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CONTROLLER PARAMETER ADAPTATION ALGORITHM USING UNFALSIFIED CONTROL THEORY AND GRADIENT METHOD [1]

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Abstract: In this paper a new controller parameter adaptation rule is proposed. The proposed adaptation rule is derived via unfalsification control theory and gradient method without imposing many assumptions on the plant. Convergence of parameter is discussed. Numerical examples and simulation results are also provided.

Keywords: Learning control, adaptive control, adaptive systems, control algorithm, uncertain dynamic systems

1. INTRODUCTION

The implementation of intelligent control algorithms such as real-time parameter adaptation, model predictive control and on-line learning algorithms become easier with the progress in computer technology, which makes computation faster and cheaper. However, the algorithms impose some assumptions on the plant that could limit the range of application of the algorithm. The typical adaptive control problems assume the following four assumptions (Ioannou and Sun, 1996; Narendra and Annaswamy, 1989): i) a plant is LTI minimum phase system, and ii) an upper bound for plant order, iii) the relative degree of the plant and iv) the sign of the high frequency gain are known. While ii) is used to choose the order of the controller and iii) to set up a reference model, i) and iv) are required to prove global stability. However, as pointed out in (Narendra and Annaswamy, 1989), adaptive control is most needed in the cases where these assumptions do not hold. There have been lots of researches trying to relax the assumptions. The assumptions ii), iii) and iv) can be relaxed at the expense of additional complexity in the control and adaptive laws (Åström, 1980; Goodwin *et al.*, 1981; Goodwin and Sin, 1981; Morse, 1985; Nussbaum, 1983; Praly, 1984).

Universal controllers were proposed that require less prior information than the aforementioned (Fu and Barmish, 1986; Martensson, 1985). They plugged one controller in closed-loop and observed the behaviour of the resulting system. They searched over a dense set of controllers (Martensson, 1985) or a finite set of controllers (Fu and Barmish, 1986) while the observed behaviours violate a performance specification. However, they assumed the plant to be linear time-invariant although it is less restrictive than i), ii), iii) and iv). Safonov *et al.* (Safonov and Tsao, 1997) proposed a method called unfalsified control theory to build a controller satisfying a

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given performance with measured data from the plant without assuming LTI plant and showed some design studies (Brugarolas *et al.*, 1998; Jun and Safonov, 1999; Tsao and Safonov, 1999). While it is similar to universal controllers by (Fu and Barmish, 1986; Martensson, 1985) in the aspect that it checks violation of given performance specification and switches to a *good* one once it observes violation, it does not do time-consuming controller search.

However, the practical problem in the unfalsified control theory is that there is no systematic way to determine an appropriate initial controller set $\mathbf{K}$. The authors admits it is essential that it begin with a sufficiently rich class of ideas for candidate controllers $\mathbf{K}$ if the theory is to lead to a non-empty set of unfalsified controllers. For instance, if one starts with a small candidate controller set $\mathbf{K}$, he can eliminate the controllers violating the performance specification without much computational load. However, it is more probable that all controllers get falsified before the system reaches steady state and stability cannot be characterized. On the other hand, if one starts with a large candidate controller set, initial computation load is too big although it has more probability to reach steady state without having all candidate falsified. In this paper, a method to combine unfalsification concept and controller parameter adaptation based on gradient method dispensing with candidate controller set $\mathbf{K}$ is proposed.

This paper is organized as follows: Preliminary theoretical background will be briefly covered in Section 2. The problem to be solved in this paper will be formulated in Section 3. The main results will be provided in Section 4. Brief discussion on the result will be given in Section 5 and some numerical examples will be presented in Section 6. Final remarks will conclude the paper in Section 7.

