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Unfalsified model reference adaptive control using the ellipsoid algorithm

Fabricio B. Cabral [1]
Instituto Militar de Engenharia, DE3, 22290-270, Rio de Janeiro, RJ, Brazil
fabricio_cabral@hotmail.com

Michael G. Safonov [2]
University of Southern California, Los Angeles, CA 90089-2563, USA
msafonov@usc.edu

Abstract

The unfalsified control paradigm, which does not require a priori assumptions on the plant (it uses, as plant information, experimental trajectories of plant signals), is applied to the model reference adaptive control problem. The use of an ellipsoid algorithm overcomes the need of gridding the parameter space used in prior applications of unfalsified control. The application of the ellipsoid algorithm produces a sequence of decreasing volume ellipsoids which contains the set of unfalsified candidates. Simulation results are provided for the control of an unstable plant.

Keywords: Unfalsified control; model reference; adaptive control; ellipsoid algorithm.

1 Introduction

The unfalsified control paradigm rules out controllers from a given class of candidates based on the check of consistency between data, performance specification and control laws ([1],[2],[3],[4],[5], [6],[7],[8],[9]). The performance specification has often been given in terms of a solution to an expanding collection of quadratic inequalities ([4],[5]), with new quadratic inequalities being continually added as time increases and measurement data accumulates. To deal with the computational problem of exactly solving this expanding set of quadratic inequalities, one approach is to approximate the unfalsified set by discarding all but one of the quadratic inequalities, viz. the most recent one. A better approach which enables exact computation of the unfalsified set involved gridding of the controller parameter space. Gridding maintains computational tractability by limiting consideration to a finite set of candidate controllers and has successfully been used in ([4],[5]).

In this work, we apply the unfalsified control paradigm to the model reference adaptive control problem. Instead of restricting attention to finitely many controllers ([4],[5]) or ignoring all but the most recent ellipsoid [10], we employ an alternative approximation that allows all ellipsoids to be considered. We limit attention to the case in which all the quadratic inequalities take the form of ellipsoids. We then approximately solve the inequalities using the ellipsoid algorithm [11]. First, we present the basic definitions of unfalsified control. The unfalsified model reference adaptive control problem is then formulated and solved, giving rise to the problem of finding a point in the intersection of a finite number of ellipsoids. A modified version of the ellipsoid algorithm which allows us to find a point in the intersection of a finite number of ellipsoids is presented. A second formulation of the model reference adaptive control problem, which takes in consideration the norm of the reference signal, is presented and solved. Simulation results are provided for the control of an unstable plant according to both formulations. Finally, we present our conclusions and comments.

2 Basic Definitions

The unfalsified control paradigm can be embedded in the behavioral framework of Willems [12]. This embedding allows us to have a model for a dynamical system given directly in terms of its experimental trajectories. Two basic definitions of this framework are used as a start point, namely the definition of a mathematical model and the definition of a data set.

Definition 2.1 *A mathematical model is a pair* $(\mathbf{U}, \mathcal{B})$, *with* $\mathbf{U}$ *the universum — its elements are called outcomes — and* $\mathcal{B} \subseteq \mathbf{U}$ *the behavior.*

Definition 2.2 *A data set is a nonempty subset* $\mathcal{D}$ *of* $\mathbf{U}$.

The main contribution of the behavioral approach concerning identification, is the definition of an unfalsified model.

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Definition 2.3 *Given a vector space of time signals* $\mathbf{U}$*, a model* $(\mathbf{U}, \mathcal{B})$*, a mapping* $\mathbf{P}_\tau : \mathbf{U} \rightarrow \mathbf{U}$ *and a data set* $\mathcal{D}_\tau \subset \mathbf{P}_\tau(\mathbf{U})$*, we say that the model* $(\mathbf{U}, \mathcal{B})$ *is* unfalsified *by the data set* $\mathcal{D}_\tau$ *if*

$$\mathcal{D}_\tau \subset \mathbf{P}_\tau(\mathcal{B}).$$

Defining a controller as a mathematical model and using the notions of interconnection of systems, an unfalsified controller is defined.

