Submitted 27 Sep 2000 (rev. 09 Apr 2001)

*Int. J. Adaptive Control and Signal Processing*, Special Issue on Control with Confidence, 2001 (to appear).

# Controller Validation*

T. F. BROZENEC†, T. C. TSAO‡, and M. G. SAFONOV§

**Abstract**

The unfalsified control concept provides a theoretical basis for directly validating closed-loop controller performance from open-loop experimental data, or even from closed-loop data acquired while another controller was in the feedback loop. It allows this to be accomplished without making any prior assumptions about the plant or noise. This approach to experimental controller validation avoids the conservatism inherent in indirect two-step validation approaches that require one to first identify a plant model and uncertainty-bounds from experimental data before analyzing control system robustness. Controller validation is shown to be the feedback generalization of open-loop model validation concepts that form the basis of various control-oriented system identification theories.

KEY WORDS: Controller identification; model validation; robust control; unfalsified control

## 1 Introduction

In all engineering disciplines, experimental data provides a most important connection between theory and practice. For control engineers, the situation is no different. Control theory makes no claims about the performance or stability of physical systems; only about their models. The engineer, who is concerned with controlling physical systems, seeks some assurance that closed-loop operation of the actual system can achieve desired performance objectives. Therefore, once a control law has been determined, by whatever method, there must be a way of evaluating if the available experimental data corroborates theoretical predictions of performance.

Robust control theory gives powerful techniques for synthesizing control laws which, theoretically, provide performance and stability guarantees for the closed-loop system. However, as is well known, these techniques require a plant model along with an upper bound on the associated uncertainty (modeling error). Usually, a conservative upper bound is hypothesized which may result in performance limitations. A more rigorous approach has recently been suggested by several researchers [1, 2, 3, 4] involving identification of modeling error using experimental data. Consideration of this problem soon led to the concept of plant model validation (unfalsification) [5, 6, 7] where a model with a hypothesized uncertainty bound is validated against input-output data with the hope that robust controllers based on the model can subsequently be applied if the uncertainty bound proves to be valid (i.e., unfalsified by the data [8]). Such controllers can be

*This research was supported in part by AFOSR Grants F49620-95-1-0095 and F49620-98-1-0026.

†Boeing Satellite Systems, El Segundo, CA, USA. Phone 1-310-416-5247. FAX 1-310-364-7663. Email thomas.f.brozenec@boeing.com.

‡T. C. Tsao was with Chinese Naval Academy, Dept. of Electrical Engineering, Kaohsiung, Taiwan 813, ROC. He is now with Boeing Satellite Systems, El Segundo, CA, USA.

§Electrical Engineering—Systems Dept., University of Southern California, Los Angeles, CA 90089-2563, USA. Phone 1-213-740-4455. FAX 1-213-740-4449. Email msafonov@usc.edu.

described as *not demonstrably unrobust*; that is, the available experimental data cannot be used to demonstrate their inability to provide the desired level of closed-loop system performance.

In control applications, the ultimate goal of model validation is to use the validated (unfalsified) plant and uncertainty models along with robust synthesis tools to design control laws which will subsequently be applied to the plant. So that they will only rarely be invalidated, the uncertainty models used are typically conservatively chosen (e.g., with unstructured uncertainty or with model error bounds that are larger than necessary). Thus, even when estimating model error from the data, conservatism may tend to limit the performance attainable with the resultant robust controllers. In addition choosing the wrong uncertainty model structure may also result in undue conservatism, as for example when a structured uncertainty is coarsely represented using multiplicative or additive plant uncertainty bounds (cf. Skogestad and Postlethwaite [9]). And finally, as we demonstrate in this paper, even if the plant and uncertainty models are invalidated, this does not with certainty demonstrate the inability of the control law in question to meet its desired performance.

The unfalsified control concept of Safonov and Tsao [10, 11, 12, 13] represents a significant improvement. Rather than validating plant models against pre-defined open-loop error bounds, controller validation (unfalsification) directly evaluates the ability of control laws to meet a given closed-loop performance criterion. The terms controller validation and controller unfalsification are used interchangeably throughout the paper.

In the present paper, we expand this theory by a study of the connections between model and controller validation. We show that controller validation is a feedback generalization of model validation and that it is less conservative to unfalsify (validate) control laws directly rather than unfalsify (validate) the plant models from which those control laws are constructed. Therefore, the ability of a controller to meet a performance criterion can be evaluated directly; no particular plant structure or property need be assumed.

The paper is organized as follows. In Section 2, a brief summary of the controller validation theory [13] is presented. A special case is used to illustrate the primary ideas. In Section 3, we show that model validation is a special case of controller validation where the controller is the nominal plant model and the performance criterion is a model error criterion. The relationship between unfalsified controllers and robust controllers is explored and the conservatism of model validation approaches is established. Finally, the conclusion summarizes the results and suggests future research directions.

## 2 Controller falsification

Consider the feedback control system of Fig. 1. The goal of applying the feedback control law

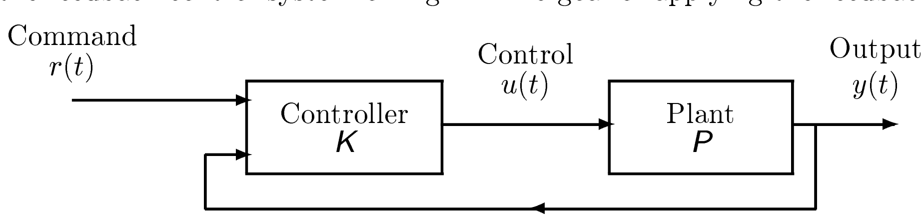


Figure 1: Feedback control system.

