

# Recent Advances in Robust Control, Feedback and Learning\*

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**Abstract.** As robust control theory has matured, a key challenge has been the need for a more flexible theory that provides a unified basis for representing and exploiting *evolving* information flows from models, noisy data, and more. Our work on *unfalsified control* is providing a foundation for the development of such a theory. The results of research in progress are expected to facilitate the design of feedback control systems with the ability to better exploit evolving real-time information flows as they unfold, thereby endowing control systems with the intelligence to adapt to unfamiliar environments and to more effectively compensate for the uncertain and time-varying effects, equipment failures and other changing circumstances.

*“The essential matter is an intimate association of hypothesis and observation.”*

*Bertrand Russell — 1949*

## 1 Introduction

The robust multivariable control theory that has evolved over the past quarter century includes methods based on the  $H_\infty$   $\mu/K_m$ -synthesis, and BMI/LMI/IQC theories. The robust control theory offers a major improvement over earlier algebraic and optimal control methods. It has enabled the design of controllers with greater tolerance of uncertainty in system model and, hence, increased reliability. Commercial computer-aided control synthesis tools like those introduced by Chiang and Safonov [14,15], Balas et al. [16] and Gahinet et al. [17] have made robust control synthesis routine, and because of this aerospace and industrial applications have now become commonplace. Further, on-going improvements based on LMI/IQC robust control problem formulations are continuing to expand the range of problems that can be cast and solved within the robust control framework (e.g., [18]).

Yet, despite the assurances of greater uncertainty tolerance and better reliability, the existing  $H_\infty$   $\mu/K_m$ -synthesis, and BMI/LMI/IQC techniques for robust control design have an Achilles heel: They are introspective theories. They derive their conclusions based on assumed prior knowledge of models and uncertainties. They are dependent of the premise that uncertainty models are reliable, and they offer little guidance in the event that experimental data either invalidates prior knowledge of uncertainty bounds or, perhaps, provides evidence of previously unsuspected patterns in the data. That is, the standard  $H_\infty$   $\mu/K_m$ -synthesis, and BMI/LMI/IQC robust control techniques fail in the all too common situation in which prior knowledge is poor or unreliable.

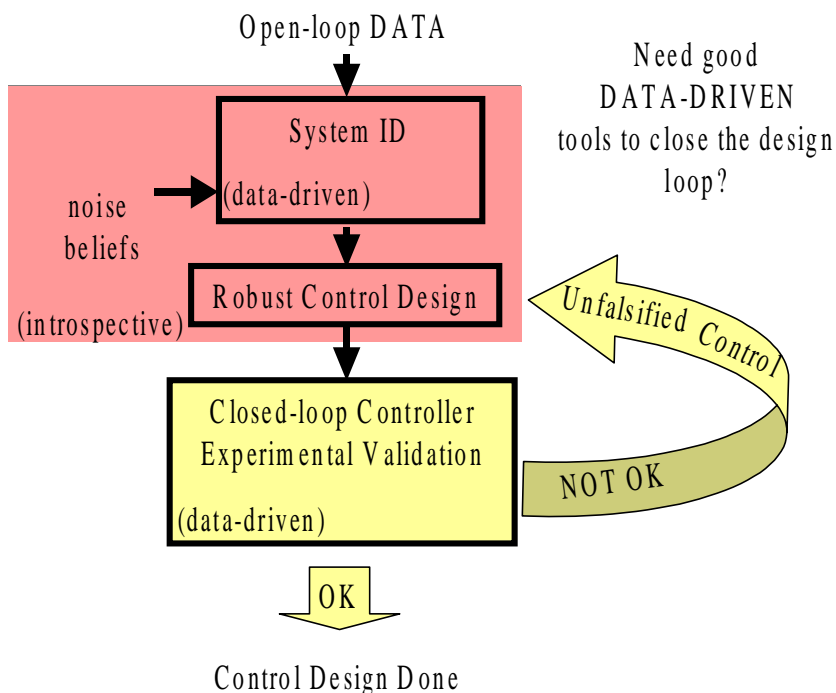
*Data-driven* design tools are needed to make the overall robust control design process more complete and reliable. Ideally, these tools should incorporate mechanisms for evaluating the design implications of each new experimental data point, and for directly integrating that information into the mathematics of the robust control design process to allow methodical update and re-design of control strategies so as to accurately reflect the implications of new or evolving experimental data. Recent thrusts in this direction are control-oriented identification theory and [19–38] and, more recently, unfalsified control [39–43]. While both theories are concerned with the difficult problem of assimilating real-time measurement data into the otherwise introspective process of robust control design, the unfalsified control theory is a particular interest because it directly and precisely characterizes the control design implications of experimental data.