Definition 2.4 *A controller is a mathematical model.*

Definition 2.5 *Given a vector space of time signals* $\mathbf{U}$*, a controller* $(\mathbf{U}, \mathcal{B}_c)$*, a desired closed loop behavior* $(\mathbf{U}, \mathcal{B}_d)$*, a mapping* $\mathbf{P}_\tau : \mathbf{U} \rightarrow \mathbf{U}$*, and a data set* $\mathcal{D}_\tau \subset \mathbf{P}_\tau(\mathbf{U})$*, we say that a controller* $(\mathbf{U}, \mathcal{B}_c)$ *is* unfalsified *by the data set* $\mathcal{D}_\tau$ *if*

$$\mathbf{P}_\tau((\mathbf{P}_\tau^{-1}(\mathcal{D}_\tau)) \cap \mathcal{B}_c) \subset \mathbf{P}_\tau(\mathcal{B}_d).$$

Let us notice that $\mathbf{P}_\tau^{-1}(\mathcal{D}_\tau)$ gives all the information consistent with the data $\mathcal{D}_\tau$. The expression $(\mathbf{P}_\tau^{-1}(\mathcal{D}_\tau)) \cap \mathcal{B}_c$ gives the behavior of the closed loop system. Finally, a verifiable statement (this explains the projection operators used in both sides of the expression) is made concerning the satisfaction the specification by the closed loop behavior.

3 Problem Formulation

3.1 The System

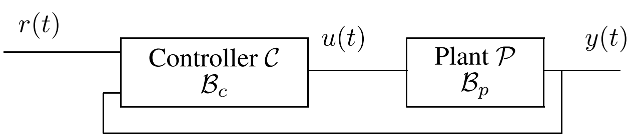

Figure 1: Feedback control system.

Consider the system in Figure 1. As in [10], we omit any arrows on the block diagram in Figure 1. This illustrates a departure from the usual input/output setting, from the processor point of view.

3.2 The Universum

Let $z = (r, y, u) \in \mathbf{U}$ where $\mathbf{U} = \mathcal{R} \times \mathcal{Y} \times \mathcal{U} = \mathcal{L}_{2e}^{n_z}$. Here $\mathcal{R} = \mathcal{L}_{2e}^{n_r}$ is the set of reference signals, $\mathcal{Y} = \mathcal{L}_{2e}^{n_y}$ and $\mathcal{U} = \mathcal{L}_{2e}^{n_u}$ are sets of plant signals, and $n_z = n_r + n_y + n_u$. In this paper we focus on the case $n_r = n_y = n_u = 1$.

3.3 The Data Set

The plant information imposes restrictions only on the past values of the signals u and y. Thus the data set is given by

$$\mathcal{D}_\tau = \{(r, y, u) \in \mathbf{P}_\tau(\mathbf{U}) \mid y = y_{data}, u = u_{data}\},$$

for some

$$(y_{data}, u_{data}) \in \mathbf{P}_\tau(\mathcal{Y} \times \mathcal{U}).$$

3.4 The Desired Behavior

Prior to introducing the performance specification, let us first introduce the norm to be used.

Definition 3.1 *Given a constant* $\sigma > 0$*, we define the exponentially-weighted truncated* L_2 *inner-products* $< x, y >_\tau$ *and norm* $\|x\|_\tau$ *by*

$$< x, y >_\tau \triangleq \int_0^\tau e^{-2\sigma(\tau - t)} y^T(t) x(t)\, dt \qquad (1)$$

$$\|x\|_\tau \triangleq \sqrt{< x, x >_\tau}. \qquad (2)$$

Given a reference model transfer function

$$W_m(s) = k_m Z_m(s)/R_m(s),$$

where, as in [13] and [14], $k_m \in \mathbb{R}\backslash\{0\}$ is a non zero real constant and $R_m(s)$ and $Z_m(s)$ are monic Hurwitz polynomials of degree n and m, respectively.

