Model Reference Adaptive Control using Multiple Controllers & Switching

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Abstract: In this paper, we develop an adaptive multi controller system to solve the MRAC problem using unfalsified control theory. Switching is done among the candidate controllers based on some suitably defined performance index, which is obtained without actually inserting the candidate controllers in the feedback loop. Though prior knowledge about the nominal plant structure or its parameters is useful to select the initial set of candidate controllers, this method makes no use nor tries to identify the plant structure or its parameters while deciding the optimal switching sequence among the candidate controllers. Simulation results are presented to indicate successful switching strategy. We also compare this approach with the switching scheme developed by Narendra for the MRAC problem in [2,7].

Key words: Adaptive Control, Supervisory Control, Switching system, Unfalsified Control, MRAC, multi controller model

1. INTRODUCTION

In the last few years, the idea of switching between various plant models has developed rapidly: Narendra et al [2,7] and Morse, Hespanha et al [5,6]. The main idea behind many of these approaches, also known as *indirect adaptive control approach*, is as follows: There are multiple candidate models of the plant to be controlled and there is a corresponding candidate controller for each candidate plant model. The correct controller identification and switching is done in two stages: first, the plant model that best represents the actual plant is identified from the set of candidate plant models and then, based on certainty equivalence principle, the corresponding controller is switched in the feedback loop.

An alternative to the above approach is a more direct adaptive control approach, also referred to as unfalsification method. Using unfalsification concept, we dispense with any assumption on the plant structure and directly identify and switch to the optimal controller among a set of candidate controller. This approach can be used to evaluate *performance* of all candidate controllers directly at every time instant without actually inserting them in the feedback loop; by *performance* of a candidate controller we mean how closely the closed loop plant would have followed the given reference model, had that candidate controller been in the feedback loop. Thus, switching between candidate controllers can be based directly and only on the performance of candidate controllers. The concept was proposed by Safonov et al [4,3,8,9]. In this paper, we apply it to the Model Reference Adaptive Control (MRAC) problem. At the end, we briefly discuss the *indirect adaptive control* approach developed by Narendra in [2,7] and compare it with the method developed in this paper.

2. STATEMENT OF MRAC PROBLEM

The statement of our problem is similar to the general MRAC control problem and hence we present it from [7] as follows: The plant to be controlled is linear and time invariant, with input u_p: $\Re+ \rightarrow \Re$ and output y_p: $\Re+ \rightarrow \Re$ related by

$$y_p = W_p(s)\, u_p \qquad (1)$$

where $W_p(s) = K_p\,(Z_p(s) / R_p(s))$ is the transfer function of the plant. Here $K_p \in \Re\backslash\{0\}$, and $R_p(s)$ and $Z_p(s)$ are monic coprime polynomials of degree n and m, respectively, with m<n, and unknown coefficients.

The only assumption made about the plant is an upper bound on the degree of the plant. Once this is fixed, this upper bound forms the upper bound on relative degree as well. Controllers suitable for all possible degrees and relative degrees can then be included in the candidate controller set. Similarly, candidate controllers can be designed for both positive and negative sign of K_p.

However, in MRAC problems, for perfect tracking and model matching, the relative degree of the reference model must be as great as the relative degree of the plant. So, choice of reference model and deciding the upper bound on relative degree of the plant has to be done accordingly.

The reference model to be followed is linear, time invariant with input r: $\Re+ \rightarrow \Re$ (which is piece continuous, uniformly bounded) and output y_m: $\Re+ \rightarrow \Re$ related by

$$y_m = W_m(s)\, r \qquad (2)$$

where $W_m(s) = K_m\,(Z_m(s) / R_m(s))$ is the transfer function of the reference model. Here $K_m \in \Re\backslash\{0\}$, and $R_m(s)$ and $Z_m(s)$ are monic coprime Hurwitz polynomials of degree n and m, respectively.

Let the control error e_c be the difference between output of the plant and the reference model:

$$e_c = y_p - y_m \qquad (3)$$

The problem is to select a controller and place it in the feedback loop that would minimize a suitably defined performance index, which has to be some norm of this error. The index examined here is of the form:

$$J(e_c,t) \triangleq \alpha\, e_c^2(t) + \beta \int_0^t \exp^{-\lambda(t-\tau)}\, e_c^2(\tau)\, d\tau \qquad (4)$$

where $\alpha, \beta, \lambda \geq 0$ are design parameters explained later. The aim of the control law is to minimize this performance index.

3. OVERVIEW OF SWITCHED SYSTEM

In this section, a brief description of unfalsification approach is presented from [4], and then the proposed methodology is discussed for general MRAC problem.