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## 2 Data-Driven Robust Control Design



**Fig. 1.** The data-driven theory of unfalsified control closes data-driven portion of the design loop by focusing squarely and precisely on the control design implications of data.

Validation — or more precisely *unfalsification* — of hypotheses against physical data is the central aspect of the process of scientific discovery. This validation process allows scientists to sift the elegant tautologies of pure mathematics in order to discover mathematical descriptions of nature that are not only for logically self-consistent, but also consistent with physically observed data. This data-driven process of validation is also a key part engineering design. Successful engineering design techniques inevitably arrive at a point where pure introspective theory and model-based analyses must be tested against physical data. But, in control engineering in particular, the validation process is one that has been much neglected by theoreticians. Here, the theory tying control designs to physical data has for the most part focused on pre-control-design ‘system identification’. Otherwise, the mathematization of the processes of post-design validation and re-design has remained relatively unexplored virgin territory. In particular, a satisfactory quantitative mathematical theory for direct feedback of experimental design-validation data into the control design process has been lacking, though this seems to be changing with the recent introduction of a theory of unfalsified control [43].

### Theory: Validation and Unfalsification

Unfalsified control is essentially a data-driven adaptive control theory that permits learning based on physical data via a process of elimination, much like the candidate elimination algorithm of Mitchell [44,45]. The theory concerns the feedback control configuration in Figure 2. As always in control theory, the goal is to determine a control law  $K$  for the plant  $P$  such that the closed-loop system response, say  $T$ , satisfies given specifications. Unfalsified control theory is concerned with the case in which the plant is either unknown or is only partially known and one wishes to fully utilize information from measurements in selecting the control law  $K$ . In the theory of unfalsified control, learning takes place when new information in measurement data enables one to eliminate from consideration one or more candidate controllers.

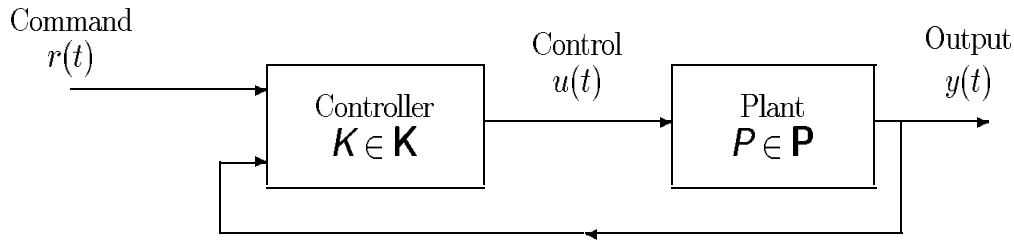


Fig. 2. Feedback control system.

The three elements that define the unfalsified control problem are (1) plant measurement data, (2) a class of candidate controllers, and (3) a performance specification, say  $T_{spec}$ , consisting of a set of admissible 3-tuples of signals  $(r, y, u)$ . More precisely, we have the following.

**Definition [43]** A controller  $K$  is said to be **falsified** by measurement information if this information is sufficient to deduce that the performance specification  $(r, y, u) \in T_{spec} \forall r \in \mathcal{R}$  would be violated if that controller were in the feedback loop. Otherwise, the control law  $K$  is said to be **unfalsified**.  $\square$

To put plant models, data and controller models on an equal footing with performance specifications, these like  $T_{spec}$  are regarded as sets of 3-tuples of signals  $(r, y, u)$  — that is, they are regarded as relations in  $\mathcal{R} \times \mathcal{Y} \times \mathcal{U}$ . For example, if  $P : \mathcal{U} \rightarrow \mathcal{Y}$  and  $K : \mathcal{R} \times \mathcal{Y} \rightarrow \mathcal{U}$  then

$$P = \left\{ (r, y, u) \mid y = Pu \right\}$$

$$K = \left\{ (r, y, u) \mid u = K \begin{bmatrix} r \\ y \end{bmatrix} \right\}.$$