Let the desired behavior be given by

$$B_d = \{(r, y, u) \mid \mathcal{J}(r, y, u, \tau) \geq 0 \; \forall \tau \in \mathcal{T}\},$$

where $\mathcal{J}(r, y, u, \tau) \triangleq \Delta(\tau) - \|y - w_m * r\|_\tau{}^2$, $w_m = \mathcal{L}^{-1}(W_m(s))$ is a time domain reference model transfer function, Δ is a function $\Delta : \mathcal{T} \rightarrow \mathbb{R}_+$ and $\mathcal{T} \subset [0, \tau]$ is a set of time instants.

3.5 The Class of Candidate Controllers

The class of candidate controllers used is the same as the one used in model reference adaptive control as in [13] and [14]. In order to define the class of candidate controllers, we first define a vector of filters as in [2] and [10]. Notice that we define a vector of filters, and not just a vector of time-domain filtered signals as in [13] and [14]. Let us define $v : \mathcal{L}_{2e} \rightarrow \mathcal{L}_{2e}^{n-1}$ by

$$\dot{v}(q) = \Lambda v(q) + l q \qquad (3)$$

$$(v(q))(0) = 0 \qquad (4)$$

where (Λ, l) is an asymptotically stable system in controllable canonical form, with

$$\lambda(s) = det(sI - \Lambda) = \lambda_1(s) Z_m(s) \qquad (5)$$

for some monic Hurwitz polynomial $\lambda_1(s)$ of degree $n^* - 1$, where n^* is the relative degree of $W_m(s)$. Let us also define

$$w(u, y) = (u, v^T(u), y, v^T(y))^T, \quad \text{and} \qquad (6)$$

$$\overline{w} = w_m * w. \qquad (7)$$

The class of controllers considered is given by $\{(\mathbf{U}, \mathcal{B}_c(\theta)) \mid \theta \in \Theta\}$, where

$$\mathcal{B}_c(\theta) = \{(r, y, u) \mid r = \theta^T w(u, y)\} \quad \text{and}$$

θ is a constant parameter vector in $\mathbb{R}^{2n}$.

4 Problem Solution

4.1 The Set of Unfalsified Controllers

Theorem 4.1 *At each time τ, the set of unfalsified controllers is given by the set of parameters*

$$\{\theta \mid \theta^T A(t)\theta - 2\theta^T B(t) + C(t) \leq 0,\ \forall t \in \mathcal{T} \cap [0,\tau]\},$$

where

$$A(\tau) = \int_0^\tau e^{-2\sigma(\tau-t)} \overline{w}_{data} \overline{w}_{data}^T dt, \quad (8)$$

$$B(\tau) = \int_0^\tau e^{-2\sigma(\tau-t)} \overline{w}_{data} y_{data} dt, \quad \textit{and} \quad (9)$$

$$C(\tau) = \int_0^\tau e^{-2\sigma(\tau-t)} y_{data}^2 dt - \Delta(\tau), \quad (10)$$

with

$$\overline{w}_{data} = w_m * w_{data} \quad \textit{and} \quad (11)$$

$$w_{data} = w(u_{data}, y_{data}) \quad (12)$$

provided that $\|u_{data}\|_\tau + \|y_{data}\|_\tau \neq 0$.

Proof. Let us first, compute the set $\mathbf{P}_\tau(\mathbf{P}_\tau^{-1}(\mathcal{D}_\tau) \cap \mathcal{B}_c(\theta))$. Notice that $\mathcal{D}(\tau) = \{(r,y,u) \in \mathbf{P}_\tau(\mathbf{U}) \mid y = y_{data}, u = u_{data}\}$,
$\mathbf{P}_\tau^{-1}(\mathcal{D}_\tau) = \{(r,y,u) \in \mathbf{U} \mid \mathbf{P}_\tau(y) = y_{data}, \mathbf{P}_\tau(u) = u_{data}\}$, $\mathbf{P}_\tau^{-1}(\mathcal{D}_\tau) \cap \mathcal{B}_c(\theta) = \{(r,y,u) \in \mathbf{U} \mid r = \theta^T w(u,y), \mathbf{P}_\tau(y) = y_{data}, \mathbf{P}_\tau(u) = u_{data}\}$ and $\mathbf{P}_\tau(\mathbf{P}_\tau^{-1}(\mathcal{D}_\tau) \cap \mathcal{B}_c(\theta)) = \{(r,y,u) \in \mathbf{P}_\tau(\mathbf{U}) \mid r = \theta^T w_{data}, y = y_{data}, u = u_{data}\}$, where w_{data} is defined by the equation (12) and by the equations (3) to (4).