$K$ to the plant $P$ is to ensure that the closed-loop system response, call it $T$, satisfies a certain performance criterion. In engineering practice, the plant is always only partially known and, therefore, the system response $\mathsf{T}$ will be uncertain. As a result, the ability of the controller $\mathsf{K}$ to ensure a desirable response is also uncertain. Controller validation asks the question, "What can be determined from the data alone?" by making a clear distinction between conclusions derived using assumptions about the plant and conclusions arrived at using only experimental data.

For the purpose of discussing controller unfalsification, the input/output description of systems due to Zames [14] is helpful. Using this approach, we view a dynamical system as a set of input/output pairs $(x, y)$ which are signals in appropriate function spaces $\mathcal{X}$, $\mathcal{Y}$. In this framework, solutions of feedback interconnections of two systems become the intersection of two graphs in the product space $\mathcal{X} \times \mathcal{Y}$. Zames and Sandberg originally used this approach to lay the foundation for much of input-output stability theory [14, 15, 16, 17]. For easy visualization, we may consider the special case where $\mathcal{X}$ and $\mathcal{Y}$ are the space of real numbers, $\mathbb{R}$. For control applications, however, it is often convenient to view the signals as elements of the extended vector space $L_{2e}$. The "behavioral system" concept of Willems [8] can be regarded as an application of these principles. Using this concept, we can view the plant and the controller as signal constraints (sets) in the product space $\mathcal{R} \times \mathcal{Y} \times \mathcal{U}$ where $r \in \mathcal{R}$, $y \in \mathcal{Y}$ and $u \in \mathcal{U}$. The plant, which is unknown, is represented by a set

$$\mathsf{P} \subset \mathcal{R} \times \mathcal{Y} \times \mathcal{U} \tag{1}$$

defined to be the set of triples $(r, y, u) \in \mathcal{R} \times \mathcal{Y} \times \mathcal{U}$ such that the pair $(u, y)$ satisfies the unknown plant's input-output relation. A single experiment represents a point $(u_0, y_0)$ in the $\mathcal{U} \times \mathcal{Y}$-plane on the graph of the unknown plant. An ensemble of several experiments could be represented by additional points on the graph of the plant. Thus, experimental data represents a (fairly small) subset of the graph of the plant and, consequently, of $\mathcal{U} \times \mathcal{Y}$. It is convenient to associate the experimental data $(u_0, y_0)$ with the following embedding in $\mathcal{R} \times \mathcal{Y} \times \mathcal{U}$

$$\mathsf{P}_{\mathrm{data}} \triangleq \{(r, y, u) \in \mathcal{R} \times \mathcal{Y} \times \mathcal{U} \mid u = u_0, y = y_0\} \tag{2}$$

which we will refer to as the plant data set. This set $\mathsf{P}_{\mathrm{data}}$ is an *inner bound* on the plant relation in the sense that $\mathsf{P}_{\mathrm{data}} \subset \mathsf{P}$.

By way of contrast, robust control problem formulations begin by *assuming* an *outer bound* $\mathsf{P}_{\mathrm{unc}}$ on the plant relation which bounds plant *uncertainty* in the sense that $\mathsf{P} \subset \mathsf{P}_{\mathrm{unc}}$. Unfortunately, such an outer bound cannot be completely validated by any finite amount of experimental evidence, unless one simply assumes that future errors will never exceed previously observed errors. Yet, though a given error bound $\mathsf{P}_{\mathrm{unc}}$ may include all previously observed experimental data, there can be no guarantee that future plant behavior will be encompassed by this bound. There is no optimal, and certainly no unique, approach to defining the bound $\mathsf{P}_{\mathrm{unc}}$ from experimental data. Experimentation allows us to view only a subset $\mathsf{P}_{\mathrm{data}}$ of possible plant behaviors whereas robust control requires the superset $\mathsf{P}_{\mathrm{unc}}$.

Like the plant, a control law is a constraint on the signals $r$, $y$, and $u$. As such, it represents a surface (a set) in the space $\mathcal{R} \times \mathcal{Y} \times \mathcal{U}$

$$\mathsf{K} \subset \mathcal{R} \times \mathcal{Y} \times \mathcal{U}. \tag{3}$$

Likewise, a performance criterion $\mathsf{T}_{perf}$ is simply another constraint on the signals $r$, $y$, and $u$ and thus represents a subset of $\mathcal{R} \times \mathcal{Y} \times \mathcal{U}$ as well

$$\mathsf{T}_{\mathrm{data}} \subset \mathcal{R} \times \mathcal{Y} \times \mathcal{U}. \tag{4}$$

This characterization of the experimental data, control law, and performance criterion as separate signal constraints provides a convenient framework for the description of the controller unfalsification problem. Just as the input-output stability of interconnections of subsystems reduces to conditions on intersecting constraints, so the controller unfalsification problem also reduces to conditions on intersecting constraints defined by the data, control law, and performance criterion.

Note that for purposes of clarity, and to avoid unnecessary details, our discussion focuses on the simplest case, where complete information is available (e.g., $(u_0, y_0) \in L_{2e}[0,\infty) \times L_{2e}[0,\infty)$) as opposed to the more general case considered by Safonov and Tsao [13] where only partial information (e.g., only past values) about $(u_0, y_0)$ is revealed by the measurements. For our simplified case, the unfalsified control theory of Safonov and Tsao states that a controller $\mathbf{K}$ is unfalsified by experimental data $\mathbf{P}_{\text{data}}$ if and only if (cf. Theorem 1 of [13])

$$\mathbf{P}_{\text{data}} \cap \mathbf{K} \subset \mathbf{T}_{\text{data}} \tag{5}$$

Eqn. (5) says that a topological separation must exist between the set $\mathbf{P}_{\text{data}} \cap \mathbf{K}$ and the complement $\overline{\mathbf{T}}_{\text{data}}$ of the set $\mathbf{T}_{perf}$; i.e.,

$$\mathbf{P}_{\text{data}} \cap \mathbf{K} \cap \overline{\mathbf{T}}_{\text{data}} = \emptyset. \tag{6}$$

The notion of topological separation underlies all of stability theory. Safonov [17] showed that both input-output and Lyapunov stability criteria can be interpreted in this context. Specifically, the stability of the feedback interconnection of two systems is assured if one can 'topologically separate' the function space containing the system's input-output relations into two regions, one containing the graph of the feedforward element and the other containing the inverse graph of the feedback element. Similarly, controller validation requires a topological separation. See [10, 11, 12, 13] for a complete development of the controller validation theory.

