burden on the mobile users as these computations can be carried out at the base station and power up/down command can be conveyed to the users by the base station.

5.5   Simulation Results

In this section, we numerically test our proposed scheme by considering a simple CDMA cell in which all users are assumed to share the same channel and are served by a single base station [SKH99]. The mobiles are uniformly distributed around the base station in a square cell of length 2000 meters.

From [FM93], we know that the controller corresponding to the DPC algorithm ($\alpha = 0$, $\beta = -1$, $\theta = 0$) produces a stable system. Hence, DPC is set as a nominal controller and the candidate controllers are chosen from all possible combinations of the parameters (α, β, θ), where

$$\alpha \in \{-0.15, 0, 0.15\}, \ \beta \in \{-1.5, -1, -0.5\}, \ \theta \in \{-0.003, 0, 0.003\},$$

which yields a total of 27 candidate controllers. As DPC is known to produce a stable system, Algorithm 1 is executed with DPC as the initial controller for all users.

The target SINR is assumed to be 1/30. The performance index used is as in (5.17) with the performance specification of (5.19), with

$$\zeta = 1, \ \delta = 1, \ \phi = 0.01.$$

The filter W_1 is given by

$$W_1(s) = \frac{s+10}{2(s+0.1)}.$$

Constant channel gains, without any fading and a path loss exponent of 4 are implemented. Receiver thermal noise is taken as 1e-6. Initial transmitted powers are set to 0.001 for all users.

As a measure of performance, we consider the number of iterations needed for all users to reach and stay within 1% of the target SINR, which is similar to the notion of settling time in control theory. The lower this number is, the faster all the components of the power vector track the desired threshold. As we assume that all mobiles are uniformly distributed within the cell, the average settling time is computed based on 1000 Monte Carlo simulations. We repeat this process for various number of users in the cell and plot the average settling time versus the number of users in the cell.

Fig. 5.5 summarizes the simulation results for the proposed switching scheme (solid curve) and for three other candidate controllers,

$$C_4(\alpha = -0.15, \beta = -1, \theta = -0.003),$$

$$C_7(\alpha = -0.15, \beta = -1.5, \theta = -0.003)$$

and

$$C_{14}(\alpha = 0, \beta = -1, \theta = 0).$$

Note that C_{14} corresponds to DPC. The curves for these three controllers are obtained by using them for all users at all times, without any switching. Responses of other controllers are not plotted as they have comparable or worse performance.
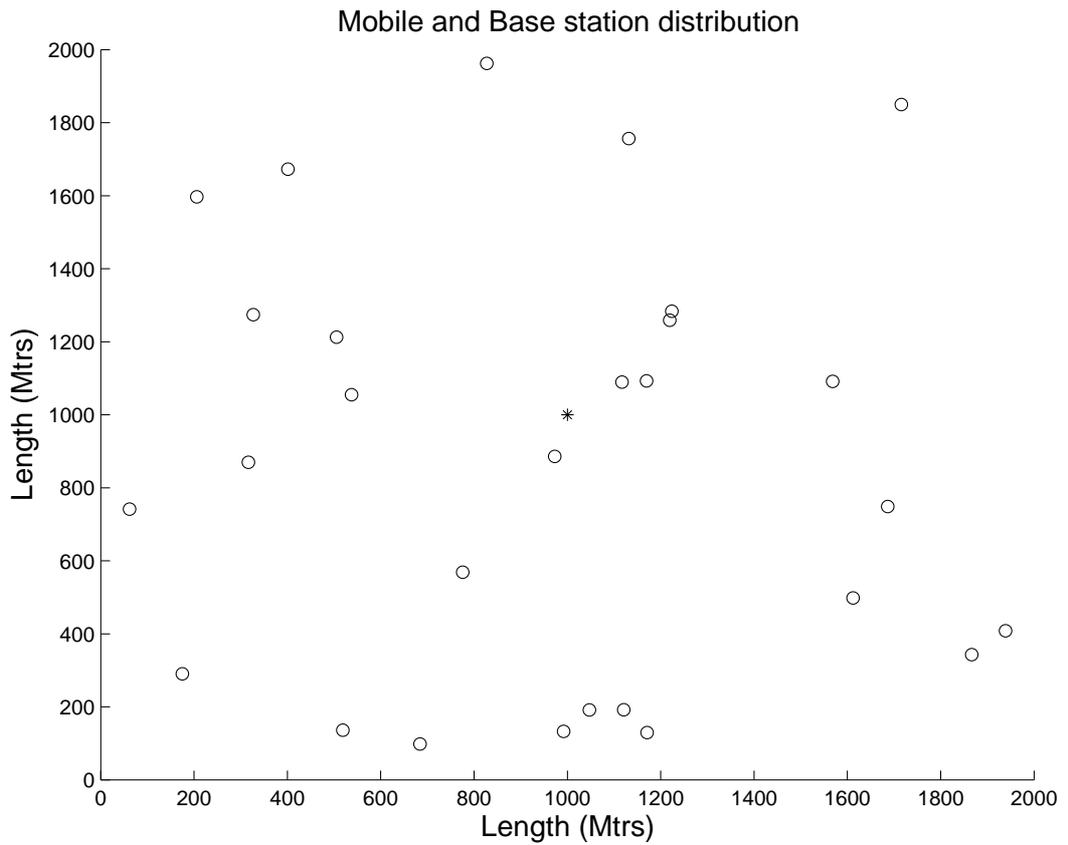
Figure 5.4: Distribution of 28 mobiles and the base station for one of the Monte Carlo simulations. The circles represent the mobiles and the star represents the base station.