A second step is to verify if the specification is satisfied, which give us the set of unfalsified controller parameters $\Theta^*(\tau)$
$= \{\theta \in \Theta \mid \|y_{data} - w_m * (\theta^T w_{data})\|_t^2 \leq \Delta(t),\ \forall t \in \mathcal{T} \cap [0,\tau]\}$
$= \{\theta \in \Theta \mid \|y_{data} - \theta^T \overline{w}_{data}\|_t^2 \leq \Delta(t),\ \forall t \in \mathcal{T} \cap [0,\tau]\}$
$= \{\theta \in \Theta \mid \theta^T A(t)\theta - 2\theta^T B(t) + C(t) \leq 0,\ \forall t \in \mathcal{T} \cap [0,\tau]\}$, where $A(\tau)$, $B(\tau)$, $C(\tau)$, $\overline{w}_{data}$, and w_{data} are defined by the equations (8) to (12). □

5 Finding a point in the intersection of a finite number of ellipsoids

5.1 An Ellipsoid Algorithm

A version of the ellipsoid algorithm is given in [11]. We will now give a modified version of this algorithm which allows us to find a point in the intersection of a finite number of ellipsoids. The basic idea of the algorithm is as follows. We start with an ellipsoid $\mathcal{E}^{(0)}$ that is obviously guaranteed to contain the intersection of all the ellipsoids. We then compute a cutting plane that passes through the center $x^{(0)}$ of $\mathcal{E}^{(0)}$. This means that we find a nonzero vector $g^{(0)}$ such that the intersection of all the ellipsoids lies in the half-space $\{z \mid g^{(0)T}(z - x^{(0)}) < 0\}$. We then know that the sliced half-ellipsoid

$$\mathcal{E}^{(0)} \cap \{z \mid g^{(0)T}(z - x^{(0)}) < 0\}$$

contains the intersection of all the ellipsoids. Now we compute the ellipsoid $\mathcal{E}^{(1)}$ of minimum volume that contains this sliced half-ellipsoid; $\mathcal{E}^{(1)}$ is then guaranteed to contain the intersection of all the ellipsoids. The process is then repeated.

5.2 Explicit description of the algorithm

As stated in [11], an ellipsoid $\mathcal{E}$ can be described as

$$\mathcal{E} = \{z \in \mathbb{R}^m \mid (z-a)^T A^{-1}(z-a) \leq 1\}$$

where $A = A^T > 0$. The minimum volume ellipsoid that contains the half-ellipsoid

$$\{z \in \mathbb{R}^m \mid (z-a)^T A^{-1}(z-a) \leq 1, g^T(z-a) \leq 0\}$$

is given by

$$\tilde{\mathcal{E}} = \{z \in \mathbb{R}^m \mid (z-\tilde{a})^T \tilde{A}^{-1}(z-\tilde{a}) \leq 1\},$$

where

$$\tilde{a} = a - \frac{A\tilde{g}}{m+1},$$

$$\tilde{A} = \frac{m^2}{m^2-1}(A - \frac{2}{m+1}A\tilde{g}\tilde{g}^T A), \quad \text{and}$$

$$\tilde{g} = \frac{g}{\sqrt{g^T A g}}.$$

The algorithm is initialized with $k = 0$, $x^{(0)}$ and $\mathcal{E}^{(0)}$ corresponding to one of the ellipsoids. The algorithm then proceeds as follows:

while $x^{(k)}$ does not belong to the intersection

compute a $g^{(k)}$ that defines a cutting plane at $x^{(k)}$

$\tilde{g} := \frac{g^{(k)}}{\sqrt{g^{(k)T} A g^{(k)}}}$

$x^{(k+1)} = x^{(k)} - \frac{A^{(k)}\tilde{g}}{m+1}$

$A^{(k+1)} = \frac{m^2}{m^2-1}(A^{(k)} - \frac{2}{m+1}A^{(k)}\tilde{g}\tilde{g}^T A^{(k)})$