A special case with a simple controller structure illustrates the basic concepts nicely. Let us consider linear time-invariant (LTI) control laws of the form

$$\mathbf{K} = \mathbf{K}_c \stackrel{\Delta}{=} \{(r, y, u) \mid u = c * (r - y)\} \tag{7}$$

where $*$ denotes convolution, $r$ is the reference command, $c$ is the impulse response of the controller, and $C(s)$ denotes the Laplace transform of $c$. Although not necessary, for purposes of simplicity, we shall assume that $c$ is *stably* invertible. Therefore, $C(s)$ is proper and minimum phase. Some examples of controller types which satisfy these assumptions are proportional-integral-derivative (PID) and lead-lag compensators.

Suppose that measurements of input $u_0(t)$ and output $y_0(t)$ are available. This data can be either open or closed-loop data. Using this data, we would like to evaluate the ability of the controller $\mathbf{K}_c$ to meet a given closed-loop performance criterion $T(r, y, u) \square 0$; e.g.,

$$\mathbf{T}_{\text{spec}} = \{(r, y, u) \mid T(r, y, u, t) \square 0 \ , \ \forall t > 0\} \tag{8}$$

where

$$T(r, y, u, t) = \|w_1 * (r - y)\|^2_{L_2[0,t]} + \|w_2 * u\|^2_{L_2[0,t]} - \|r\|^2_{L_2[0,t]}, \tag{9}$$

and $w_1(t)$ and $w_2(t)$ are the impulse responses of given stable minimum phase weighting transfer functions $W_1(s)$ and $W_2(s)$ respectively, $r$ is the input reference signal, $y$ is the plant output signal, and $u$ is the control signal. This criterion says that the error signal $r - y$ and the control signal $u$ should be "small" compared to the command signal $r$; the dynamical weights $w_1$ and

$w_2$ determine what is small. Further, if our unknown plant $\mathsf{P}$ were to be assumed linear time-invariant (LTI) , then requiring that (8)–(9) holds for all $r$ would be equivalent to a weighted mixed-sensitivity performance criterion

$$\left\| \begin{bmatrix} W_1 S \\ W_2 CS \end{bmatrix} \right\|_{H_\infty} \square\ 1 \tag{10}$$

where $S$ is the sensitivity $S \stackrel{\Delta}{=} 1/(1+GC)$ (cf. Skogestad and Postlethwaite [9]). The assumption of an LTI plant would also permit us to deduce more from the data, e.g., that all linear combinations of time-delayed data also must satisfy the plant relation, which in turn would lead us to replace (8)–(9) with Toeplitz matrix conditions, exactly as has been done for LTI model validation by Poolla *et al.*. [7]. But, since we make no assumptions whatsoever about the plant, the LTI Toeplitz theory does not apply.

The only thing we know about the plant is that it generated the experimental data $(u_0, y_0)$; we will assume nothing else. Therefore, the plant input-output behavior is constrained by the fact that measured data is in the graph $\mathsf{P}$ of the plant. Combining this information with the controller relation $u = c * (r - y)$, we have that, if the candidate controller $c$ were in the loop, a triple satisfying the closed-loop system equations would be

$$(r, y, u) = (\tilde{r}_c, y_0, u_0) \tag{11}$$

where

$$\tilde{r}_c \stackrel{\Delta}{=} \hat{c} * u_0 + y_0 \tag{12}$$

where $\hat{c}$ denotes the impulse response of the inverse system, i.e., the signal $\hat{c}$ has Laplace transform $1/C(s)$. The physical interpretation is as follows: The reference command

$$\tilde{r}_c = \hat{c} * u_0 + y_0, \tag{13}$$

when applied to the closed loop system with $\mathsf{K}_c$ as controller, would also have generated the measured data $(u_0, y_0)$. We refer to this signal as the *fictitious reference signal.*

It now becomes clear that if $T(\tilde{r}_c, y_0, u_0) > 0$, we have proof that the controller $\mathsf{K}_c$ cannot meet the performance criterion. For a reference command has been found (namely $\tilde{r}_c$) for which the closed-loop performance specification would be violated were $\mathsf{K}_c$ in the loop. If the inequality $T(\tilde{r}_c, y_0, u_0) > 0$ is satisfied, we say that the controller $\mathsf{K}_c$ is invalidated or falsified. If not, the controller remains unfalsified or validated.

*Remarks:*

a). The analysis above requires no assumptions whatsoever about the plant. Only the data $(u_0, y_0)$ is used to draw conclusions about the ability of the control law $\mathsf{K}_c$ to meet the performance criterion $\mathsf{T}_{\text{spec}}$.

b). The data may provide no information at all. Suppose, for example, that $u_0(t) = y_0(t) = 0$ , $\forall t$. Then, we have $\tilde{r}_c = 0$ and no controller can be invalidated via the analysis above (i.e., all controllers remain unfalsified by the data in this case). Therefore, not surprisingly, there are certain conditions which the experimental data much satisfy in order to supply information about the plant. These conditions are akin to the well-known persistence of excitation conditions which are commonly found in the adaptive control literature (cf. [18]).

c). Real plants may be subject to disturbances and sensor noise. Our controller validation theory evaluates performance *a posteriori* with respect to the actual data, whether noisy or not. The effect of noise will typically be that more controllers will be falsified, since noise reduces achievable control precision. However, it would be incorrect to suppose that the presence of plant noise or other uncertainty should otherwise change or complicate the controller validation theory or its application.