2. PRELIMINARIES

In this section, a brief overview of unfalsified control theory is described. For further details about unfalsified control theory, refer to (Safonov and Tsao, 1997) and references therein. The formal definitions of *unfalsification* and *falsification* are as follows:

Definition 1. (Safonov and Tsao, 1997) A controller is said to be *falsified* by measurement information if this information is sufficient to deduce that the performance specification $(r, y, u) \in T_{spec}$ $\forall r \in \mathbb{R}$ would be violated if that controller were in the feedback loop. Otherwise, the controller is said to be *unfalsified.*

With the definitions of *unfalsification* and *falsification* one can state the following theorem in order to solve unfalsified control problem. Let the symbol K denote the set of triples (r, y, u) that satisfy the equations that define the behavior of controller. Denote by P_{data} the set of triples (r, y, u) consistent with past measurements of (u, y) – cf. (Safonov and Tsao, 1997).

Theorem 2. (Safonov and Tsao, 1997) A control law K is unfalsified by measurement information set P_{data} if, and only if, for each triple $(r_0, y_0, u_0) \in P_{data} \cap K$, there exists at least one pair (u_1, y_1) such that

$$(r_0, y_1, u_1) \in P_{data} \cap K \cap T_{spec} \qquad (1)$$

Fictitious reference signals occupy an important position in unfalsified control theory. Given measurements of plant input-output signals u and y, there may correspond for each candidate controller, say K_i, one or more fictitious reference signals $\tilde{r}_i(t)$. The $\tilde{r}_i$'s are hypothetical signals that would have exactly reproduced the measured data (u, y) if the candidate controller K_i had been in the feedback loop during the *entire* time period over which the measured data (u, y) was collected. Because the data (u, y) may have been collected with a controller other than K_i in the feedback loop, the fictitious reference signal $\tilde{r}_i$ is in general not the same as the actual reference signal $r(t)$. A candidate controller K_i is called *causally-left-invertible* if unique past values for its fictitious reference signal $\tilde{r}_i(t)$ are determined by past values of the open-loop data $u(t)$ and $y(t)$.

3. PROBLEM FORMULATION

Suppose that the control laws $K \in \mathbf{K}$ are linearly parameterized by an unspecified vector $\theta \in \mathbb{R}^n$ and

$$K(\theta) = \{(r, y, u) \mid u = K(\theta)(r - y)\}. \qquad (2)$$

Furthermore, assume that all control laws in $\mathbf{K}$ are causally-left-invertible. Therefore, there exists unique fictitious reference signal $\tilde{r}$ such that

$$\tilde{r}(t) = y(t) + K(\theta)^{-1}u(t). \qquad (3)$$

Suppose that at each time t, the performance specification set T_{spec} consists of the set of triples (r, y, u) satisfying an integral performance inequality of the form

$$\begin{aligned} J(\tau) \triangleq -\rho(\tau) + \int_0^\tau T_{spec}(r(\xi), y(\xi), u(\xi))d\xi \\ \leq 0, \quad \forall\, \tau \in [0, t] \end{aligned} \qquad (4)$$

where $\rho(\tau) \geq 0$ and $T_{spec}(\cdot, \cdot, \cdot)$ are chosen by the designer. By Theorem 2, the i-th candidate controller $K_i \in \mathbf{K}$ is unfalsified at time t by plant data $u(\tau), y(\tau)$, $(\tau \in [0,t])$ if, and only if,

$$\tilde{J}(\theta,\tau) \le 0, \quad \forall \tau \in [0,t] \tag{5}$$

where

$$\tilde{J}(\theta,\tau) \triangleq -\rho(\tau) + \int_0^\tau T_{spec}(\tilde{r}_i(\theta,\xi), y(\xi), u(\xi))d\xi. \tag{6}$$

$u(\xi)$ and $y(\xi)$, $(\xi \in [0,t])$ are measured past plant data, and $\tilde{r}_i(\theta,\xi)$ denotes the fictitious reference signal for the i-th controller K_i.

The condition (5) requires all past plant data and memory space for it grows as time increases. Therefore, the condition (5) is relaxed for finite time window with maximum length T, that is, the i-th candidate controller $K_i \in \mathbf{K}$ is unfalsified at time t by plant data $u(\tau), y(\tau)$, $(\tau \in [t_0,t])$ if, and only if,

$$\tilde{J}(\theta,\tau) \le 0, \quad \forall \tau \in [t_0,t] \tag{7}$$

where $t_0 = \max(0, t-T)$.