Consider a plant ***P*** for which we need to determine a control law **K** such that the closed loop system response, say ***T,*** satisfies a specification requiring that, for all command inputs $r \in R$, the triple (r,y,u) be in the given specification set T_{spec}.

Definition [Safonov and Tsao, 4]: A controller $K \in \mathbf{K}$ is said to be *falsified* by measurement information if this information is sufficient to deduce that the performance specification $(r,y,u) \in T_{spec}\ \forall\ r \in R$ would be violated if the controller were

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in the feedback loop. Otherwise the control law K is said to be unfalsified.

Let $\mathbf{K}$ be a given a class of admissible control laws and let $\boldsymbol{P}_{\text{data}}$ be the set of triples (r,y,u) consistent with past measurement of (u,y)

Theorem 1 [Safonov and Tsao, 4]: *A control law $K \in \mathbf{K}$ is unfalsified by measurement information $\boldsymbol{P}_{data}$ if, and only if, for each triple $(r_0, y_0, u_0) \in \boldsymbol{P}_{data} \cap K$, there exists at least one pair (u_1, y_1) such that*

$$(r_0, y_1, u_1) \in \boldsymbol{P}_{data} \cap K \cap T_{spec} \qquad (5)$$

General Methodology: The proposed system has finite set of N candidate controllers, denoted by K_i, $i \in \boldsymbol{I}=\{1,2,\ldots N\}$. These candidate controller models can be selected or designed offline before the process starts, based on some nominal model of the process. Then at every instant, one of the candidate controllers is selected from this set, based on some suitably defined performance criterion, and switched into the feedback loop.

Candidate Controllers: The unknown plant is assumed to belong to a compact set $\boldsymbol{S}$. First, N nominal plant models, evenly distributed in the set $\boldsymbol{S}$, are chosen. When less prior knowledge of the plant is available, this set can be made larger to include all possibilities. Corresponding to each plant model, we choose a candidate controller that can meet performance requirements of that plant. The actual plant can take any of the infinite number of values within the compact set $\boldsymbol{S}$ and not necessarily match exactly with the initially chosen N number of plant models for which the candidate controllers were designed. The objective is to select the optimal controller among the set of candidate controllers, which would make the performance index $J(e_c, t)$ minimum.

Fictitious Reference Signal: Given a set of past plant input/output data $\{u_p(t),\ y_p(t)\}$, we now define fictitious reference input $\tilde{r}(K_i, y_p, u_p)$ for the candidate controller K_i. This is a hypothetical command signal that would have produced exactly the measured data $(u_p(t),\ y_p(t))$ had the candidate controller K_i been in the feedback loop with the unknown plant during the entire time period over which the measured data (u_p, y_p) were collected.. Note that this signal is not the actual reference signal, hence the name *fictitious*.

For example, for the system in figure (1), the fictitious reference signal for the i^{th} candidate controller model with controller parameter as $K_{a\,i}$ and $K_{b\,i}$ would be:

$$\tilde{r}(K_i, u_p, y_p) \triangleq (u_p - K_{ai}\, y_p)\ K_{bi}^{-1} \qquad (6)$$

If the i^{th} controller K_i is actually in the loop during which plant i/o data $(u_p,\ y_p)$ were collected, then the i^{th} fictitious reference signal would be same as the actual reference signal r; else it would be different from the actual reference signal.

Remark: When the controller is in the feedback loop the problem of determining the fictitious reference signal is relatively easy. However, if the controller is placed in the feed forward loop, then to uniquely determine the fictitious reference signal, the controller has to be *causally-left-invertible*, that is, from the past and present output of the controller, one should be able to uniquely determine its present input. All controllers with biproper transfer function, including the PID controller and the controller structure employed in our MRAC problem have this property.

Corresponding to each candidate controller, a fictitious output $\tilde{y}\,(K_i, u_p, y_p)$ can be defined as

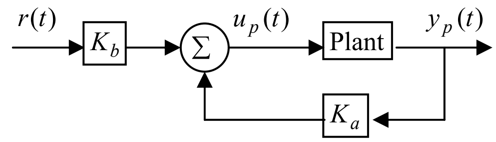


Figure – 1

$$\tilde{y}(K_i, u_p, y_p) \triangleq W_m\ \tilde{r}(K_i, u_p, y_p) \qquad (7)$$

where W_m is the given reference model.

