Book Review

A Course in Robust Control Theory: A Convex Approach—Geir E. Dullerud and Fernando G. Paganini (New York, NY: Springer-Verlag, 1999).
Reviewed by Michael G. Safonov

A dominant aspect of 1990's robust control research was the emergence of a new analysis/synthesis paradigm centered around convex optimization and, in particular, on the Linear Matrix Inequality (LMI). Lyapunov and Riccati equations H_2/H_∞ problem formulations that dominated the literature of the 1980's came to be seen as special cases of more flexible LMI problem formulations. The interplay of functional analysis and state- space optimal control theory proved to be seductively exotic, reinforcing and giving new impetus to the close links between control and advanced mathematics. Because progress in LMI robust control theory has been explosive, only books published in the past 3 or 4 years can hope to adequately document the phenomenon. The textbook of Dullerud and Paganini rises admirably to the challenge, starting from the basics of linear algebra and system theory and leading the reader through the key 1990's breakthroughs in LMI robust control theory. To keep things simple, the authors relegate the issue of robustness against nonlinear uncertainties to the citations, focusing attention squarely on the linear case.

The book begins with a brief Chapter 0 that should be useful to mathematicians having little or no control engineering background. This chapter introduces concerns and notation of control engineers, and gives an example of an engineering problem with uncertain dynamics as justification for robust control. The remainder of the book is pure mathematics, presented in a clean and simple theorem-proof style. Chapter 1 reviews the most basic elements of matrix theory and linear algebra, eigenvalues, singular values as well as analysis concepts such as convexity and topological separations that are crucial to understanding LMI's and their role in stability and robustness analysis. Chapter 2 covers the standard elements of state space system theory, including controllability/observability, realization, observers and observer-based controllers. Chapter 3 develops the relevant function space background, including the Banach algebras, Hilbert spaces, adjoint operators, and the Hardy spaces H_∞ and H_2. With the function space background in hand, the Chapter 4 compactly develops classical state-space concepts such as minimality, observers, Hankel operators, balanced realizations and model reduction. The notation here, and throughout the book, is tidy and terse, facilitating a focus on concepts. Systems and relations are compactly represented throughout the book by block system matrix notation such as

$$\hat{G}(s) = \left[\begin{array}{c|cc} A & B_1 & B_2 \\ \hline C_1 & D_{11} & D_{12} \\ C_2 & D_{21} & 0 \end{array}\right], \quad \hat{K} = \left[\begin{array}{c|c} A_K & B_K \\ \hline C_K & D_K \end{array}\right].$$

The development of LMI theory begins in earnest with Chapter 5, which opens with a derivation of a state-space LMI condition that provides necessary and sufficient conditions the existence of stabilizing controllers for a given plant. This is then tied to the Youla parameterization of all stabilizing controllers, including the necessary discussion of coprime matrices over the ring RH_∞ of stable rational transfer functions. Chapters 6 and 7, respectively, develop H_2 and H_∞ feedback synthesis in an LMI setting. The relationship between the LMI solution of the H_∞ controller synthesis and the Riccati inequality solution of Sampei, Mita and Nakamichi is treated via an exercise at the end of Chapter 7.

In the second half of their book, Dullerud and Paganini treat the topic of robustness analysis and controller synthesis for systems with structured uncertainty. Following standard practice in robust control theory, they cast robust control problem in the canonical framework of Fig. 1. Here

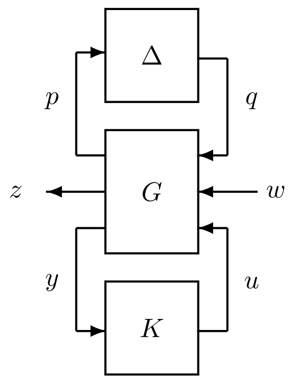

Fig. 1. Feedback system with uncertainty $\Delta = \text{diag}(\Delta_1, \ldots, \Delta_n)$.

G is the nominal plant model, K is the feedback controller, and $\Delta = \text{diag}(\Delta_1, \ldots, \Delta_n)$ is a block-diagonal 'structured uncertainty' which Dullerud and Paganini take to be linear and possibly time-varying.

In Chapter 8, stability robustness is examined from the 'topological separation of graphs' perspective. The problem of structured uncertainty is treated via the standard optimal 'diagonal-scaling' approach, which is known to be conservative for linear *time- invariant* Δ. However, the authors retrieve a measure of mathematical elegance by emphasizing some discoveries from the early 1990's by A. Megretski and S. Treil, and by J. Shamma, that allow one to regard diagonal-scaling as completely nonconservative in the case of linear *time-varying* Δ.