$k := k+1$

Notice that if $x^{(k)}$ does not belong to the ellipsoid

$$\{\theta \mid \theta^T A\theta - 2\theta^T B + C \leq 0\}$$

then a cutting plane at $x^{(k)}$ is given by $g^{(k)} = Ax^{(k)} - B$.

The previous recursion generates a sequence of ellipsoids that are guaranteed to contain the intersection of all the ellipsoids. As stated in [11], it turns out that the volume of these ellipsoids decreases geometrically:

$$vol(\mathcal{E}^{(k)}) \leq e^{-\frac{k}{2m}} vol(\mathcal{E}^{(0)}).$$

Thus, the algorithm presented either finds a point in the intersection of a finite number of ellipsoids and an ellipsoid which contains that intersection, or determines if the volume of that intersection is smaller than a given value.

6 Another problem formulation dealt with the ellipsoid algorithm

Another problem formulation is also possible, if we take in consideration the norm of the reference signal when specifying the desired behavior in an unfalsified control problem [10]. Thus, let the desired behavior be given by

$$B_d = \{(r, y, u) \mid \mathcal{J}(r, y, u, \tau) \geq 0 \; \forall \tau \in \mathcal{T}\},$$

where r is a reference signal, (y, u) are plant signals, $\mathcal{J}(r, y, u, \tau) \triangleq \alpha(\tau)\|r\|_\tau{}^2 - \|(y - w_m * r)\|_\tau{}^2$, $w_m = \mathcal{L}^{-1}(W_m(s))$ is a time domain reference model transfer function, α is a function $\alpha : \mathcal{T} \to \mathbb{R}_+$ and $\mathcal{T} \subset [0, \tau]$ is a set of time instants. The set of unfalsified candidates is, in this case, given by the set of parameters expressed in the following theorem.

Theorem 6.1 *If*

$$\mathcal{J}(r, y, u, \tau) \triangleq \alpha(\tau)\|r\|_\tau{}^2 - \|y - w_m * r\|_\tau{}^2.$$

At each time τ, the set of unfalsified controllers is given by the set of parameters $\{\theta \mid \theta^T A(t)\theta - 2\theta^T B(t) + C(t) - \alpha(t)\theta^T D(t)\theta \leq 0, \; \forall t \in \mathcal{T} \cap [0, \tau]\}$, where $A(\tau)$ and $B(\tau)$ are given by expressions (8) and (9),

$$C(\tau) = \int_0^\tau e^{-2\sigma(\tau - t)} y_{data}^2 dt - \Delta(\tau), \quad and \quad (13)$$

$$D(\tau) = \int_0^\tau e^{-2\sigma(\tau - t)} w_{data} w_{data}^T dt, \quad (14)$$

$$(15)$$

with $\overline{w}_{data}$ and w_{data} given by expressions (11) and (12), provided that $\|u_{data}\|_\tau + \|y_{data}\|_\tau \neq 0$.