d). Prior beliefs about noise such as knowledge of plant noise probabilities and plant noise models are another matter, raising issues distinct from those raised by the posterior effects of real physical noise on data. Noise probabilities may well affect one's choices of candidate controllers $\boldsymbol{K} \in \mathbf{K}$ or even one's choice of the goal $\boldsymbol{T}_{\text{spec}}$. But plant and sensor noise have no direct impact on the process of validation of given controllers with respect to given goals. Controller models are validated directly against observed data which, *a posteriori*, is always deterministic. The controller models being validated are, likewise, deterministic. And ultimately, the performance specification is, *a posteriori*, a deterministic function of the data — even though at first sight it may seem to be otherwise in cases in which we have performance specified indirectly in terms of a probabilistic conditional expectation of unmeasured signals internal to the unknown plant, computed via prior beliefs about stochastic modeling. Thus, perhaps contrary to academic intuition, stochastic noise and probability theory have no essential role to play in controller validation theory or its application so long as validation criteria are functions of only measured data, as we think they should be.

e). Initial conditions are part of the specification of a controller, just as are other parameters such as gains. Whether or not a controller is invalidated depends in general on all of its defining parameters, including its initial condition parameter if nonzero initial conditions are allowed. In our example involving the convolution controllers (7), there is no initial condition parameter because the initial state of a convolution operator as defined by (7) is, by definition, zero.

# 3 Connections with model validation and robust control

There is a connection between controller validation and model validation which will be clarified in this section. A typical model validation setup is illustrated in Fig. 2 where the notation is chosen to illustrate the connection between the two. Model validation asks the question "is there an element of the plant model set such that the observed input-output data $(r_0, y_0)$ is produced exactly?" Typically a plant model set is specified by a nominal plant model $\boldsymbol{G}_0 \subset \mathcal{R} \times \mathcal{Y}$ along with an associated model error criterion $\boldsymbol{E}_{err} \subset \mathcal{R} \times \mathcal{E}$. One example of an error criterion which may be used for model validation is given by

$$\boldsymbol{E}_{err} = \left\{ (r, e) \;\middle|\; \|e\|_{L_2[0,t]} < \epsilon \|r\|_{L_2[0,t]} \;,\; \forall t > 0 \right\} \tag{14}$$

where $e = y - \boldsymbol{G}_0 r$, for some $\epsilon > 0$. Model validation concerns whether a model is able to describe a physical system (just as controller validation concerns whether a control law is able to achieve a specified level of performance).

**Definition 1 (Plant Model Validation)** *A plant model $\boldsymbol{G}_0$ is said to be* **invalidated** *(falsified) if plant input-output data $(r_0, y_0)$ can be used to prove that it violates model error criterion $\boldsymbol{E}_{err}$. If the data $(r_0, y_0)$ cannot prove such a violation, $\boldsymbol{G}_0$ is said to be* **validated** *(unfalsified).* □

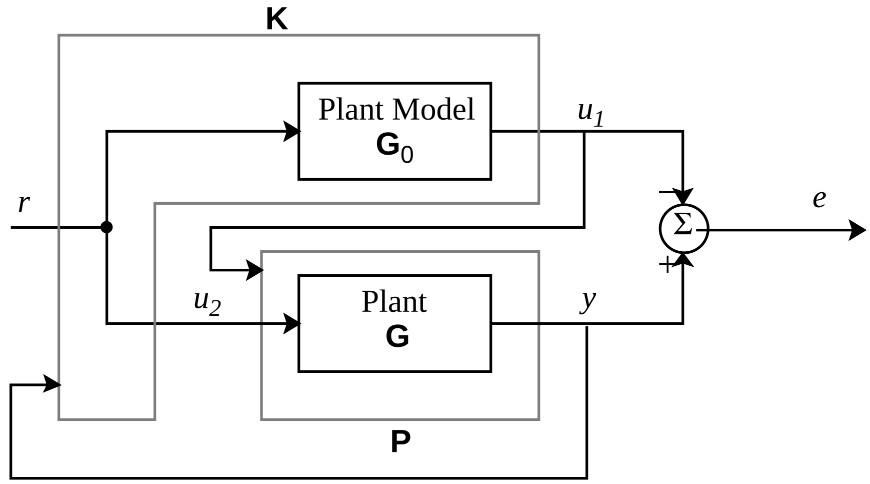


Figure 2: Model validation as a special case of controller validation.

Though it emphasizes the connection with controller validation, the foregoing definition is consistent with others in the literature—cf. (Poolla *et al.* [7], Smith and Doyle [5, 6]). The model validation problem may be posed formally in the following way.

**Problem 1 (Model Validation)** *Given*

*a). Measurements of plant input-output signals* $(r_0, y_0) \in \mathcal{R} \times \mathcal{Y}$.

*b). A nominal plant model* $\mathsf{G}_0 \subset \mathcal{R} \times \mathcal{Y}$.

*c). A model error criterion* $\mathsf{E}_{err} \subset \mathcal{R} \times \mathcal{E}$.

*Determine whether or not the ability of the plant model* $\mathsf{G}_0$ *to satisfy the model error criterion is falsified by the data.* □

We then see that controller validation becomes model validation in the special case in which the constraint set defined by the data is

$$\mathsf{P}_{\text{data}} = \left\{ (r, y, u) \,\middle|\, u \triangleq \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}, u_2 = r_0, y = y_0 \right\}, \tag{15}$$

the controller is

$$\mathsf{K} = \left\{ (r, y, u) \,\middle|\, u \triangleq \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}, (r, u_1) \in \mathsf{G}_0, u_2 = r \right\}, \tag{16}$$

and the performance criterion is

$$\mathsf{T}_{\text{data}} = \left\{ (r, y, u) \,\middle|\, u \triangleq \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}, (r, y - u_1) \in \mathsf{E}_{err} \right\}. \tag{17}$$

Compare equations (15), (16), and (17) with equation (5). The experimental data $(r_0, y_0)$ defines a "plant data set" $\mathsf{P}_{\text{data}}$ and the nominal plant model $\mathsf{G}_0$ defines, via (16), a "controller" $\mathsf{K}$ (which, since it does not utilize the available signal $y$, provides no feedback); see Fig. 2. Furthermore, the model error criterion $\mathsf{E}_{err}$ defines, via (17), a "performance criterion" $\mathsf{T}_{\text{data}}$.