Many researchers would consider the seminal breakthroughs for LMI robust control synthesis to be the works

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of Packard *et al.* [1], [2], [3], who developed the basic methods used by to reduce standard robust controller synthesis problems to linear or, in some cases, bilinear matrix inequalities. A highlight of the book is Chapter 9, which provides a lucid development of the ideas of [1], [2] in a continuous-time setting, with emphasis on the deriving the LMI

$$\begin{bmatrix} \bar{N}_c & 0 \\ 0 & I \end{bmatrix}^* \begin{bmatrix} AY+YA^* & YC_1^* & B_1\Theta^{-1} \\ C_1Y & -\Theta^{-1} & D_{11}\Theta^{-1} \\ \Theta^{-1}B_1^* & \Theta^{-1}D_{11}^* & -\Theta^{-1} \end{bmatrix} \begin{bmatrix} \bar{N}_c & 0 \\ 0 & I \end{bmatrix} < 0$$

where $\bar{N}_c = \mathrm{null}(\begin{bmatrix} B_2^* & D_{12}^* \end{bmatrix})$ and the LMI variables are the matrices $Y = Y^* > 0$, $\Theta^{-1} = \mathrm{diag}(\theta_1, \ldots, \theta_m)^{-1}$. This LMI is both necessary and sufficient for the existence of robust controller for the plant $\hat{G}$ in the case of time-varying diagonally-structured uncertainty Δ when the measurement y is the full state of the plant G [2].

Chapters 10 and 11 of the book introduce the Integral Quadratic Constraint (IQC) perspective and examine robust synthesis for parameter varying systems, tying these topics to the LMI framework via the *S-procedure* of Yakubovich. These final chapters are followed by three appendices that amplify on several technical issues, and by a detailed index.

The book also has an extensive bibliography, though history buffs may find it somewhat difficult to navigate. Citations in the text are limited to a terse *Notes and References* section at the end of each chapter. A key matrix inequality result popularly associated with the names Finsler and Parrott (cf. [4]) is assigned no name; it is called simply Lemma 7.2. The authors cite the 1994 papers [5], [3] as the principal references for the continuous-time LMI H_∞ results of Chapter 7, but they do not cite the parallel work of Iwasaki and Skelton [6] nor do they trace the historical roots of the derivations to the parallel discrete-time derivations that appear in the 1991 paper of Packard *et al.* [1]. For further perspective, consider the edited anthology of El Ghaoui and Niculescu [7].

The book would make an excellent text for a two-semester or two-quarter course for first year graduate students beginning with no prior knowledge of state-space methods. Alternatively, for control students who already have a state-space background, one quarter would be sufficient for a course emphasizing the main concepts in the latter chapters of the book. Though engineering design examples are conspicuously absent, instructors wishing to emphasize theory should like the extensive set of well-designed mathematical exercises at the end of each chapter. More application-oriented competitors to this book are the 1998 books of Skelton, Iwasaki and Grigoriadis [8] and of Sánchez-Peña and Sznaier [9]; these cover some of the same ground, but without quite the terse mathematical simplicity and comprehensive scope that distinguishes the book of Dullerud and Paganini.

References

[1] A. Packard, K. Zhou, P. Pandey, and G. Becker. A collection of robust control problems leading to LMI's. In *Proc. IEEE Conf. on Decision and Control*, volume 2, pages 1245–1250, Brighton, England, December 11–13, 1991.

[2] A. Packard, K. Zhou, P. Pandey, J. Leonhardson, and G. Balas. Optimal, constant I/O similarity scaling for full- information and state-feedback problems. *Systems and Control Letters*, 19:271–280, 1992.

[3] A. Packard. Gain scheduling via linear fractional transformations. *Systems and Control Letters*, 22:79–92, 1994.

[4] J. Ly, M. G. Safonov, and F. Ahmad. Positive real Parrott theorem with application to LMI controller synthesis. In *Proc. American Control Conf.*, pages 50–52, Baltimore, MD, June 29–July 1, 1994.

[5] P. Gahinet and P. Apkarian. A linear matrix inequality approach to H_∞ control. *Int J. Robust and Nonlinear Control*, 4(4):421–448, July–August 1994.

[6] T. Iwasaki and R. E. Skelton. All Controllers for the General H_∞ Control Problem: LMI Existence Conditions and State Space Formulas. *Automatica*, 30(8):1307–1317, August 1994.

[7] L. El Ghaoui and S. Niculescu, editors. *Recent Advances in LMI Theory for Control.* SIAM, Philadelphia, PA, 2000.

[8] R. E. Skelton, T. Iwasaki, and K. Grigoriadis. *A Unified Algebraic Approach to Linear Control Design.* Taylor and Francis, London, 1998.

[9] R. S. Sánchez-Peña and M. Sznaier. *Robust Systems Theory and Applications.* Wiley, New York, 1998.