For each candidate controller, a fictitious error signal can be defined as the error between its fictitious output and the actual plant output. Hence, i^{th} fictitious error signal would be:

$$\tilde{e}(K_i, u_p, y_p) \triangleq y_p - \tilde{y}(K_i, u_p, y_p) \qquad (8)$$

To keep notation simple, we represent signals for i^{th} controller as : $\tilde{r}_i = \tilde{r}(K_i, u_p, y_p)$, $\tilde{y}_i = \tilde{y}(K_i, u_p, y_p)$, $\tilde{e}_i = \tilde{e}(K_i, u_p, y_p)$

Performance index: The fictitious output $\tilde{y}_i(t)$ is the output of the reference model with the i^{th} fictitious reference signal $\tilde{r}_i(t)$ as the command signal (7). And $y_p(t)$ is the output of the actual process with K_i controller in the loop and with the i^{th} fictitious reference signal $\tilde{r}_i(t)$ as the command signal (follows from the definition of fictitious reference signal). Hence, the error $\tilde{e}_i(t)$ would have been the control error (3), had the i^{th} candidate controller K_i been in the feedback loop during the entire time period over which the measured data (u, y) were collected, with plant input/output data as (u,y) and fictitious reference signal $\tilde{r}_i(t)$ as the reference signal. Thus, the error $\tilde{e}_i(t)$ is a measure of effectiveness of the i^{th} candidate controller if it is placed in the loop, i.e. how closely will the feedback system with i^{th} controller in the loop follow the given reference model. So, the switching among the candidate controllers should be based on some norm of this error.

The performance index examined here is:

$$\tilde{J}_i(t) \triangleq \alpha\, \tilde{e}_i^2(t) + \beta \int_0^t \exp^{-\lambda(t-\tau)}\ \tilde{e}_i^2(\tau)\, d\tau \qquad (9)$$

where i is the candidate controller index, and design parameters α, β, $\lambda \geq 0$. The non-negative forgetting factor λ determines the weight of past error. It also ensures the boundedness of the integral terms. The parameter α penalizes instantaneous errors while β penalizes accumulated past error. These design parameters can be adjusted depending on the given problem. For example, for a rapidly time varying plant, more weighting can be given to instantaneous error with a small time windowing; for a time invariant or slowly varying plant equal weighting can be given to the two error terms.

Given a set of candidate controllers K_i, $i \in \boldsymbol{I}=\{1,2,\ldots N\}$, the problem is to identify and switch to the optimal controller $K^*(t)$, that would minimize this performance index at each instant:

$$K^*(t) = \arg \min_{k\,i,\, i \in \{1,..N\}} \tilde{J}_i(t) \qquad (10)$$

Also, the controllers can be ordered on the basis of their performance depending on this index; the lower is this performance index for a candidate controller, the more is the likelihood of this controller giving better performance. By theorem 1, the i^{th} candidate controller K_i is unfalsified at time t by plant input/output data $(u_p(t),\ y_p(t))$, if and only if the performance index for the i^{th} controller satisfies

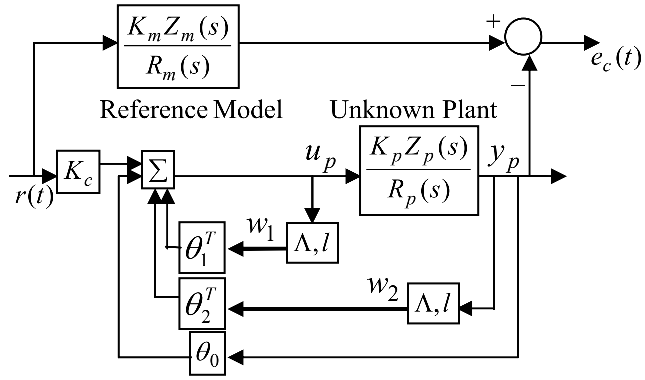


Fig: 2, Controller realization in MRAC problem [1], Controller parameters: θ_0, θ_1, θ_2, K_c

$$\tilde{J}_i(t) < \gamma \qquad (11)$$

where γ is a positive threshold and design parameter; if for a candidate controller, the performance index exceeds this threshold at any time, it is not suitable for the actual unknown plant and hence falsified and taken out of the candidate controller set. Switching is done among till now unfalsified candidate controllers only.

Falsifying the candidate controllers enables us to monotonically reduce the candidate controller set with the evolving plant data (u,y); controllers which are not remotely suitable for the plant gradually gets falsified & thus eliminated from the candidate controller set. Thus, even if we start with a very large set of candidate controllers, the computational burden decreases rapidly with the decrease in number of candidate controllers. The spectral richness of reference signal (somewhat analogous to persistent excitation in system identification of adaptive control problem) and the threshold γ will determine how fast the *bad* controllers are eliminated from the unfalsified candidate controller set. However, for time varying plants, the idea of falsifying a candidate controller does not hold, as a candidate controller not suitable for the current plant may work for in future.