Proof. Since the set $\mathbf{P}_\tau(\mathbf{P}_\tau^{-1}(\mathcal{D}_\tau) \cap \mathcal{B}_c(\theta))$, is the same as in the previous theorem, we can proceed directly to the verification step,
$\Theta^*(\tau)$
$= \{\theta \in \Theta \mid \|y_{data} - w_m * (\theta^T w_{data})\|_t{}^2 \leq \alpha(t)\|\theta^T w_{data}\|_t{}^2, \; \forall t \in \mathcal{T} \cap [0, \tau]\}$
$= \{\theta \in \Theta \mid \|(y_{data} - \theta^T \overline{w}_{data})\|_t{}^2 \leq \alpha(t)\|\theta^T w_{data}\|_t{}^2, \; \forall t \in \mathcal{T} \cap [0, \tau]\}$
$= \{\theta \in \Theta \mid \theta^T(A(t) - \alpha(t)D(t))\theta - 2\theta^T B(t) + C(t) \leq 0, \; \forall t \in \mathcal{T} \cap [0, \tau]\}$,
where $A(\tau)$, $B(\tau)$, are defined by the equations (8) to (9), and $C(\tau)$, $D(\tau)$, are defined by the equations (13) to (14) whereas $\overline{w}_{data}$, and w_{data} are defined by the equations (11) to (12). □

Notice that $A(t) \geq 0$ with the inequality being strict when the disturbance is "persistently exciting" for $\bar{w}(t)$. Consequently, provided we choose $\alpha(t)$ sufficiently small so that $A(t) - \alpha(t)D(t) > 0 \; \forall t \in [0, \tau]$, then the solution to the problem is given by an intersection of ellipsoids exactly as in Section 4.

7 Practical Considerations

The essential idea behind the computational procedure to be used is to apply the ellipsoid algorithm to find a candidate which is the center of an ellipsoid that contains the whole set of unfalsified candidates.

Notice that if we have a poor excitation or a specification that is too relaxed we may have an intersection of sets that are not ellipsoids. The essential point to have in mind is that if we have a convex set then there is a cutting plane and if this convex set is an ellipsoid then a cutting plane is very easily computed.

8 Example

In this section, we present simulation results for the unfalsified model reference control problems formulated in sections 3 and 6. Let us choose $W_m(s) = \frac{1}{s+1} = \frac{s+1}{(s+1)^2}$. Choosing the second form we have that $n = 2$. Additionally, let us choose $\Lambda = -1$, $l = 1$ and $\sigma = 0.01$. The filter w defined in [10] is then given by $w(u, y) = (u, w_m * u, y, w_m * y)^T$ and the class of candidate controllers is given by $\{(\mathbf{U}, \mathcal{B}_c(\theta)) \mid \theta \in \Theta\}$, where $\mathcal{B}_c(\theta) = \{(r, y, u) \mid r = \theta^T w(u, y)\}$ and θ is a constant parameter vector in $\mathbb{R}^4$. For purposes of this simulation let "the true but unknown plant" be given by

$$\begin{aligned} \dot{x} &= \begin{bmatrix} 0 & -1 \\ 1 & 1 \end{bmatrix} x + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u + \begin{bmatrix} n_1 \\ n_2 \end{bmatrix} \\ y &= \begin{bmatrix} 0 & 1 \end{bmatrix} x \end{aligned}$$

where n_1 and n_2 are uncorrelated normally distributed random signals with mean zero and variance one. Let the reference signal $r(t) = sign(cos(0.1\pi t)) \; \forall t \geq 0$. We obtain (y_{data}, u_{data}) by closing the loop with the initial controller associated to the parameter $\theta(0) = [\; 1 \quad 0 \quad 0.1 \quad -0.1 \;]^T$ and the initial plant state given by $x = 0$. Thus we are able to use our theory to compute a new controller parameter $\theta(t)$ based on the data available at any given time $t = \tau$. Controller adaptation is achieved by repeating this operation periodically as time τ evolves and (y_{data}, u_{data}) accumulates, in order to update the controller parameter θ. This procedure was used to update the controller parameter vector $\theta(t)$ every 5 sec starting at time $\tau = 5$.

For the first problem formulation with $\Delta(t) = 0.1t$, we obtained the simulation results shown in figure 2, while for the second problem formulation with $\alpha(t) = .0001$ we obtained the simulation results shown in figure 3. Let us notice that we plotted the graphic of the determinant of A_k versus time, since the volume of the corresponding ellipsoid is equal to the product of a constant, which depends on the dimension of the parameter space, times the square root of that determinant.