Therefore, we can think of model validation as a special case of controller validation. Model validation in H-infinity such as considered by (Poolla *et al.* [7]) can also be considered a special case if the plant is assumed to be linear time-invariant.

In Figure 2, the same signal $r$ is an input to both the actual plant and the nominal plant model generating signals $u_1$ and $y$. If the difference between these two is small, as defined by the model validation criteria, the model is validated. This is one specific case of the more general controller validation setup which is illustrated by overlaying the controller and plant blocks from Figure 1 on Figure 2. The "controller" in this case is the nominal plant model which generates $u_1$ (available, but not input to the plant in this special case) and $u_2$, the actual plant input. And there is no feedback since the "controller" does not process the plant output. And the performance criteria is just constraints on the signals $r$, $u$, and $y$ as in the more general case of Figure 1. Controller validation complements robust control. Robust controllers represent the best one can do with only *a priori* knowledge. Models, uncertainty structure, and uncertainty bounds embody the available *a priori* information. Robust control allows optimal and efficient use of all the available *a priori* information to produce an initial guess for an unfalsified controller. Some methods, like $H^\infty$ (sub)optimal control, give as an initial guess a whole set of candidate unfalsified controllers (via the well-known Youla parameterization). Controller unfalsification allows us to evaluate the performance capabilities of these robust controllers directly using experimental data, without trying them in the loop. More importantly, when future plant data violates conjectured *a priori* uncertainty bounds, the controller unfalsification theory gives a precise (i.e., nonconservative) characterization of how the set of robust controllers shrinks.

Controller validation provides an alternative to model validation for testing the consistency of hypotheses with data. Model based methods implicitly extrapolate the experimental data using the modeling assumptions. This extrapolation provides, in effect, new data measurements which are then used to invalidate a larger set of plants and, thus, controllers. Controller validation makes a clear distinction between controllers falsified by the actual data and controllers falsified by assumptions.

The non-conservative nature of the controller validation approach can be quickly demonstrated. As already discussed above, in model validation we start with a nominal plant model and an assumed model uncertainty set. A robust controller is one which provides the desired level of performance for any plant model, say $\mathsf{P}_{model}$ in a somewhat larger uncertainty set $\mathsf{P}_{\text{unc}}$, i.e.,

$$\mathsf{P}_{\text{unc}} \cap \mathsf{K}_{\text{robust}} \subset \mathsf{T}_{\text{spec}}. \tag{18}$$

Model validation asks whether or not the experimental data $\mathsf{P}_{\text{data}}$ (embedded in $\mathcal{R} \times \mathcal{Y} \times \mathcal{U}$) is consistent with the assumed uncertainty set. If $\mathsf{P}_{\text{data}} \subset \mathsf{P}_{\text{unc}}$, the model is validated; if not, the model is invalidated by the data.

Obviously, model validation can be used to demonstrate that a controller is *not-demonstrably unrobust*. It does so using the following argument:

$$(\mathsf{P}_{\text{data}} \subset \mathsf{P}_{\text{unc}}) \;\&\; (\mathsf{P}_{\text{unc}} \cap \mathsf{K}_{\text{robust}} \subset \mathsf{T}_{\text{spec}}) \tag{19}$$

implies

$$\mathsf{P}_{\text{data}} \cap \mathsf{K}_{\text{robust}} \subset \mathsf{T}_{\text{spec}}. \tag{20}$$

But it is clear (because of the possibly strict inclusion of the data set $\mathsf{P}_{\text{data}}$ within the model set $\mathsf{P}_{\text{unc}}$) that equation (20), which is the controller validation condition, can be satisfied even if (19) is not. Thus, the model validation approach to robust controller design requires both conditions of (19) be satisfied to conclude that a controller is *not-demonstrably unrobust* while controller validation approach requires only the weaker condition in (20). Consequently, the controller validation approach to robust control design is simpler and more direct. It is also less conservative.

# 4 Example

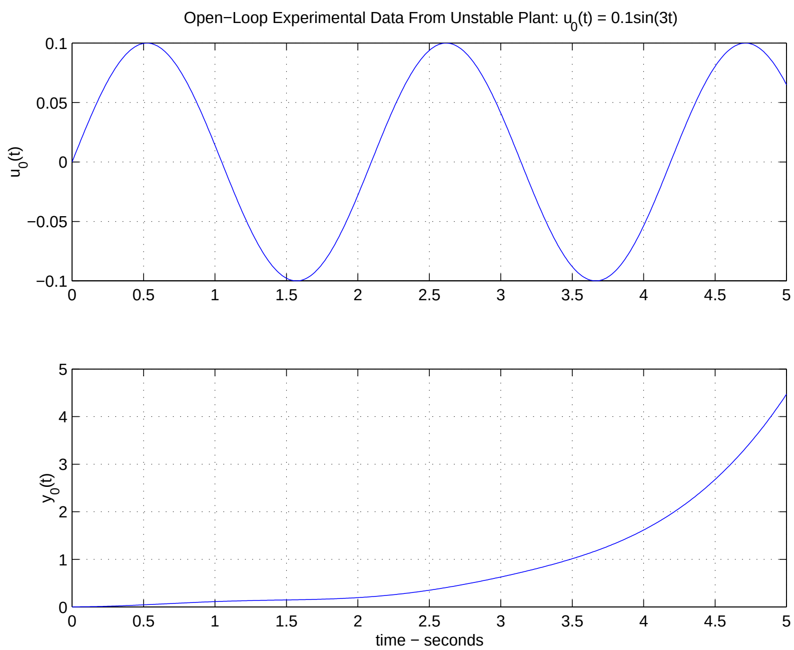


Figure 3: Input-output data from an open-loop unstable plant.