And, if  $J(r, y, u)$  is a given loss-function that we wish to be non-positive, then the performance specification  $T_{spec}$  would be simply the set

$$T_{spec} = \left\{ (r, y, u) \mid J(r, y, u) \leq 0 \right\}. \quad (1)$$

On the other hand, experimental information from a plant corresponds to *partial* knowledge of the plant  $P$ . Loosely, data may be regarded as providing a sort of an “interpolation constraint” on the graph of  $P$  — i.e., a ‘point’ or set of ‘points’ through which the infinite-dimensional graph of dynamical operator  $P$  must pass.

Typically, the available measurement information will depend on the current time, say  $\tau$ . For example, if we have complete data on  $(u, y)$  from time 0 up to time  $\tau > 0$ , then the *measurement information* is characterized by the set [43]

$$P_{data} \triangleq \left\{ (r, y, u) \in \mathcal{R} \times \mathcal{U} \times \mathcal{Y} \mid P_\tau \begin{bmatrix} (u - u_{data}) \\ (y - y_{data}) \end{bmatrix} = 0 \right\} \quad (2)$$

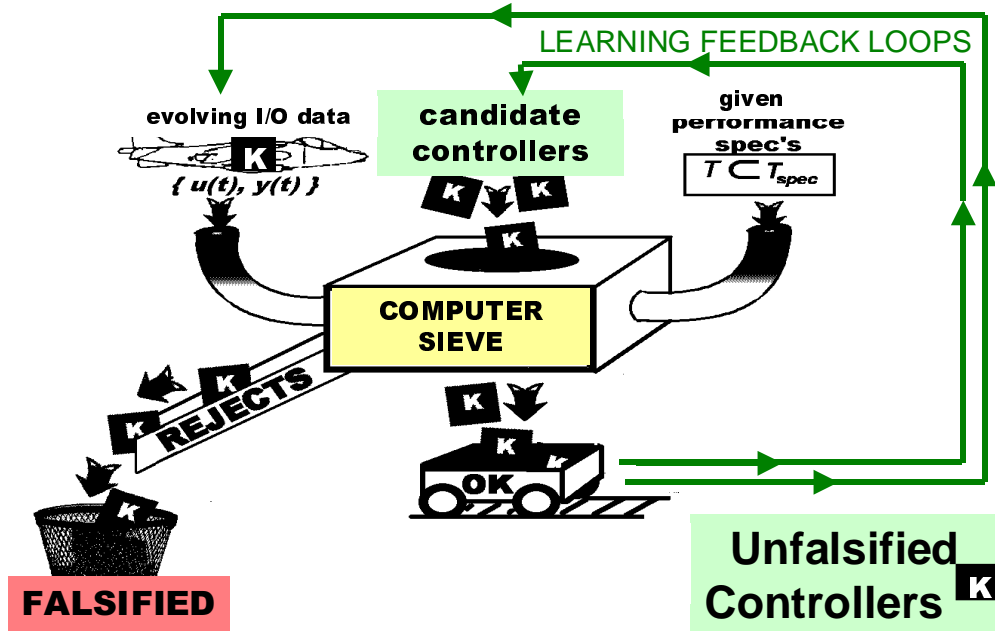
where  $P_\tau$  is the familiar time-truncation operator of input-output stability theory (cf. [46,47]), viz.,

$$[P_\tau x](t) \triangleq \begin{cases} x(t), & \text{if } 0 \leq t \leq \tau \\ 0, & \text{otherwise.} \end{cases}$$

The main result of unfalsified control theory is the following theorem which gives necessary and sufficient conditions for past open-loop plant data  $P_{data}$  to falsify the hypothesis that controller  $K$  can satisfy the performance specification  $T_{spec}$ .