From Fig. 5.5, it is seen that when the number of users is low ($N \leq 10$), the DPC algorithm (C_{14}) outperforms other candidate controllers when used by all users without any switching. On the other hand, when the cell has a larger number of users, other candidate controllers perform better. The switched system utilizes DPC mainly when the number of users is low and uses other candidate controllers when the number of users is high, and thus performs better than most of the candidate controllers for all possible user configurations. For example, with 28 users distributed uniformly in the cell, it is seen that C_7 gives good performance (low settling time). In simulation, it is observed that most users converge to this controller when the proposed switching
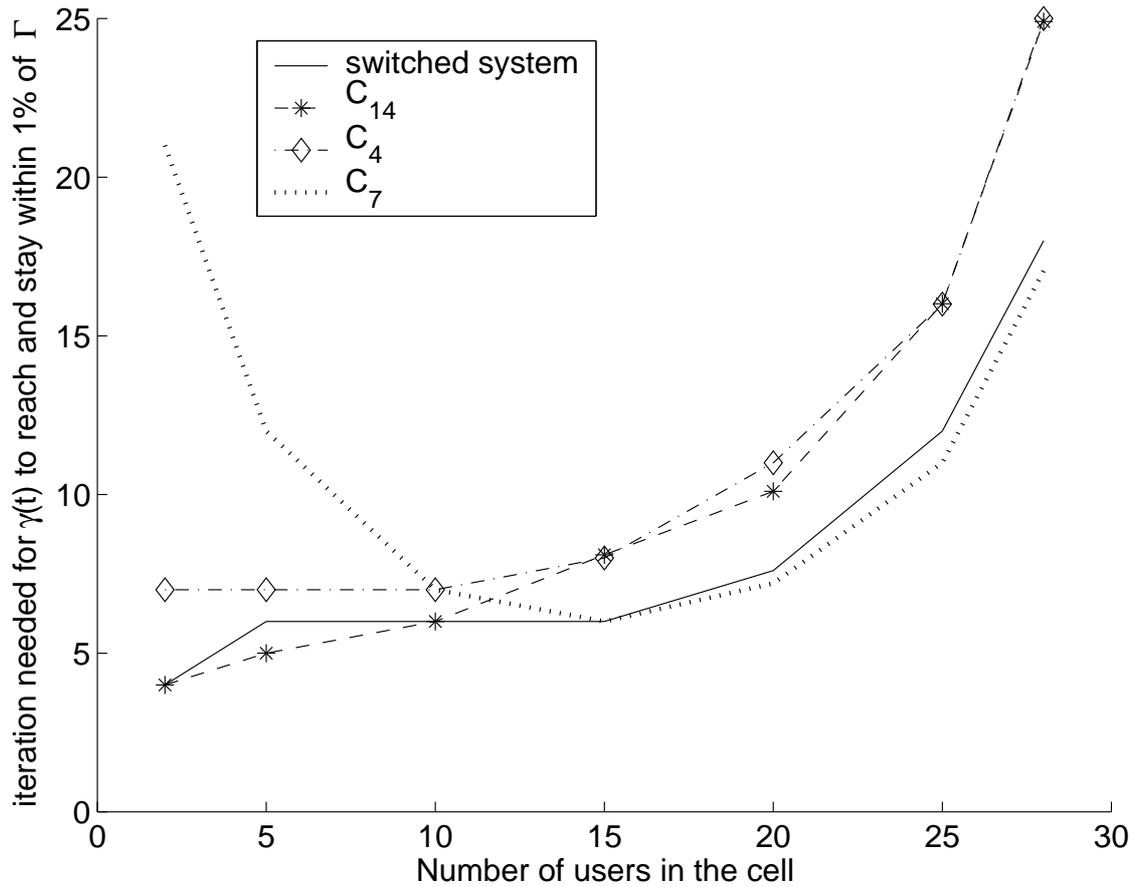
Figure 5.5: Average number of iterations required for all users to reach and stay within 1% of the target SINR.