In order to achieve the performance given in Eq. (7), controller parameters should be adjusted so as to make the performance specification $\tilde{J}(\theta,t)$ negative. In other word, controller parameters should be adapted so as to lead the performance specification $\tilde{J}(\theta,t)$ to decrease when it is positive. The problem to be solved in this paper can be stated as follows:

Problem 3. Given $K(\theta)$ and $\tilde{J}(\theta,t)$, find a controller parameter adaptation rule which drives the value of performance specification (5) to decrease when it is positive.

4. CONTROLLER PARAMETER ADAPTATION

4.1 *Adaptation Rule*

Since our objective is to adjust controller parameters so as to satisfy the given performance specification (7), controller parameters should be adjusted in direction that the performance specification $\tilde{J}(\theta,t)$ decreases, viz.,

$$\dot{\theta} = -\gamma \nabla \tilde{J}(\theta,t) \tag{8}$$

where

$$\nabla \tilde{J}(\theta,t) \triangleq \left[\frac{\partial \tilde{J}(\theta,t)}{\partial \theta_1} \; \frac{\partial \tilde{J}(\theta,t)}{\partial \theta_2} \; \cdots \; \frac{\partial \tilde{J}(\theta,t)}{\partial \theta_n} \right]^T$$

is the gradient of $\tilde{J}(\theta,t)$ with respect to θ and $\gamma > 0$ is an arbitrary design constant. From Eq. (6), $\nabla \tilde{J}(\theta,t)$ can be evaluated as

$$\nabla \tilde{J}(\theta,t) = \int_0^t \frac{\partial T_{spec}(\tilde{r}(\theta,\xi), y(\xi), u(\xi))}{\partial \tilde{r}} \cdot \nabla \tilde{r}(\theta,\xi) d\xi \tag{9}$$

where

$$\nabla \tilde{r}(\theta,\xi) = -K(\theta)^{-1} \nabla K(\theta) \cdot K(\theta)^{-1} u(\xi),$$
$$\nabla K(\theta) \triangleq \left[\frac{\partial K(\theta)}{\partial \theta_1} \; \frac{\partial K(\theta)}{\partial \theta_2} \; \cdots \; \frac{\partial K(\theta)}{\partial \theta_n} \right]^T$$

since $u(t) = K(\theta)(\tilde{r}(t) - y(t))$.

Therefore, controller parameter adaptation rule can be expressed as

$$\dot{\theta} = \gamma \int_0^t \frac{\partial T_{spec}}{\partial \tilde{r}} K(\theta)^{-1} \nabla K(\theta) \cdot K(\theta)^{-1} u(\xi)\, d\xi. \tag{10}$$

If the Eq. (7) is convex with respect to θ, controller parameters converge to the value that achieves global minimum of $\tilde{J}(\theta,t)$ if the step size γ is sufficiently small. However, our objective is not to find the controller parameter which lead to the global minimum of $\tilde{J}(\theta,t)$ but to find a controller parameter which provides the given performance, that is, leads to the condition (7). Therefore, we stop controller parameter adaptation when $\tilde{J}$ is negative. Our controller parameter adaptation rule can be summarized as follows:

Controller Parameter Adaptation Rule:

$$\dot{\theta} = \begin{cases} \gamma \displaystyle\int_0^t \frac{\partial T_{spec}}{\partial \tilde{r}} K(\theta)^{-1} \nabla K(\theta) \cdot K(\theta)^{-1} u(\xi)\, d\xi, & \\ & \text{if } \tilde{J}(\theta,\tau) > 0,\ \exists \tau \in [t_0,t] \\ 0, & \text{if } \tilde{J}(\theta,\tau) \le 0,\ \forall \tau \in [t_0,t] \end{cases}. \tag{11}$$

where $t_0 = \max(0, t-T)$.

The following is a simple example of parameter adaptation law for proportional gain controller.