A brief example illustrates the salient points. Consider again the setup of Figure 1. Suppose that we have open-loop input-output measurements $(u_0, y_0)$ from a plant which, though its precise characteristics are uncertain, is known to be unstable. For purposes of illustration let the uncertain plant be LTI and given by

$$P(s) = \frac{51s + 950}{(s + 1000)(s - 1)} \tag{21}$$

Plots of the input-output data are shown in Figure 3. Further, suppose we have controller (known to provide closed-loop stability)

$$C(s) = \frac{9.9s + 23}{s + 0.16}, \tag{22}$$

and we wish to use the available data to evaluate its ability to meet the performance specification in (9) with the frequency weightings given as

$$W_1(s) = \frac{s^2 + 2 * 0.707 * 2s + 2^2}{s^2 + 2 * 0.85 * 0.2s + 0.2^2}, W_2(s) = 0. \tag{23}$$

We have assumed the second frequency weighting is zero to simplify the example. This performance specification requires that the control error be small for frequencies below about 1 rad/sec. Specifically, we desire $|S(j\omega)| \;\Box\; |W_1^{-1}(j\omega)|, \forall\omega$ where $S$ is the sensitivity. The frequency response of $W_1$ is shown in Figure 4.

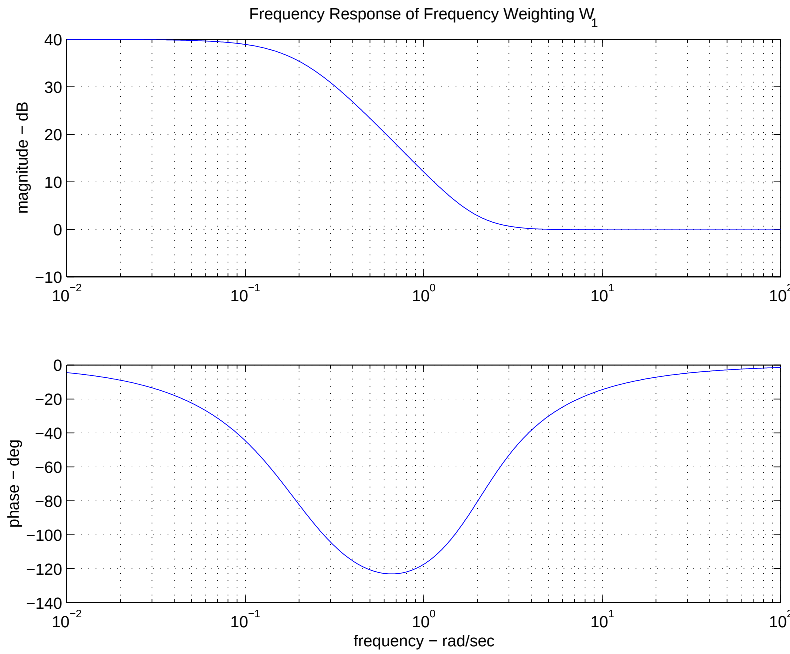


Figure 4: Performance specification frequency dependent weighting.

There are two possible approaches to validating the ability of the controller in (22) to meet the given performance specification. First, we could use the more traditional model validation approach summarized in equations (19) and (20). To do so, we assume that the uncertain plant of (21) can be modeled using a nominal plant model $P_0$ and associated uncertainty structure

$$P(s) = (1 + W_p(s)\Delta(s))\, P_0(s) \equiv \left(1 + \frac{0.1(s+1)}{0.001s+1}\Delta(s)\right)\frac{1}{s-1}. \tag{24}$$

The uncertainty is modeled as unstructured and multiplicative. To validate the model of (24) using the available experimental data, we check

$$\|w_p^{-1} * (y_0 - p_0 * u_0)\|^2_{L_2[0,t]} \;\Box\; \|p_0 * u_0\|^2_{L_2[0,t]}, \forall t > 0 \tag{25}$$

where $w_p^{-1}$ and $p_0$ denote the impulse responses of $W_p^{-1}(s)$ and $P_0(s)$, respectively. Figure 5 shows the results of the model validation test summarized by (25). The data validates the model and associated uncertainty structure; this establishes the first condition of (19). Application of the performance robustness theorem [19] establishes, however, that the second condition of (19) is not satisfied. This is easily seen by inspecting the plot of the structured singular value of the appropriately augmented system shown in Figure 6 where the maximum value is nearly two (performance robustness would require a peak value less than unity). At this point, one might mistakenly conclude that the experimental data has invalidated the controller (22). This is not the case, as is shown by a direct test of the controller using the data alone; i.e., the one-step controller validation process. The controller validation test for this example is

$$\|w_1 * c^{-1} * u_0\|^2_{L_2[0,t]} \square \|c^{-1} * u_0 + y_0\|^2_{L_2[0,t]}, \forall t > 0 \tag{26}$$

The results of this test are shown in Figure 7. The data does not invalidate this controller. Indeed, for the actual plant this controller results in the sensitivity being fully bounded by $W_1^{-1}(j\omega)$; see Figure 8. The model validation approach rejects the controller even though, in this example, it is fully capable of meeting the desired performance specification. It does so because the *assumed* uncertainty structure of (24), though not invalidated by the data (because the true plant falls within the structure and bounds), allows the potential of many input-output data pairs that the plant itself cannot produce. It is these *assumed* input- output pairs, not the actual data, that in this example exclude the controller when using the two-step model validation approach. For example, with $\Delta(s) = (s - 500)/(s + 501)$ in (24), whose magnitude is bounded by unity, the controller in question does not meet the performance requirement.