**Unfalsified Control Theorem [43]** A control law  $K$  is unfalsified by measurement information  $P_{data}$  if, and only if, for each triple  $(r_0, y_0, u_0) \in P_{data} \cap K$ , there exists at least one pair  $(\hat{u}_0, \hat{y}_0)$  such that  $(r_0, \hat{y}_0, \hat{u}_0) \in P_{data} \cap K \cap T_{spec}$ .  $\square$

The unfalsified control theorem says simply that controller falsification can be tested by computing an intersection of certain sets of signals. A noteworthy feature of the unfalsified control theory is that a controller need not be in the loop to be falsified. Broad classes of controllers can be falsified with open-loop plant data or even data acquired while other controllers were in the loop. Adaptive control is achieved within this framework by using the unfalsification process as the key element of a supervisory controller (cf. [48,49]). The supervisor switches an unfalsified controller into the feedback loop whenever the current controller in the loop is amongst those falsified by observed plant data — see Fig. 3.



**Fig. 3.** The unfalsification process mathematically sifts controllers to find those that are consistent with both performance goals and physical data. It plays the role of a ‘supervisor’ that chooses one of the currently unfalsified controllers to put in the aircraft’s control loop.

### Conceptual Challenges and Controversies

*“Heavier-than-air flying machines are impossible.”*

*Lord Kelvin, President, British Royal Society, 1895*

A typical initial response from knowledgeable academic researchers has been to dismiss unfalsified control theory out of hand as a sort of mathematical ‘snake oil’. The claim that unfalsified control permits control design without a plant models has tended to be regarded as too outlandish to be taken seriously. Certainly it is true that unfalsified control theory has its limitations — and that the theory needs improvement. But, these ‘snake oil’ objections to unfalsified control have been fallacious — based on intuition derived from inappropriate analogies. However, thoughtful control theorists have been genuinely surprised and impressed by the simplicity and power of the unfalsified control theory as a mathematical basis for explaining feedback and learning, as a practical method for designing more reliable adaptive controllers, and as a data-driven technique for off-line tuning of non-adaptive feedback control gains.

Following are some typical examples of fallacious objections to unfalsified control:

1. *Unfalsified control seems unacceptably weak in its conclusion of mere unfalsification, given that familiar theories of control seem to offer stronger predictions like global stability and optimality derived deductively through the analysis of models even without the aid of validating data.*

However, this objection fails to recognize the fundamental distinction between conclusions deduced from model or assumptions and conclusions obtained via a mathematical analysis of experimental data: Beliefs about the validity of models and assumptions, and therefore any conclusions based in whole or in part on mathematical models, are not necessarily scientific truths — they might be falsified by future physical data. Unfalsified control augments introspective model-based robust control design methods by providing a quantitative methodology for closing the loop on the control design process when, at the experimental validation stage, the model-based robust control design proves to be unsatisfactory.

2. *Unfalsified control theory incorporates no sensor noise models, and therefore must perform poorly.*

In unfalsified control, control system performance criteria are framed directly in terms of observed vari-

ables. This runs counter to established intuition for some control theorists who have grown overly comfortable with traditional control problem formulations that characterize physical measurements as noise-corrupted observations of the unseen internal ‘reality’ of a ‘true’ model. Given this tradition of regarding models and noise beliefs as having more truth content than physical observations, it has been easy to succumb to the temptation to assume that the models and their noise are the ‘true’ explanation of observed physical data. The fact is that unfalsified control does accommodate noise quite well when suitably ‘soft’ performance criteria are employed. And, moreover, the performance criteria used in unfalsified control are quite flexible, and may even be associated with, and derived from, traditional stochastic noise hypotheses.

3. *Unfalsified adaptive controllers are claimed to be quick and sure-footed in discovering good control gains, even for non-minimum-phase plants. This is too good to be true, and so must be false.*

This erroneous belief apparently arises from knowledge that popular model reference adaptive control schemes are relatively sluggish and have been proved to fail for non-minimum-phase plants. But, unfalsified control is not model reference control and does not suffer its limits. It is fast and sure-footed because, unlike other adaptive schemes such as model reference control, unfalsified control theory is based on a precise analysis of the mathematical constraints induced by (1) performance criteria, (2) physical data and (3) the control law.