scheme is implemented. However, for certain distribution of the mobiles, *e.g.*, when one or several users are very close to the base station, and with different initial transmitted powers, C_7 may not always give the best performance. Even in such cases, we have observed that the proposed switching scheme selects different candidate controllers for different users, thereby still improving the overall performance. The achieved SINR of the two worst case users (who take the longest time to reach 99% of the target SINR), out of 28 randomly distributed mobile users, with the proposed switching scheme, is shown in Fig. 5.6; the settling time is seen to be 20. When DPC (C_{14}) is used for all

Figure 5.6: Achieved SINR (solid line –) of the two worst case users (who take the longest time to reach 99% of the target SINR), out of 28 randomly distributed users using the proposed switched system. Dashed line (- -): 99% and 101% of the target SINR. Settling time = 20 iterations.

users without any switching for the same mobile distribution, the settling time is found to be 25 (Fig. 5.7).

5.6   Conclusions

In this chapter, we have described an adaptive technique to tune the gains of a PID–type controller to be used for power control in a wireless network. The proposed algorithm selects controllers in a distributed fashion using the available data only, without any prior knowledge about the existing channel conditions. Simulation results show that the
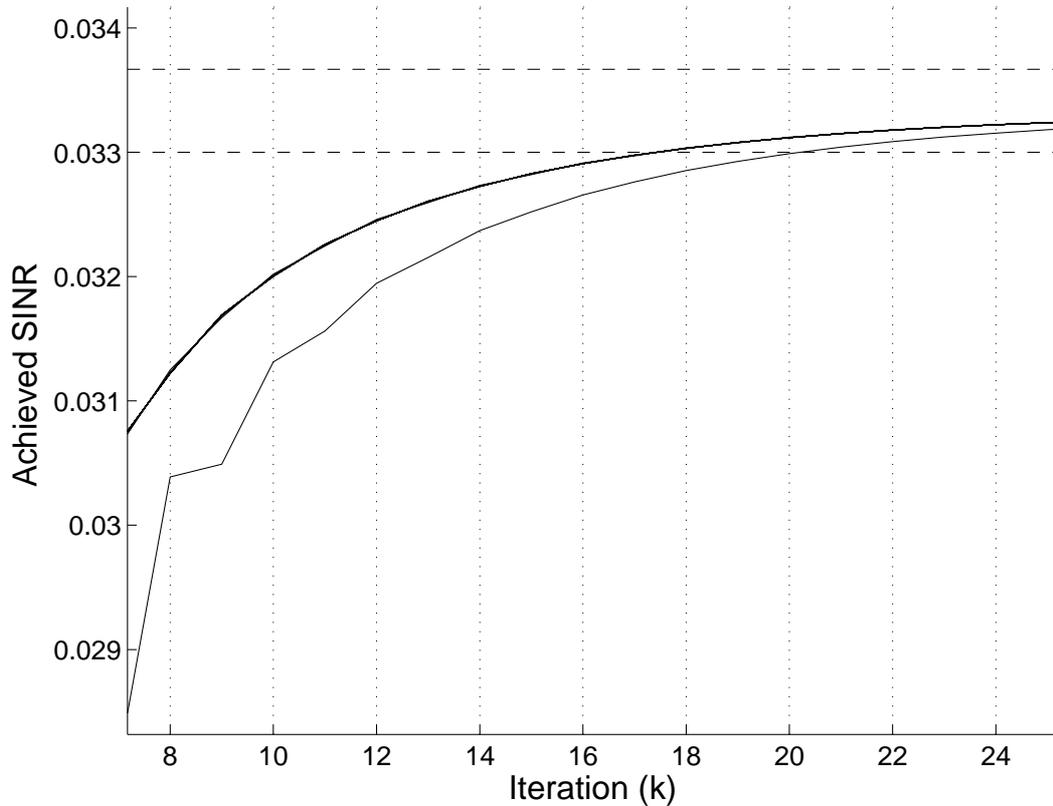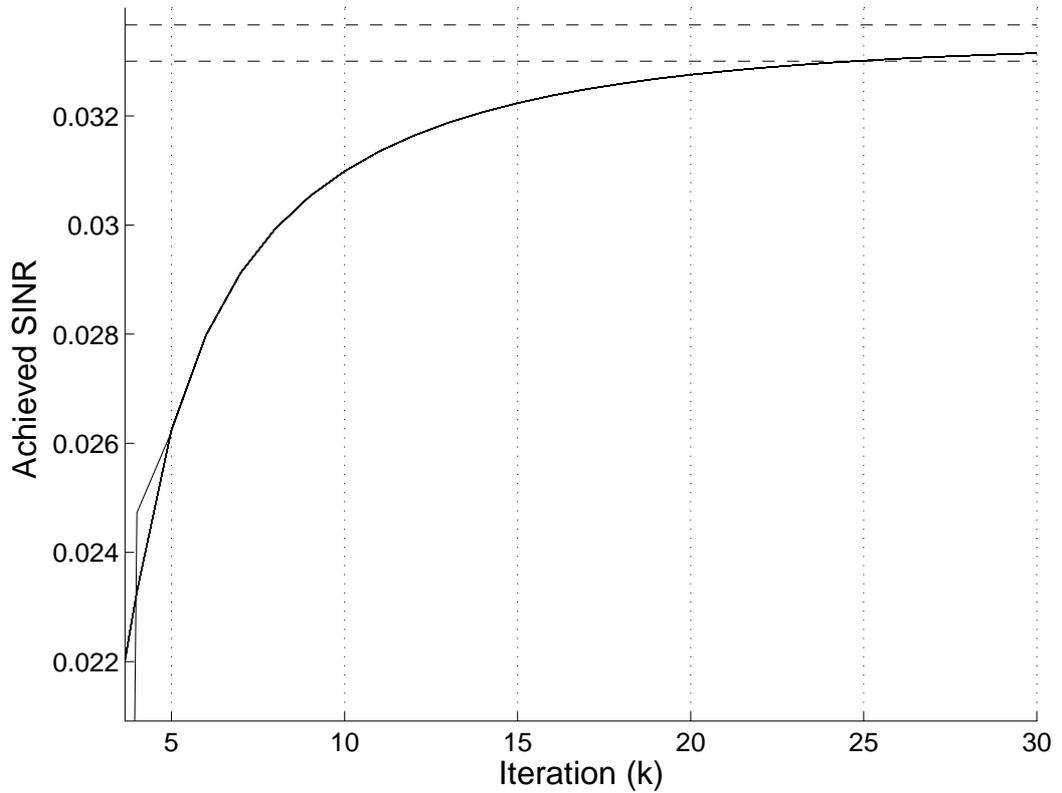
Figure 5.7: Achieved SINR (solid line –) of the two worst case users (who take the longest time to reach 99% of the target SINR), out of 28 users having the same mobile distribution as in Fig. 5.6 and with all of them using only C_{14} (DPC). Dashed line (--): 99% and 101% of the target SINR. Settling time = 25 iterations.

algorithm performs better than most of the available candidate controllers (including the DPC for higher number of users in the cell) and can adapt to different number of users in the cell by using different candidate controllers.

Chapter 6

Conclusion and Future Research

6.1 Dissertation Summary

The primary focus of this dissertation is to develop theory and practical applications of the Multi–Controller Adaptive Control (MCAC) methods. In particular, the contributions are:

- Develop necessary conditions for cost detectability of the performance index for MCAC methods. It was shown that cost detectability is a function of the candidate controller set and the performance index used and thus it imposes some restriction on the class of candidate controller set that can be used. However, it should be noted that the restrictions imposed on candidate controllers are still *plant-assumption-free*, i.e. to satisfy the them, one does not have to assume anything about the plant; these restrictions can easily be satisfied by judiciously choosing the candidate controllers.