Example 4. Let $T_{spec} = (\tilde{r} - y)^2/2$, $\rho(\tau) = 0$ and $K(\theta) = \theta > 0$. Then, we have

$$\tilde{J}(\theta,\tau) = \frac{1}{2\theta^2} \int_0^\tau u(\xi)^2\, d\xi \tag{12}$$

Therefore, from Eq. (10), controller parameter adaptation rule can be said

$$\dot{\theta} = \begin{cases} \dfrac{\gamma}{\theta^3} \displaystyle\int_0^t u(\xi)^2\, d\xi, & \tilde{J}(\theta,\tau) > 0,\ \exists \tau \in [t_0,t] \\ 0, & \tilde{J}(\theta,\tau) \le 0,\ \forall \tau \in [t_0,t] \end{cases}. \tag{13}$$

In this example, we can notice that Eq. (12) is convex with respect to θ regardless of the value of measured data $u(t)$. So, we can say that controller parameter converges in the steady state with the adaptation rule (13) *if* there exists a controller parameter that satisfies the given performance specification.

Remark 5. Another performance specification with different structure from Eq. (6) can be used. For example, if

$$\tilde{J}(\theta,\tau) \triangleq -\rho(\tau) + T_{spec}(\tilde{r}_i(\theta,\tau), y(\tau), u(\tau)) \tag{14}$$

is used, then the adaptation rule without integral applies.

4.2 *Convergence*

Usually typical adaptive control problems such as model reference adaptive control (MRAC) and self-tuning regulator (STR) put some assumptions on the plant such as LTI minimum phase plant or known information on relative plant order and so on in order to guarantee convergence of parameters or global stability. On the other hands, goal of unfalsification disables us for claims about the plant's future behaviour, e.g., asymptotic stability (Safonov, 1996).

Some assumptions on the measured plant input-output data and performance specification are imposed for convergence issue. For convergence, the set

$$\Theta(t) = \left\{\theta \mid \tilde{J}(\theta, \tau) \le 0, \;\; \theta \in \mathbb{R}^n, \; \tau \in [t_0, t]\right\} \tag{15}$$

should not be empty for all $t \in \mathbb{R}$. However, this condition is *not a priori condition* since the future behaviour of the plant is not known, thus, it cannot be checked without measured plant data since the basis of this paper lies in unfalsification concept but such limits on our powers of clairvoyance would seem to be inherent in any unprejudiced scientific analysis of feedback control problems based solely on data.

5. DISCUSSION

The proposed controller parameter adaptation algorithm does not require the plant to be minimum phase and relative plant degree to be known while the adaptation rule in MRAC or STR adaptive control problems do. Furthermore, such assumptions were relaxed without additional complexity in the control and adaptive law. Our adaptation rule is very simple and easy to implement. However, there is loss by relaxing assumptions on the plant. It is that there is possibility that the given performance might not be achieved in the steady state. In other words, we cannot guarantee convergence of controller parameters before practice as mentioned in previous section. The undesirable situation – divergence of controller parameters – can be cured by changing the controller structure $K(\theta)$ or performance specification $\tilde{J}(\theta, \tau)$ *not* by imposing impractical assumptions on the plant.

The idea to use gradient tuning when the performance specification is not satisfied and to stop adaptation when it is satisfied is not new. There are many adaptation rules by gradient method. However, the approach in this paper differs from other gradient tuning algorithm in that the cost function is a function of measured plant data, not a function of some of plant parameters.

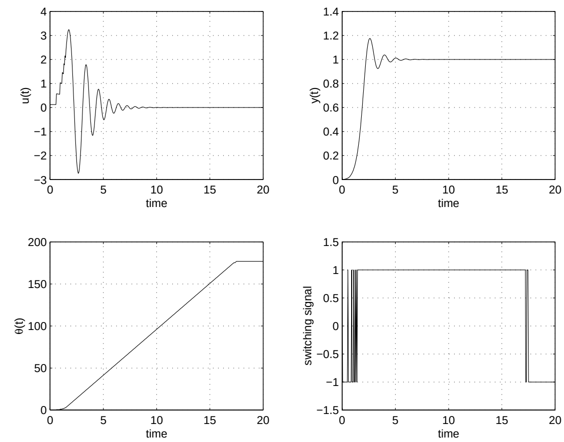

Fig. 1. Simulation results with $\theta(0) = 0.1$ and $\tilde{J}(\theta, t) = -e^{-1.5t} + \int_0^t \frac{(\tilde{r}(\xi) - y(\xi))^2}{2} d\xi$.