This example, though a bit simplistic and contrived, serves to illustrate the non-conservatism of the controller validation approach. It also shows that controller validation allows the control designer to understand what verdict the available data alone, as opposed to assumptions, renders about a potential control law and its ability to meet desired performance specifications.

## 5 Conclusions

This paper expands upon the unfalsified control concept by making a study of the connections with model validation and controller validation. Model validation is seen as a special case of controller validation. Controller validation is shown to provide a non-conservative approach to testing controller robustness hypotheses against data. Irrespective of whether noise considerations may have played a prior role in forming hypotheses to be validated, the validation process itself remains nevertheless deterministic since, *a posteriori*, the performance goal, the data, and the controller are deterministic. One of the broader implications of the theory is that experimental validation of candidate control laws is possible without a plant model and without the risk or inconvenience of physically connecting these controllers to the plant.

REFERENCES

1. A. Helmicki, C. Jacobson, and C. Nett. Control oriented system identification: A worst-case/deterministic approach in $H_\infty$. *IEEE Trans. Autom. Control*, 36(10):1163–1176, 1991.

2. R.L. Kosut, M.K. Lau, and S.P. Boyd. Set membership identification of systems with parametric and nonparametric uncertainty. *IEEE Trans. Autom. Control*, 37(7):929–941, 1992.

3. G.C. Goodwin, M. Gevers, and B. Ninness. Quantifying the error in estimated transfer functions with application to model order selection. *IEEE Trans. Autom. Control*, 37(7):913–928, 1992.

4. R. L. Kosut. Uncertainty model unfalsification: A system identification paradigm compatible with robust control design. In *Proc. IEEE Conf. on Decision and Control*, volume 1, pages 3492–3497, New Orleans, LA, December 13–15, 1995. IEEE Press, New York.

5. R. Smith and J. Doyle. Model validation – a connection between robust control and identification. *IEEE Trans. Autom. Control*, 37(7):942–952, 1992.

6. R. Smith and J. Doyle. Model validation and parameter identification for systems in $H_\infty$ and $l_1$. In *Proc. American Control Conf.*, pages 2852–2856, 1992.

7. K. Poolla, P. Khargonekar, A. Tikku, J. Krause, and K. Nagpal. A time-domain approach to model validation. *IEEE Trans. Autom. Control*, 39(5):951–959, 1994.

8. J.C. Willems. Paradigms and puzzles in the theory of dynamical systems. *IEEE Trans. Autom. Control*, AC-36(3):259–294, 1991.

9. S. Skogestad and I. Postlethwaite. *Multivariable Feedback Control.* Wiley, New York, 1996.

10. M.G. Safonov and T.C. Tsao. The unfalsified control concept: A direct path from experiment to controller. In B.A. Francis and A. Tannenbaum, editors, *Feedback Control, Nonlinear Systems, and Complexity.* Springer-Verlag, New York, 1995.

11. T. C. Tsao. *Set Theoretic Adaptor Systems.* PhD thesis, University Of Southern California, Los Angeles, CA, December 1993.

12. Thomas F. Brozenec. *Controller Invalidation, Identification, and Learning.* PhD thesis, University Of Southern California, Los Angeles, CA, December 1996.

13. M.G. Safonov and T.C. Tsao. The unfalsified control concept and learning. *IEEE Trans. Autom. Control*, AC-42(6):843–847, 1997. IEEE Press, New York.

14. G. Zames. On the input–output stability of time-varying nonlinear feedback systems — Parts I and II. *IEEE Trans. Autom. Control*, AC-15(2 and 3):228–238 and 465–467, April and July 1966.

15. I.W. Sandberg. On the $L_2$-boundedness of solutions of nonlinear functional equations. *Bell System Technical Journal*, 43(4):1581–1599, 1964.

16. I.W. Sandberg. A frequency domain condition for the stability of feedback systems containing a single time-varying nonlinear element. *Bell System Technical Journal*, 43(4):1601–1608, 1964.

17. M. G. Safonov. *Stability and Robustness of Multivariable Feedback Systems.* MIT Press, Cambridge, MA, 1980.

18. P.A. Ioannou and A. Datta. Robust adaptive control: Design, analysis and robustness bounds. In P.V. Kokotovic, editor, *Foundations of Adaptive Control*, pages 71–152. Springer-Verlag, New York, 1991.

19. J. C. Doyle, J. Wall, and G. Stein. Performance and robustness analysis for structured uncertainty. In *Proc. IEEE Conf. on Decision and Control*, pages 629–636, Orlando, FL, December 8–10, 1982. IEEE Press, New York.

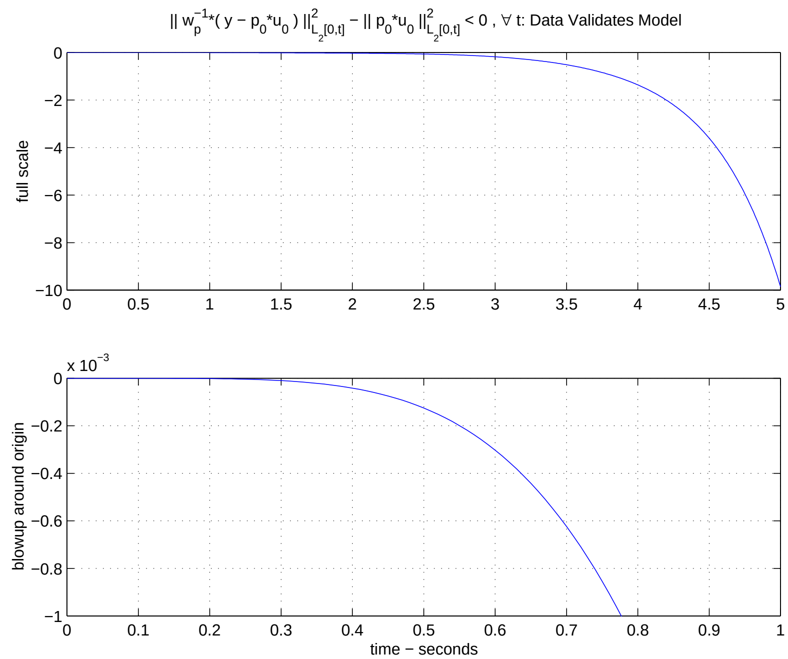


Figure 5: Model validation results for example.