Switching Strategy: At every instant, switching is done to the till now unfalsified candidate controller with minimum performance index. Other robust switching strategies can be found in [5,6] and references therein

4. STRUCTURE OF CONTROLLER IN MRAC

We use the same structure of the controller in MRAC problem as in [7]. However, we would not use the plant parameterization given there as unlike [7], our scheme is based on multi models of controller and not of the plant. Hence, we just show that the controller structure used has enough degree of freedom to satisfy the requirements of MRAC problem [1].

Construct w_1, w_2: $\Re+ \rightarrow \Re^{n-1}$ by:

$$\dot{w}_1 = \Lambda w_1 + l u_p$$
$$\dot{w}_2 = \Lambda w_2 + l y_p \qquad (12)$$

where (Λ, l) is an asymptotically stable system in controllable canonical form, with

$$\lambda(s) \triangleq \det(sI - \Lambda) = \lambda_1(s) Z_m(s) \qquad (13)$$

for some monic Hurwitz polynomial $\lambda_1(s)$ of degree n-m-1.
The control input is given by:

$$u_p(t) = \theta^T(t) \underline{w}(t) \qquad (14)$$

where $\underline{w} \triangleq (r, w_1^T, y_p, w_2^T)^T \in \Re^{2n}$ (15)

$$\theta \triangleq (K_c, \theta_1^T, \theta_0, \theta_2^T)^T \in \Re^{2n} \qquad (16)$$

where $K_c, \theta_0 \in \Re$, $\theta_1, \theta_2 \in \Re^{n-1}$ are the controller parameters. The controller structure is shown in fig(2). Let us define:

$$\frac{C(s)}{\lambda(s)} = \theta_1^T (sI - \Lambda)^{-1} l \text{ and } \frac{D(s)}{\lambda(s)} = \theta_0 + \theta_2^T (sI - \Lambda)^{-1} l \qquad (17)$$

Then overall transfer function of the feedback system is:

$$W_o(s) = \frac{K_c K_p Z_p(s) \lambda_1(s) Z_m(s)}{R_p(s) [\lambda(s) - C(s)] - K_p Z_p(s) D(s)} \qquad (18)$$

Since $Z_p(s)$ and $R_p(s)$ are coprime, using Bezout Identity, unique polynomials $C^*(s)$ and $D^*(s)$ can be found such that the denominator of (18) equals to $Z_p(s)\lambda_1(s)R_m(s)$. If $K_c^* = K_m/K_p$, then the over all transfer function of (18) becomes equal to the transfer function of the given reference model. Hence,

$$R_p(s) [\lambda(s) - C^*(s)] - Kp\, Zp(s)\, D^*(s) = Zp(s)\lambda_1(s) R_m(s)$$
$$K_c^* = K_m / K_p \qquad (19)$$

So, the problem is to identify this $C^*(s)$, $D^*(s)$ and K_c^* which are related to the optimal controller parameter θ^* by (17) and (16), such that the closed loop plant transfer function given by (18) equals the given reference model.

The N candidate controller are given by θ_i, $i=1,\ldots N$

where $\theta_i = (k_{ci}, \theta_{1i}^T, \theta_{0i}, \theta_{2i}^T)^T \in \Re^{2n}$ (20)

and it is assumed that there is at least one controller in the candidate controller set which is close to θ^* (so that it can fulfill the MRAC objectives); the task of the switching system is to identify the controller nearest to or equal to this θ^*.

The control input u(t), if i^{th} candidate controller θ_i is in the loop, is given by: $u(t) = \theta_i^T \underline{w}(t)$ (21)
where $\underline{w}(t)$ is given in (15).

5. SWITCHING STRATEGY

From the control law given in (14), the fictitious reference signal for i^{th} candidate controller θ_i is given by:

$$\tilde{r}_i(t) = \frac{1}{K_{ci}} \left(u_p(t) - \theta_{1i}^T w_1 - \theta_{0i} y_p - \theta_{2i}^T w_2 \right)$$

Which can be written as: $\tilde{r}_i(t) = \underline{\theta}_i^T w$ (22)

$$\text{Where } \underline{\theta}_i^T \triangleq \left(\frac{1}{K_{ci}}, -\frac{\theta_{1i}^T}{K_{ci}}, -\frac{\theta_{0i}}{K_{ci}}, -\frac{\theta_{2i}^T}{K_{ci}} \right) \in \Re^{2n} \qquad (23)$$

$$w \triangleq (u_p(t), w_1(t), y_p(t), w_2(t)) \qquad (24)$$

It is seen that all candidate controllers share the same regression vector w for generating the fictitious reference signal.

The fictitious output corresponding to i^{th} candidate controller is given by

$$\tilde{y}_i(t) = W_m \tilde{r}_i(t) \qquad (25)$$

The error signal corresponding to the i^{th} candidate controller would be: $\tilde{e}_i(t) = y_p(t) - \tilde{y}_i(t)$ (26)

As per justification given in section 3, this error is an indication of performance of i^{th} controller and hence, the performance index must be based on this error.