4. *Unfalsified adaptive controllers make use of the inverse of the controller transfer function, so they must be sensitive to plant model error and noise.*

Apparently some control theorists are confused by the thought of controller inversion, since it triggers unrelated memories concerning known difficulties with controllers that rely on inverting the plant itself. In any case, the controller inversion is not an essential part of theory but merely one of several conceivable ways to perform the computations that are necessarily associated with any logically correct test of consistency a controller hypothesis against performance goals and physical data.

5. *Anyone who claims to be able to design a controller without a mathematical model of the plant must be a charlatan, ergo unfalsified control must be the product of charlatans.* A seasoned reviewer of our paper [43] put this objection very eloquently, saying

“Modern science is model-based. If to abandon models is not to abandon mathematical science. . .”

In such arguments, one may perhaps glimpse elements of the conflict between the introspective belief-driven methods of ancient Platonic science and the observation-driven methods of post-Galileo experimental science. Such fallacious views are based on the knowledge that established control theories (like the theories of Plato) are heavily introspective, relying on models and assumptions to deduce such things as stability or optimality. They fail to take account of the fact that it may be possible to devise dependable data-driven methods for discovering good control designs without models via careful mathematical analysis of the logical implications of experimental observation alone. They also fail to notice that models play a key role in the unfalsified control method, but that the models used in unfalsified control are models of candidate controller hypotheses, not plants. Indeed, a careful comparison of unfalsified control theory and system identification theory shows that they are conceptually the same, except that in unfalsified control one identifies controller models (not plant models) and performance criteria involve closed-loop control errors (not open-loop plant model errors).

The foregoing are representative of fallacious criticisms that have been levied by experienced nay sayers who were convinced that unfalsified control is a heavier-than-air theory that could not possibly fly. They were wrong. Unfalsified control theory is taking off. It is proving to be a legitimate and useful vehicle for data-driven control design, even if the initial flights have been somewhat ungainly and short.

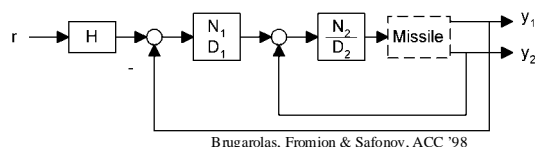
## Design Studies

The 1895 declaration of the Lord Kelvin notwithstanding, the Wright brothers flew in 1903. As for unfalsified control theory, accumulating case study evidence is likewise proving the intuition of experienced nay sayers to be wrong. Over the past three years several design studies have confirmed the theoretical expectation that unfalsified control can be useful in closing the outer data-driven loop on the control design process. Unfalsified

control theory has proved effective in applications involving both off-line controller gain tuning and in real-time adaptive control design studies. These initial design studies have helped us to better understand the potential of the unfalsified control theory, as well as limitations of the current theory.

**Missile Autopilot** One design study that we conducted involved using an unfalsified controller to robustly discover PID controller gains for an adaptive missile autopilot ‘on the fly’ in real-time [9]. Figure 4 summarizes the results of the missile design. In all trials, the response of the adaptive loops was swift and sure-footed — in stark contrast to what would be expected from traditional model reference adaptive methods.

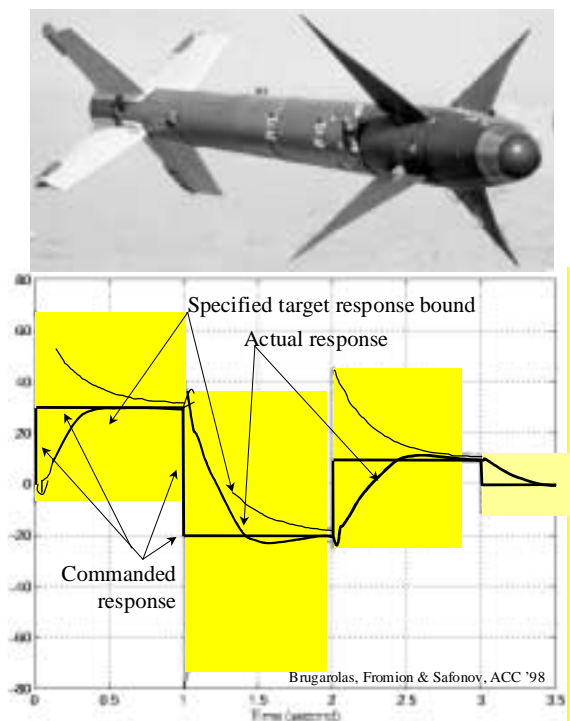
- Learns control gains
- Adapts quickly to compensate for damage & failures
- Superior performance