- Apply MCAC to solve the Model Reference Adaptive Control (MRAC) problem. It is shown that standard MRAC problem, using the MMAC scheme by Narendra

et al [NB94, Nar97] is a dual to the problem of identifying a controller directly using the MCAC framework.

- Problem of transmitting optimal powers by mobile users in a wireless network has been studied. An algorithm, employing MCAC technique has been developed for the above problem. The proposed algorithm provides an adaptive technique to tune the gains of a PID–type controller and can adapt to different number of mobile users by employing different candidate gains. This line of research provides a step towards applying adaptive feedback control systems for transmitting power by a mobile user in a dynamic wireless environment.

6.2   Future Directions

There are several theoretical and practical issues which might be addressed in future:

- Extend the results of Chapter 3 to derive necessary and sufficient condition for cost detectability of the performance index and subsequently stability of MCAC techniques. Some results in this direction can be found in [WPSS06].

- The MCAC switching scheme assumes that there is at least one stabilizing controller in the candidate controller set for the switched system to be stable (referred to as the feasibility condition). In our opinion, this assumption is less restrictive than the standard assumptions found in traditional adaptive control [NA89]. Issues concerning choosing the initial set of candidate controller, to satisfy the feasibility assumption, might be a future research topic.

- In Chapter 5, a practical control problem has been examined for controlling transmitted power in a wireless environment. The controller used has a simple PID structure. Keeping in mind the structure of the system matrix in the problem (which are M–Matrices) [PASM04a, PASM04b, PASM05, FM93, UJK00], future research might focus on finding a bound on stabilizing PID gains for all possible mobile user distribution (provided the power control problem is a feasible one). Once this bound is known, choosing the candidate PID gains would be easier as then these gains can be evenly distributed in the range of the stabilizing gains.

Reference List

[ABD04] T. Alpcan, T. Başar, and S. Dey. A power control game based on outage probabilities for multicell wireless data networks. In *Proc. of American Control Conference*, pages 1661–1666, Boston, MA, June 30 - July 2, 2004.

[Aei73] J. Aein. Power balancing in systems employing frequency reuse. *COMSAT Tech. Rev.*, 3(2):277–300, Fall 1973.

[Aka04] M. Akar. Improved transient performance in sliding modes using multiple models and switching. In *Mediterranean Conference on Control and Autumation*, accepted for presentation. Kusadasi, Turkey, June 2004.

[AM04] D. Angeli and E. Mosca. Adaptive switching supervisory control of non-linear systems with no prior knowledge of noise bounds. *Automatica*, 40(3):449–457, March 2004.

[Bal96] J. Balakrishnan. *Control Systems Design Using Multiple Models, Switching, and Tuning*. PhD thesis, Department of Electrical Engineering, Yale University, New Haven, CT, 1996.

[CS96] F.B. Cabral and M.G. Safonov. A falsification perspective on model reference adaptive control. In *35th IEEE conf. on Decision and Control*, volume 3, pages 2982 – 2985, Japan, Dec. 1996.

[CZM01] E. G. Collins Jr., Y. Zhao, and R. Millett. A genetic search approach to unfalsified PI control design for a weigh belt feeder. *International Journal of Adaptive Control and Signal Processing*, 15(5):519–534, August 2001.

[FB86] M. Fu and B. R. Barmish. Adaptive stabilization of linear systems via switching control. *IEEE Transactions on Automatic Control*, 31:1097–1103, June 1986.

[FM93] G. J. Foschini and Z. Miljanic. A simple distributed autonomous power control algorithm and its convergence. *IEEE Transactions on Vehicular Technology*, 42(4):641–646, November 1993.

[Fu96] M. Fu. Minimum switching control for adaptive tracking. In *IEEE Conference on Decision and Control*, pages 3749–3754, December 1996.

[GGB99] F. Gunnarsson, F. Gustafsson, and J. Blom. Pole placement design of power control algorithms. In *IEEE 49th Vehicular Technology Conference*, volume 3, pages 2149–2153, May 1999.