Implementation Issues: The above control law can be implemented in both continuous and discrete time. However, as we monitor all the above signals in continuous fashion, we choose to implement it in a continuous fashion. The performance index of (9) needs some modification. The state space of (9) can be achieved easily as:

$$\dot{x}_i(t) = -\lambda\, x_i(t) + \tilde{e}_i^2(t)$$
$$\tilde{J}_i(t) = x_i(t) + \alpha\, \tilde{e}_i^2(t) \qquad (27)$$

where $x_i(t)$ is the state of i^{th} filter that generates the i^{th} performance index, with $\tilde{e}_i(t)$ forming the input to the filter.

Algorithm 1 (our method): *INITIAL SETTING:*

- *A finite set of N number of candidate controllers, given by θ_i, $i \in \mathbf{I} \triangleq \{1,\ldots.N\}$; where $\theta_i \in \Re^{2n}$ given in (20)*
- *Initial performance index $\tilde{J}_i(0)=0$, initial state of filter in (27) be $x_i(t)=0$; $i=1,2\ldots N$*
- *Let the resulting switching index of candidate controllers in the loop be given by $\tilde{i}^*(t)$. Set $\tilde{i}^*(0) = N$, that is, place the N^{th} candidate controller in the loop at t=0.*

PROCEDURE:

(1) *For each $i \in \{1,\ldots.N\}$: Continuously update the following signals: fictitious reference signal $\tilde{r}_i(t)$, fictitious output $\tilde{y}_i(t)$, fictitious error signal $\tilde{e}_i(t)$ and finally i^{th} performance index $\tilde{J}_i(t)$ using (22), (25), (26) and (27).*

(2) *Continuously identify the current optimal controller index by $\tilde{i}^*(t) = \arg \min_{i \in I=\{1,..N\}} \tilde{J}_i(t)$ and switch $\tilde{\theta}^*(t) = \theta_{\tilde{i}^*}$ controller in the loop.*

Remark: **(a)** $\tilde{\theta}^*(t)$ is the resulting switching sequence of Algorithm 1.

(b) We could have falsified any unsuitable candidate controllers using (11) and incorporated it after step (1) of the Algorithm procedure as follows:

*(1a) If $\tilde{J}_i(t) > \gamma$, the i^{th} candidate controller is falsified by evolving plant i/o data $u_p(t)$ and $y_p(t)$. Delete the controller index element i from candidate controller set **I**. If the set **I** is empty, then all the controllers have been falsified, hence terminate the algorithm; else continue.*

However, later in the paper, we would like to compare our algorithm with a switched system developed by Narendra who hasn't used the falsification concept; hence to make the comparison fair, we didn't incorporate the falsification concept in the above algorithm.

6. NARENDA'S PLANT MODEL BASED SWITCHED SYSTEM

The fictitious output of the i^{th} candidate controller in (25) can be written as:

$$\tilde{y}_i(t) = W_m \frac{1}{K_{ci}} \left(u_p(t) - \theta_{1i}^T w_1 - \theta_{0i}\, y_p - \theta_{2i}^T w_2 \right)$$
$$= W_m \underline{\theta}_i^T w$$
$$= \underline{\theta}_i^T \overline{w} \text{ (as } \underline{\theta}_i^T \text{ is constant for a given candidate controller)} \qquad (28)$$

where $\overline{w} \triangleq W_m(s)\, I_{2n}\, w$

and
$$\underline{\theta}_i^T \triangleq \left(\frac{1}{K_{ci}}, -\frac{\theta_{1i}^T}{K_{ci}}, -\frac{\theta_{0i}}{K_{ci}}, -\frac{\theta_{2i}^T}{K_{ci}} \right) \in \Re^{2n} \qquad (29).$$

When $\underline{\theta}_i = \underline{\theta}^*$ the actual plant output can be represented by:

$$y_p = \underline{\theta}^{*\,\mathrm{T}}\, \overline{w} \qquad (30)$$

Hence, the actual plant can be parameterized by this $\underline{\theta}^*$, with $\overline{w}$ forming the input and $\underline{\theta}^*$ forming the plant parameters.