Brugarolas, Fromion & Safonov, ACC '98

Unfalsified adaptive missile autopilot:

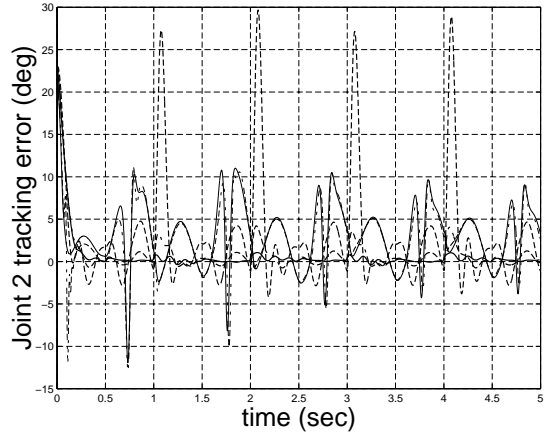
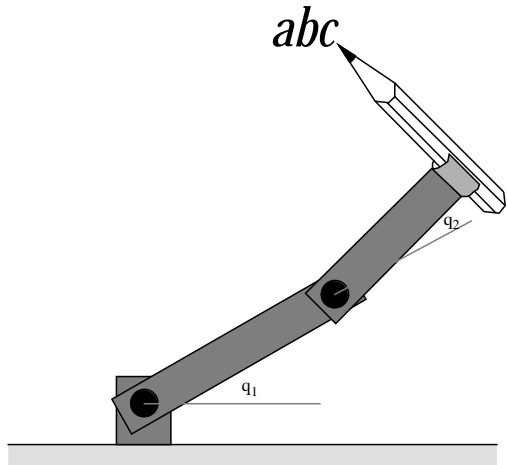
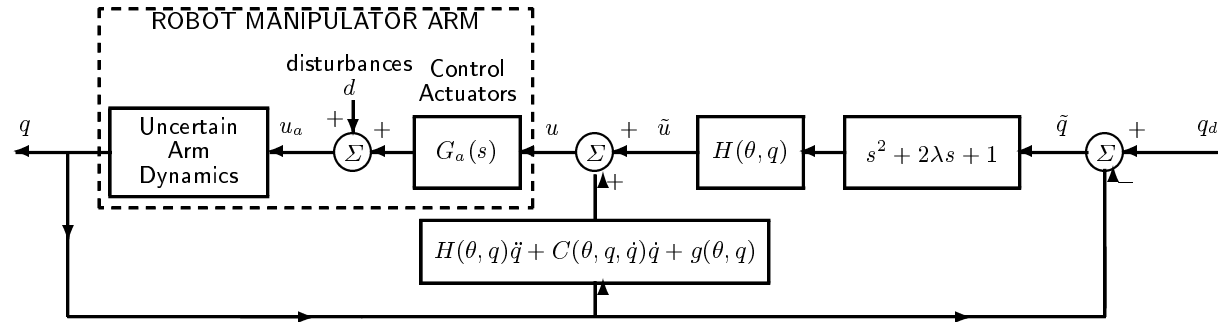
- discovers stabilizing control gains as it flies, nearly instantaneously
- maintains precise sure-footed control



**Fig. 4.** A data-driven unfalsified missile controller would have abilities to adaptively discover solutions in real-time to compensate for sudden in-flight changes and damage.

**Robot Manipulator Arm** We used to unfalsified methodology to adaptively tune the parameters of a nonlinear ‘computed-torque’ controller for a robot manipulator arm [50,5] — see Fig fig:robot. The arm proved to be capable of a quick and reliable control response despite large and sudden variations in load mass. Again, the controller performed with precision, despite noise, dynamical actuator uncertainties and without prior knowledge of the plant model or its parameters. Results for the robot design were surefooted and precise, with the controller maintaining an order of magnitude more precise control than a similar model-reference adaptive controller during widely fluctuating manipulator load variations; the controller was also more robust in that it was capable of maintaining precise control even during load variations that destabilized a similarly structured model-reference adaptive controller.