[GRVZ93] S. A. Grandhi, D. J. Goodman R. Vijayan, and J. Zander. Centralized power control in cellular radio systems. *IEEE Transactions on Vehicular Technology*, 42(4):466–468, November 1993.

[Gun00]   Fredrik Gunnarsson. *Power Control in cellular radio systems: Analysis, Design and Estimation.* PhD thesis, Department of Electrical Engineering, Linkopings University, Linkoping, Sweden, 2000.

[Hes01]   J. Hespanha. Tutorial on supervisory control. In *Workshop at IEEE Conference on Decision and Control*, Orlando, Florida, Dec. 2001.

[IS95]   P. A. Ioannou and J. Sun. *Robust Adaptive Control.* Prentice–Hall, Englewood Cliffs, NJ, 195.

[JH98]   H. Ji and C. Huang. Noncooperative uplink power control in cellular radio systems. *Wireless Networks*, 4(3):233–240, April 1998.

[JS99]   M. Jun and M. G. Safonov. Automatic PID tuning: An application of unfalsified control. In *Proc. of IEEE Int. Sym. on Computer Aided Control System Design*, pages 328–333, Hawaii, August 1999.

[JZM01]   E. G. Collins Jr., Y. Zhao, and R. Millett. Unfalsified PI control design for a weigh belt feeder using genetic search. In *American Control Conference*, volume 5, pages 4116–4121, Arlington, VA, June 2001.

[KR96]   S. R. Kulkarni and P. J. Ramadge. Model and controller selection policies based on output prediction errors. *IEEE Transactions on Automatic Control*, 41(11):1594–1604, Nov. 1996.

[Mar86]   B. Martensson. *Adaptive stabilization.* PhD thesis, Lund Institute of Technology, Lund, Sweden, 1986.

[MD91]   D. Miller and E. J. Davison. An adaptive controller which provides an arbitrary good transient and steady–state response. *IEEE Transaction On Automatic Control*, 36(1):68–81, Jan. 1991.

[MGHM88] R. H. Middleton, G. C. Goodwin, D. J. Hill, and D. Q. Mayne. Design issues in adaptive control. *IEEE Transaction On Automatic Control*, 31(1):50–58, Jan. 1988.

[MMG92]   A. S. Morse, D. Q. Mayne, and G. C. Goodwin. Application of hysteresis switching in parameter adaptive control. *IEEE Transaction On Automatic Control*, 37(9):1343–1354, Sept. 1992.

[Mor96]   A.S. Morse. Supervisory control of families of linear set-point controllers-part 1: Exact matching. *IEEE Transactions on Automatic Control*, 41(10):1413–1431, October 1996.

[NA89]   K.S. Narendra and A.M. Annaswamy. *Stable Adaptive Systems.* Prentice–Hall, Englewood Cliffs, NJ, 1989.

[Nar97]   K. S. Narendra. Adaptive control using multiple models. *IEEE Transaction on Automatic Controls*, 42(2):171–187, Feb 1997.

[NB94]   K. S. Narendra and J. Balakrishnan. Improving transient response of adaptive control systems using multiple models and switching. *IEEE Transactions on Automatic Control*, 39(9):1861–1866, September 1994.

[NS97]   K. S. Narendra and R. N. Shorten. A sufficient condition for the existence of a common Lyapunov function for two second order linear systems. *IEEE Conf on Decision and Control*, pages 3521–3522, December 1997.

[Nus83]   R. D. Nussbaum. Some remarks on the conjecture in parameter adaptive control. *Systems and Control Letters*, 3:243–256, 1983.

[PAMS04]   A. Paul, M. Akar, U. Mitra, and M. G. Safonov. A switched system model for stability analysis of distributed power control algorithms for cellular network. *Proceedings of the American Control Conference*, Boston, MA, June 2004.

[PASM04a]   A. Paul, M. Akar, M. G. Safonov, and U. Mitra. Necessary and sufficient conditions for stability of a class of second order switched systems. *Proceedings of the American Control Conference*, Boston, MA, June 2004.

[PASM04b]   A. Paul, M. Akar, M. G. Safonov, and U. Mitra. Power control for wireless networks using multiple controllers and switching. *Proceedings of the American Control Conference*, Boston, MA, June 2004.