Now, in [2,7], Narendra had used a group of N candidate plant models P_i, $i \in \{1,\ldots N\}$,each with a corresponding candidate controller C_i, $i \in \{1,\ldots N\}$ model. The candidate controllers were designed so as to meet the control objective of the corresponding candidate plant models. The candidate plant models were either fixed or tuned using some suitable tuning algorithm. Here, we consider the case where Narendra had considered fixed candidate plant models (2) (No tuning of candidate plant model parameters involved). The candidate plant model, which best represents the actual plant, was identified at each instant and the corresponding controller was switched in the loop. To identify the correct plant model, N identifiers (corresponding to each plant model), each with output $\hat{y}_i$, were proposed. Narendra has used equations similar to (28) in [1,2,7] to parameterize the plant and construct the N identifiers, with elements of $\underline{\theta}_i$, $i \in \{1,\ldots N\}$ forming the set of N candidate plant model parameters and the signal vector $\overline{w}$ forming the input to the identifier; if $\underline{\theta}(t)=\underline{\theta}^*$, the output of the identifier would then be same as the plant output. The output of the i^{th} identifier (same as 26) was given by

$$\hat{y}_i = \underline{\theta}_i^T\, \overline{w} \qquad (31).$$

Each candidate plant model with parameters $\underline{\theta}_i$ had θ_i as the corresponding controller parameters, so that the i^{th} plant model, with corresponding controller in the loop, produces zero control error. Each identifier had a corresponding identification error

$$\hat{e}_i = \hat{y}_i - y_p \qquad (32)$$

This error was an indication of how close the i^{th} plant model was to the actual plant and the switching was based on performance index $\hat{J}_i(t)$, which was some norm of this error. ,

$$\hat{J}_i(t) \triangleq \alpha\, \hat{e}_i^2(t) + \beta \int_0^t \exp^{-\lambda(t-\tau)}\, \hat{e}_i^2(\tau)\, d\tau \qquad (33)$$

A state space equation of (33) can also be developed in line with (27).

The problem is then to identify from the candidate plant set P_i, $i=\{1,..N\}$ at each instant the optimal plant model $P^*(t)$ that is closest to the actual plant and switch the corresponding candidate controller set in the feedback loop.

Algorithm 2 :(Narendra's method, [2,7]): *INITIAL SETTING:*

- *Given N fixed candidate plant model (No tuning involved), parameterized by $\underline{\theta}_i$, $i \in \{1,\ldots N\}$; $\underline{\theta}_i^T \in \Re^{2n}$ as in (29). Also given N identifiers corresponding to each plant model, each having structure as in (31).*
- *Given N candidate controller models, parameterized by θ_i, $i=1,\ldots N$, as in (20) such that each θ_i is able to meet the performance specification for the corresponding plant model $\underline{\theta}_i$, so that when θ_i is placed in feedback loop with plant $\underline{\theta}_i$, it results in zero control error.*
- *Initial performance index $\hat{J}_i(0)=0$, $i=1,2\ldots N$, initial state of state space representation of (33) is 0.*
- *The resulting index of candidate controllers in the loop be given by $\hat{i}^*(t)$, Set $\hat{i}^*(0) = N$, that is, place the N^{th} candidate controller in the loop at t=0.*

PROCEDURE:

(1) *For each* $i \in \{1,\ldots N\}$*: Continuously update the following signals: output of* i^{th} *identifier* $\hat{y}_i(t)$*, identification error* $\hat{e}_i(t)$ *and finally* i^{th} *performance index* $\hat{J}_i(t)$ *using (31), (32) and state space representation of (33).*

(2) *Continuously compute the index of the optimal plant model, the plant model which is closest to the actual plant:* $\hat{i}^*(t) = \arg \min_{i \in I=\{1,..N\}} \hat{J}_i(t)$. *Now,* $\hat{\theta}^*(t) = \theta_{\hat{i}^*}$ *and switch the corresponding candidate controller* $\hat{\theta}^*(t)$ *in the loop.*

Remark: **(1)** Let us denote $\hat{\theta}^*(t)$ as the resulting switching sequence of Algorithm 1.

7. COMPARISON

Assumptions: ***(a)*** Let the candidate controller set in algorithm 1 be selected such that they meet the control objectives for the corresponding candidate plant models selected in algorithm 2, that is, let the candidate controller set and their index in the above two algorithm be the same. ***(b)*** Let the initial candidate controller (when t=0) in both algorithms be same. Also, the reference signal applied and the initial condition of the plant in the above two algorithms be same.

The above assumptions are essential to compare the performance of the two algorithms.