**Industrial Process Control** Although very few researchers other than ourselves have as yet examined unfalsified control methods, those who have taken this step have predictably confirmed the effectiveness of unfalsified control methods in several industrial process control applications. For example, Kosut [35] examined unfalsified controller for direct data-driven off-line control gain tuning under the assumption of



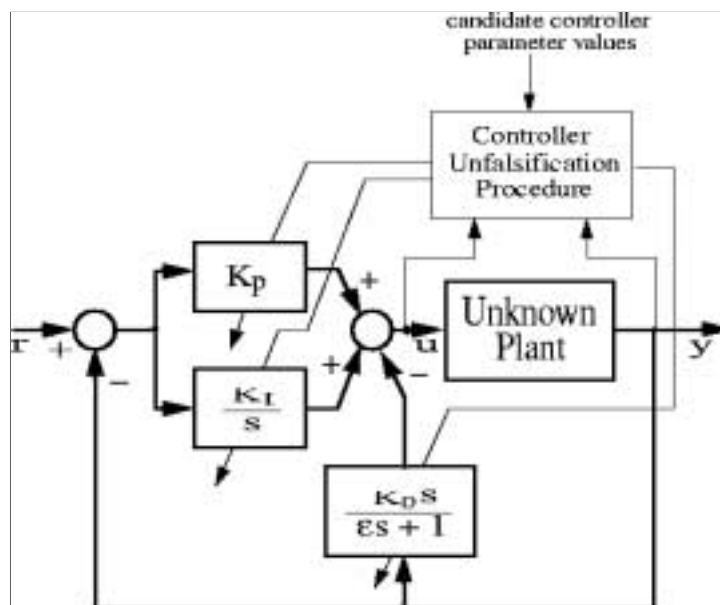
**Fig. 5.** Unfalsified control produced superior results for a nonlinear two-link robot manipulator subject to uncertain dynamics, noisy disturbances and abrupt changes in load mass. The two sluggishly smooth traces large amplitude signals in the plot are with a conventional adaptive controller used to adjust control gain-vector  $\theta(t)$ , and the two very low amplitude traces are for the unfalsified controller. The unfalsified controller had a much quicker, sure-footed and precise response without increased control effort.

a noise-free linear-time-invariant plant. Woodley, How and Kosut and used the theory with good result for data-driven discovery of good control gains for a laboratory control problem involving two spring-connected masses. Also, Collins and Fan [36] successfully used the unfalsified control methodology in a run-to-run setting to tune gains off-line in an industrial weigh-belt feeder control design study. More recently, there have been some promising adaptive control applications to machine control by Razavi and Kurfess [37,38] based on the unfalsified control methodology.

**Universal PID Controller** One application of the theory involved implementing a PID-based adaptive ‘universal’ controller implemented as MATLAB Simulink block based on the unfalsified theory [10] — see Fig 6. The controller was capable of sifting through a bank of candidate controllers in real-time, stabilizing an open-loop unstable plant without knowledge of the plant model despite sensor noise and without noticeable transients.

### 3 Summary

The main goal of unfalsified control theory has been to close the loop on the adaptive and robust control design processes by developing data-driven methods to complement traditional model-based methods for the design of robust control systems. The theory produced so far has served to strengthen the theoretical foundation



**Fig. 6.** In one study, we designed an adaptive ‘universal controller’ having a PID structure and based on the unfalsified control theory. Simulations using MATLAB Simulink showed that the adaptive unfalsification loops were so fast that the controller was able to stabilize an unstable plant without prior plant knowledge and without appreciable transients.

of control theory, paving the way for carefully reasoned extensions of feedback control theory into the realm of intelligent and learning systems. Unfalsified control theory lays the foundation for a more complete and rigorous understanding of feedback that focusing squarely on the quantitative design implications of physical data, thereby enabling the design of robust control systems with the ability to autonomously enhance both their robustness and their performance by exploiting real-time data as it unfolds. Such designs will be better able to compensate for uncertain and time-varying effects, equipment failures and other changing circumstances.

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