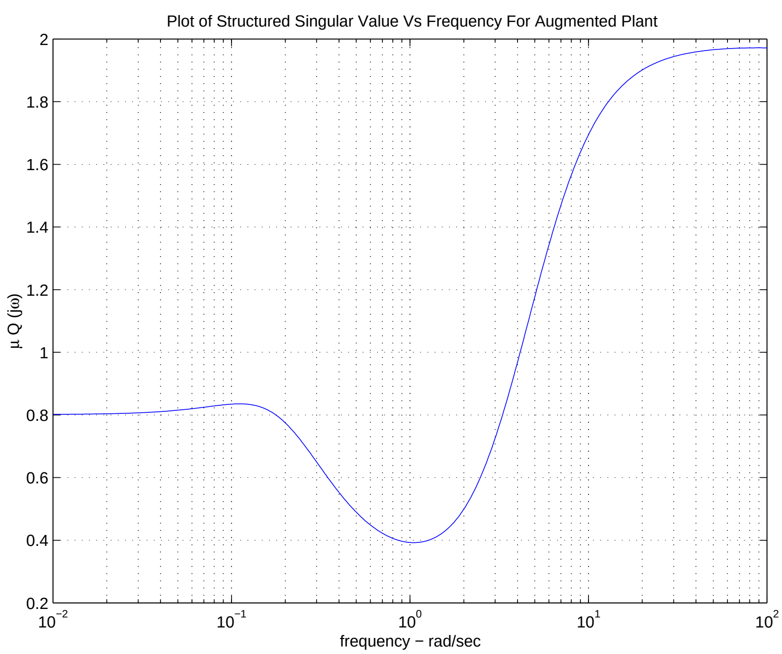


Figure 6: Structured singular value vs. frequency for performance robustness check.

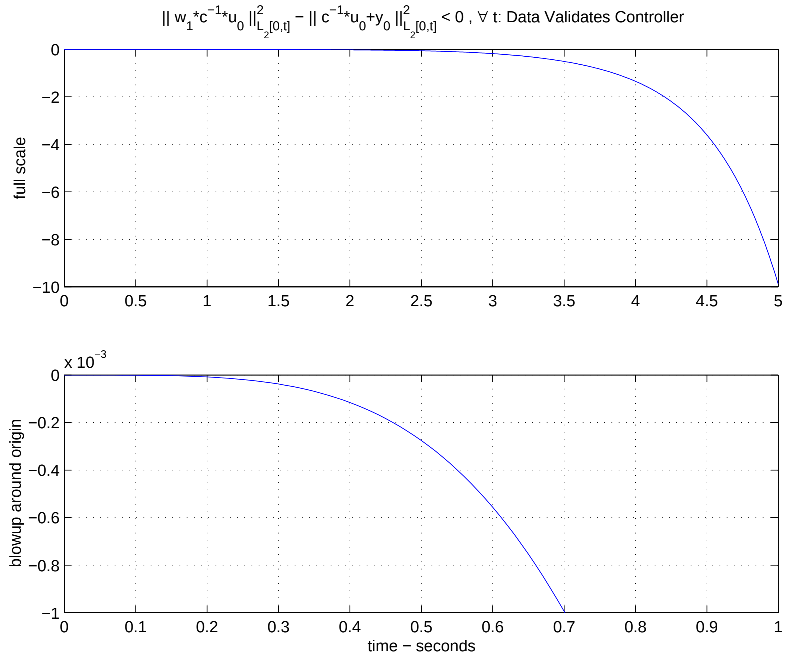


Figure 7: Controller validation results for example.

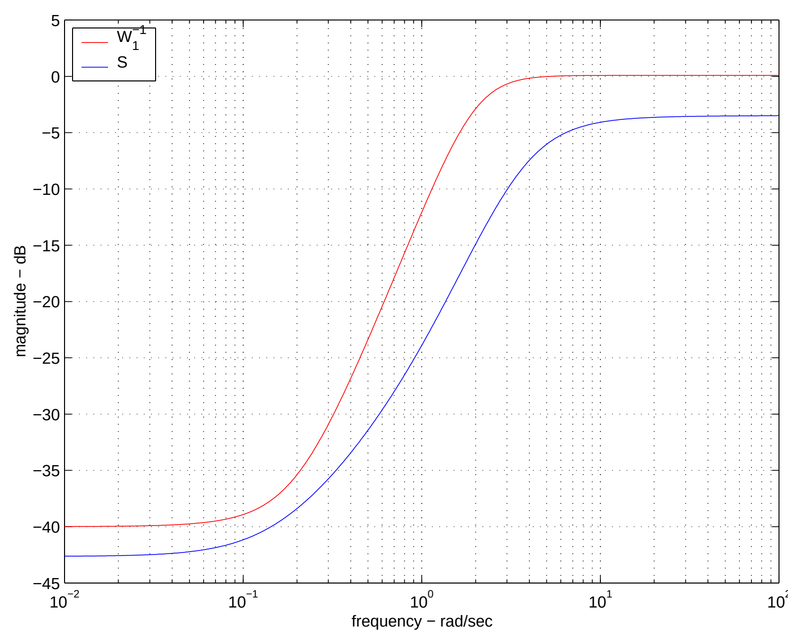


Figure 8: Sensitivity and bound for example.