Theorem 2: *Subject to satisfaction of the above assumptions, (a) the fictitious error* $\tilde{e}_i(t)$ *of* i^{th} *candidate controller in algorithm 1 and the Identification error* $\hat{e}_i(t)$ *of the* i^{th} *candidate plant/controller pair in algorithm 2 are equal:* $\hat{e}_i(t) = \tilde{e}_i(t)$ *for all t and i = 1,…N and*
(b) the switching sequences of the two algorithms are equal: $\tilde{\theta}^*(t) = \hat{\theta}^*(t)$*) for all t.*

Proof: The i^{th} fictitious output in Algorithm 1 is $\tilde{y}_i(t)$ and the output of i^{th} identifier in algorithm 2 is $\hat{y}_i$. As per justification given in the beginning of section 6, $\hat{y}_i(t) = \tilde{y}_i(t)$. Hence, the fictitious error of algorithm 1 and the identification error of algorithm 2 for corresponding models are equal, that is: $\tilde{e}_i(t) = \hat{e}_i(t)$. Hence, the performance index of the i^{th} candidate controller in algorithm 1 would be identical to the index of i^{th} candidate plant/controller pair in algorithm 2. Since switching is based on this performance index, the switching sequences for the two algorithms are equal. □

So, in this paper, we arrive to the same results as Narendra's, but using a completely different approach. Narendra has used his method (given in algorithm (2)) to identify the candidate plant model, which is closest to the actual plant, and then switch to the corresponding controller. And in this paper, we directly identify and switch to the candidate controller based on the fictitious error concept. So, we can say that in [1,2,7], in the process of selecting the best plant model representing the actual plant, Narendra has also selected the controller, which would have given *best* performance, if placed in the loop with the unknown plant. This similarity is due to the special parameterization of the plant used by Narendra to construct the identifiers.

Narendra proved the validity of the special parameterization of the plant and structure of the identifier [7]; here, using a different approach, we give a different motivation behind using such parameterization and structure.

However, parameterizing the plant in some other way and constructing some other observers may not result in this similarity between the two conceptually different approaches.

The beauty of our method is it is a plant model free approach, that is, we do not try to evaluate the plant structure or its parameters. The switching is based only and directly on the performance of the candidate controllers. Hence, it can be applied to a broader class of problems, where estimating a plant model might prove difficult; though prior knowledge about the nominal plant structure or its parameters is always useful to select the initial set of candidate controllers.

8. SIMULATION

The actual plant to be controlled has a transfer function of the form: $W_p(s) = k_p / (s^2 + as + b)$. The sign of K_p is unknown. The parameters K_p, a, b are known to lie in the compact set given by $\boldsymbol{S} = \{1 \le |Kp| \le 0.1, -.5 \le a \le 2, -1.5 \le b \le 1.5\}$.

Here, for simplicity, we impose these assumptions on the plant. When no or less prior information about the plant is available, the candidate controller set has be made larger to accommodate all possible plant structures; but this set can be reduced monotonically with the unfalsification concept in (11).

The reference model is given by $W_m(s) = 1/(s^2+1.4s+1)$ (same as in [7]). The reference input r(t) is given by a square wave, with unit amplitude and period 10 units. The candidate controllers are chosen for plants models, evenly distributed in the set $\boldsymbol{S}$, given by $\{K_p: -1, -0.7, -0.4, -0.1, 0.1, 0.4, 0.7, 1\} \times \{a: -0.5, 0, 0.5, 1, 1.5, 2\} \times \{b: -1.5, -1, -0.5, 0, 0.5, 1, 1.5\}$.

The parameter of the performance index in (15) is taken as: $\lambda = 0.05$, α, $\beta = 1$. The simulations were carried out with a 0.1 mean, 2 variance disturbances in the input of plant, applied after 40 seconds of starting of the simulation. So,

$$d(t) = N(0.1,2)\, u(t-40) \qquad (34)$$

Case:1 An unstable plant given by: $W_p(s)=.4/(s^2 + s - 1)$. The plant parameters match exactly with one of the nominal plant model for which there exists a matching candidate controller. The correct controller, as expected, is determined almost immediately and the switching stops thereafter.

Case 2: The plant is given by: $W_p(s) = -0.5/(s^2 + 1.1s + 1.2)$. The candidate controller set does not have a perfect controller for this plant; however, it selects the *best* available controller among the set for the plant to minimize the control error. The system reacts to the disturbance (35) in this case by switching between two candidate controllers.

9. CONCLUSION

In this paper, we described a way to apply the direct controller model based approach to build a switched system for the MRAC problem. This is a plant model free approach; we do not try to estimate the unknown plant; though prior knowledge about the plant is useful to construct the initial candidate controller set. Like all other fixed model based switching system, the success of this approach depends on the selection of candidate controller set. Making this set large assures better result (less control error), but increases the computational burden. However, using the unfalsification concept, the unsuitable controllers (candidate controllers that