[PASM05]   A. Paul, M. Akar, M. G. Safonov, and U. Mitra. Adaptive distributed power control in cellular communication networks. *IEEE Transactions on Neural Network, Special Issue on Adaptive Learning Systems in Communication Networks, Accepted for publication*, 2005.

[PS03]   A. Paul and M. G. Safonov. Model reference adaptive control using multiple controllers and switching. In *IEEE Conference on Decision and Control*, Hawaii, December 2003.

[PWSS05]   A. Paul, R. Wang, M. Stefanovic, and M. G. Safonov. Cost detectability of performance index for unfalsified multi-controller adaptive control. In *IEEE Conference on Decision and Control, Submitted to*, Seville, Spain, December 2005.

[RK01]   H. A. Razavi and Thomas R. Kurfess. Force control of a reciprocating surface grinder using unfalsification and learning concept. *International Journal of Adaptive Control and Signal Processing*, 15(5):508–518, Aug. 2001.

[RVAS85]   C. Rohrs, L. Valavani, M. Athans, and G. Stein. Robustness of continuous-time adaptive control algorithms in the presence of unmodeled dynamics. *IEEE Transactions on Automatic Control*, 30(9):881–889, Sept. 1985.

[SB89]   S. Sastry and M. Bodson. *Adaptive Control: Stability, Convergence, and Robustness*. Prentice Hall, Englewood Cliffs, N.J., 1989.

[SF01]   M.G. Safonov and F.B.Cabral. Fitting controllers to data. *Systems and Control Letters*, 43(4):299–308, 2001.

[SKH99]   A. Sampath, P. S. Kumar, and J. M. Holtzman. Power control and resource management for a multimedia CDMA wireless systems. In *Proceedings of the IEEE Int. Symp. on Personal, Indoor and Mobile Radio Communications*, pages 21–25, 1999.

[SP00]   S. Skogestad and I. Postlethwaite. *Multivariable Feedback Control*. John Wieley and Sons, West Sussex, United Kingdom, 2000.

[ST97]   M. G. Safonov and T. C. Tsao. The unfalsified control concept and learning. *IEEE Transactions on Automatic Control*, 42(6):843–847, June 1997.

[SWS04]   M. Stefanovic, R. Wang, and M. G. Safonov. Stability and convergence in adaptive systems. *Proceedings of the American Control Conference*, Boston, MA, June 2004.

[TI93]   G. Tao and P. A. Ioannou. Model reference adaptive control for plants with unknown relative degree. *IEEE Transactions on Automatic Control*, 38(6):976–982, June 1993.

[TS01]   T. C. Tsao and M. G. Safonov. Unfalsified direct adaptive control of a two-link robot arm. *Int. J. Adaptive Control and Signal Processing*, 15:319–334, May 2001.

[Tsa94]   T. Tsao. *Set Theoretic Adaptor Systems*. PhD thesis, Department of Electrical Engineering, University of Southern California, Los Angeles, CA, 1994.

[UJK00]   Z. Uykan, R. Jantti, and H. Koivo. A PI-power control algorithm for cellular radio systems. *Proceedings of the 6th IEEE Int. Symp. on Spread-Spectrum Tech. and Applications*, pages 782–785, September 2000.

[WB84]   J. C. Willems and C. I. Byrnes. Global adaptive stabilization in the absence of information in the sign of the high frequency gain. *Proceedings of the 6th INRIA conference on analysis and optimization of systems*, pages 549–554, France, June 1984.

[WPSS06]   R. Wang, A. Paul, M. Stefanovic, and M. G. Safonov. Cost detectability of performance index for unfalsified multi-controller adaptive control. In *Systems and Control Letters, Submitted to*, 2006.

[WSar]   R. Wang and M. G. Safonov. Stability of unfalsified adaptive control using multiple controllers. In *American Control Conference*, Portland, OR, June 8–10, 2005, to appear.

[WSS04]   R. Wang, M. Stefanovic, and M. Safonov. Unfalsified direct adaptive control using multiple controllers. In *AIAA Guidance, Navigation, and Control Conference*, Rhode Island, USA, August 16-19, 2004. Article No. AIAA-2004-5223, 7 pages.

[Zan92]   J. Zander. Performance of optimum transmitter power control in cellular radio systems. *IEEE Transactions on Vehicular Technology*, 41(1):57–62, February 1992.