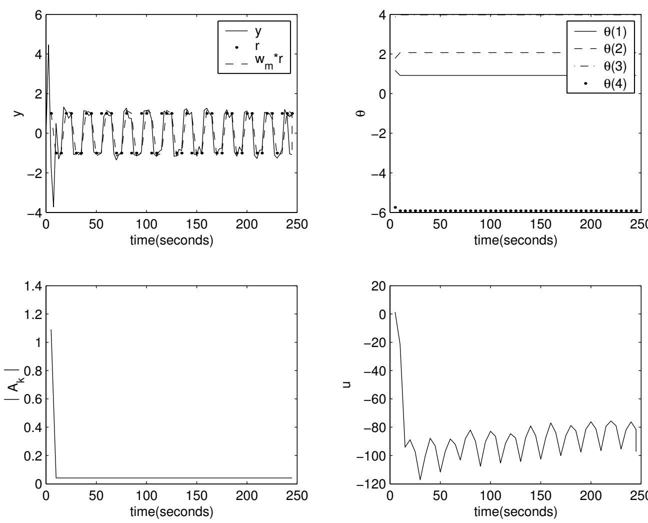

Figure 2: Simulation results for $B_d = \{(r, y, u) \mid \|y - w_m * r\|_\tau{}^2 \leq 0.1\tau \, \forall \tau \in \mathcal{T}\}$

9 Concluding Remarks

A formulation of the model reference adaptive control problem according to the unfalsified control paradigm was given. This formulation used a performance specification given in terms of a quadratic inequality which should be satisfied at a set of time instants (multi-ellipsoid cost).

The need for gridding the parameter space in order to solve the ellipsoid intersection problem in unfalsified control was overcome through the application of the ellipsoid algorithm to approximate the unfalsified set. This algorithm was used to determine a candidate which is the center of an ellipsoid which contains the set of unfalsified candidates.

Let us also notice that the sequence of decreasing volume ellipsoids which contains the set of unfalsified controllers

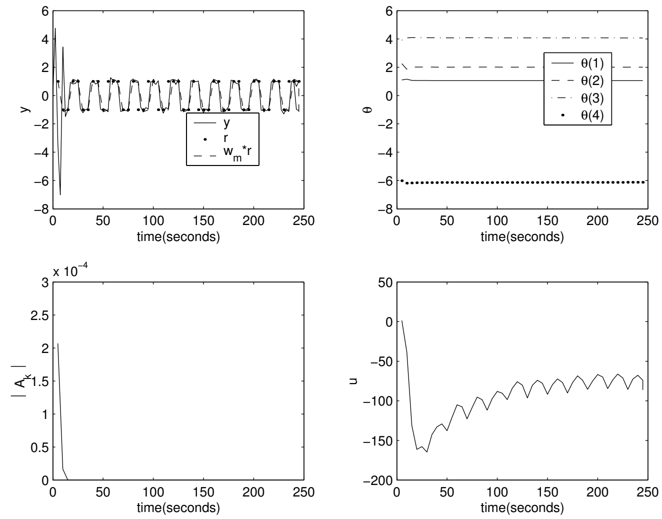

Figure 3: Simulation results for $B_d = \{(r, y, u) \mid \|y - w_m * r\|_\tau{}^2 \leq 0.0001\|r\|_\tau{}^2 \;\; \forall \tau \in \mathcal{T}\}$

gives a characterization of learning in model reference adaptive control in terms of a sequence of recursively data-based computable ellipsoids of controllers.

Regarding the ellipsoid algorithm used, it is also important to notice that deeper cuts would be possible if the cuts were not required to pass through the center of the prior ellipsoid. This would accelerate the determination of an unfalsified candidate and give a point deep in the intersection. The computational simplicity and properties of the ellipsoid algorithm with center cuts were a major factor when choosing the computational procedure to be used. For several simple modifications to the basic ellipsoid algorithm to improve its rate of convergence we refer to [15].

Simulation results provided for the control of an unstable plant illustrated the potential using the ellipsoid algorithm for model reference adaptive control.

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