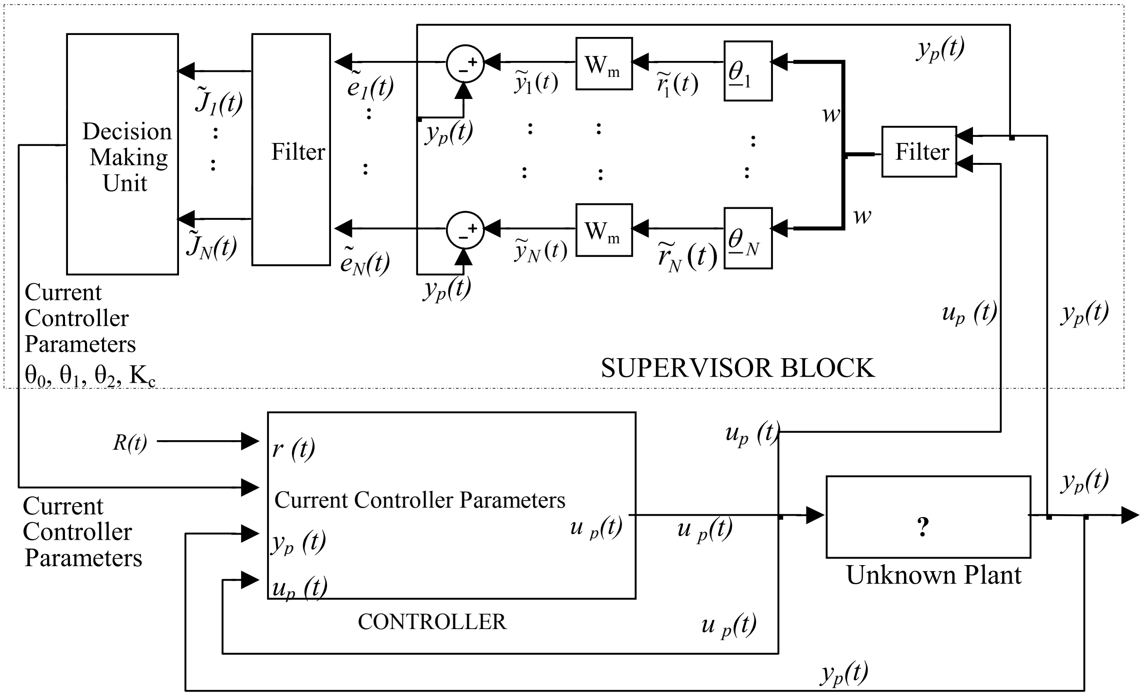
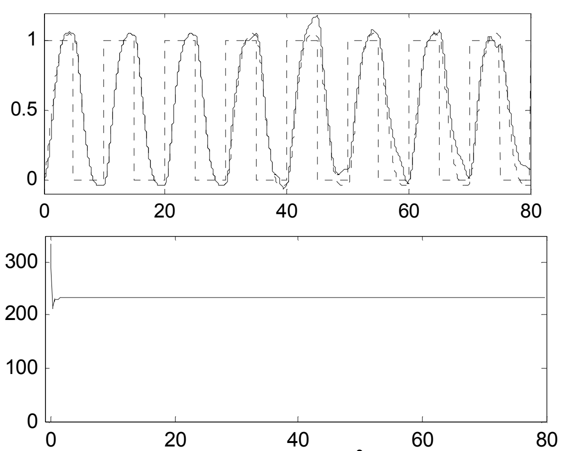
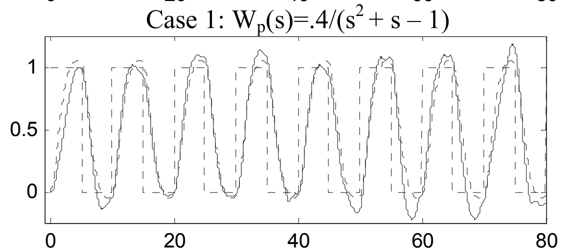


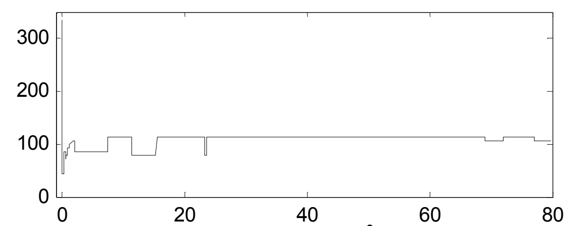
Figure 4: Architecture for control using *N* candidate controllers θ_i, $i = 1,\ldots N$ for MRAC problem

would have produced large control error, if inserted in the loop) can be rapidly eliminated from the candidate controller set, thus reducing the computational burden. We have also pointed out the similarity between our switching strategy and that of Narendra's. These two conceptually different approaches produce same result; the similarity owes to the special parameterization used by Narendra to construct the identifiers.



Case 1: $W_p(s)=.4/(s^2 + s - 1)$





Case 2: $W_p(s) = - 0.5/(s^2 + 1.1s +1.2)$

Fig 3. Plant response $y_p(t)$ (solid line), reference output $y_m(t)$ (dotted line), reference input r(t) (dotted line) and corresponding switching sequence

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