6. SIMULATION

In this section, two numerical examples will be presented which show how the proposed controller parameter adaptation rule (11) can be used.

Example 6. Consider the framework of the Example 4. The followings are used in the simulation: $T_{spec} = (\tilde{r} - y)^2/2$, $K(\theta) = \theta > 0$, $\gamma = 5$, $T = 10$ and $\rho(t) = e^{-1.5t}$. Thus, the adaptation rule in Eq. (13) is applied. The *unknown* plant is $G(s) = \frac{1}{s^2+2s}$ and the initial value for $\theta(t)$ is $\theta(0) = 0.1$. The simulation results are in Figure 1. The right-bottom plot shows switching signal that enables adaptation when it is 1 and stops adaptation when it is -1. The set $\Theta(t)$ is expressed by

$$\begin{aligned}\Theta(t) &= \{\theta \mid \tilde{J}(\theta, \tau) \le 0, \;\; \theta > 0, \; \tau \in [t_0, t]\} \\ &= \left\{\theta \;\middle|\; \frac{1}{\theta^2} \int_{t_0}^{\tau} u(\xi)^2 d\xi \le e^{-2\tau}, \right. \\ &\qquad\qquad \left. \theta > 0, \;\; \tau \in [t_0, t]\right\}\end{aligned}$$

and it can be noticed that the set $\Theta(t)$ is dependent on measured data $u(t)$, thus, emptiness of the set is dependent on it. The convergence of controller parameter cannot be guaranteed even though controller adaptation seems to converge to 177 approximately after $t = 17$ since the control signal $u(t)$ might have peaks at some time due to perturbation of the plant or disturbance noise and begin adaptation again.

Example 7. Consider the same plant as in the previous example but use different performance specification. The performance specification in the form of Eq. (14) is used in this example. The followings are also used in the simulation: $T_{spec} = (\tilde{r} - y)^2$, $K(\theta) = \theta > 0$, $\gamma = 3/2$,

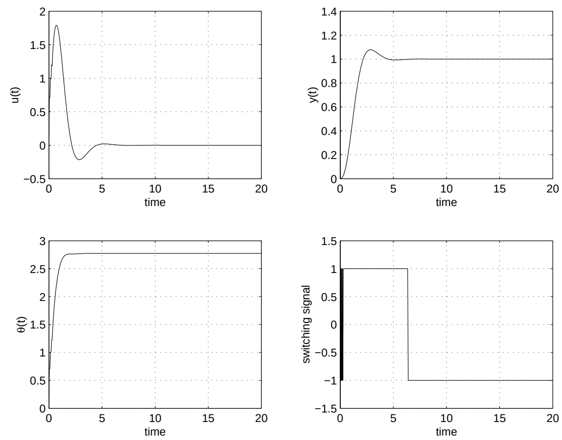

Fig. 2. Simulation results with $\theta(0) = 0.1$ and $\tilde{J}(\theta, t) = -e^{-1.5t} + (\tilde{r}(t) - y(t))^2$.

$T = 10$ and $\rho(t) = e^{-1.5t}$. Our objective in this example is to reduce error not cumulative error as in Example 6. The initial value for $\theta(t)$ is $\theta(0) = 0.1$ which is the same value as in the previous example. The simulation results are in Figure 2. The simulation shows that no more controller parameter adjustment happens after about $t = 6.7$ and the resulting value of controller parameter is $\theta = 2.77$.

7. CONCLUDING REMARKS

In this paper, we proposed a new controller parameter adaptation rule in conjunction with unfalsified control theory. The proposed algorithm does not impose many assumptions on the plant. A noteworthy feature of the algorithm is its flexibility and simplicity of implementation. Convergence of controller parameters is also discussed with some assumptions on the performance specification and parameter set but not on the plant although it cannot be checked *a priori*, which is inherent in any unprejudiced scientific analysis of feedback control problems based solely on data.

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