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RECENT ADVANCES IN ROBUST CONTROL THEORY

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Abstract

The problem of modeling-uncertainties that may not conform to assumed prior bounds is considered from an adaptive control perspective, but without the standard assumptions of adaptive control. A supervisory control architecture is employed, based on the data-driven logic of unfalsification. The supervisory controller modifies or replaces controllers when sensor data falsifies the hypothesis that the currently active controller is insufficiently robust to satisfy performance goals. The resultant unfalsified adaptive control law responds rapidly and precisely with guaranteed convergence. No assumptions about the plant are required beyond the availability of at least one candidate controller capable of robustly meeting performance goals. Potential applications include aircraft stability augmentation systems, highly maneuverable aircraft design, missile guidance systems, and precision pointing and tracking systems. Such designs will be better able to more reliably compensate for battle damage, equipment failures and other changing circumstances.

> *"It is a capital mistake to theorize before one has data. Insensibly one begins to twist facts to suit theories instead of theories to suit facts."*
>
> Sherlock Holmes
> Arthur Conan Doyle

INTRODUCTION

Though the robust multivariable control theory that has evolved over the past quarter century offers a major improvement over earlier algebraic and optimal control methods, it cannot produce reliable control designs unless reliable prior bounds on plant uncertainty are available. This applies to methods based on the H_∞ μ/K_m-synthesis, and BMI/LMI/IQC theories [1]–[6]. These robust control design methods all have an Achilles heel: They are dependent of the premise that uncertainty models are reliable, and they offer little guidance in the event that experimental data either invalidates prior knowledge of uncertainty bounds or, perhaps, provides evidence of previously unsuspected patterns in the data. That is, the standard H_∞ μ/K_m-synthesis, and BMI/LMI/IQC robust control techniques fail in the all too common situation in which prior knowledge is poor or unreliable.

To correct this, reliable *data-driven* adaptive design techniques are needed. Ideally, these techniques should incorporate mechanisms for evaluating the design implications of each new experimental data point, and for directly integrating that information into the mathematics of the robust control design process to allow methodical update and re-design of control strategies so as to accurately reflect the implications of new or evolving experimental data. Examples of recent thrusts in this direction are indirect controller tuning/adaptation methods based on control-oriented identification theory and [7]–[26] and, more recently, related direct methods that bypass plant identification based on controller unfalsification [27]–[56]. While both control-oriented identification theory and unfalsified control theory are concerned with the difficult problem of assimilating real-time measurement data into the otherwise introspective process of robust control design, the unfalsified control approach is a particular interest because it directly and precisely characterizes the control design implications of experimental data.

DATA-DRIVEN ROBUST CONTROL

Validation — or more precisely *unfalsification* — of hypotheses against physical data is the central

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aspect of the process of scientific discovery. This validation process allows scientists to sift the elegant tautologies of pure mathematics in order to discover mathematical descriptions of nature that are not only for logically self-consistent, but also consistent with physically observed data. This data-driven process of validation is also a key part engineering design. Successful engineering design techniques inevitably arrive at a point where pure introspective theory and model-based analyses must be tested against physical data. But, in control engineering in particular, the validation process is one that has been much neglected by theoreticians. Here, the theory tying control designs to physical data has for the most part focused on pre-control-design 'system identification'. Otherwise, the mathematization of the processes of post-design validation and re-design has remained relatively unexplored virgin territory. In particular, a satisfactory quantitative mathematical theory for direct feedback of experimental design-validation data into the control design process has been lacking, though this seems to be changing with the recent introduction of a theory of unfalsified control [31].

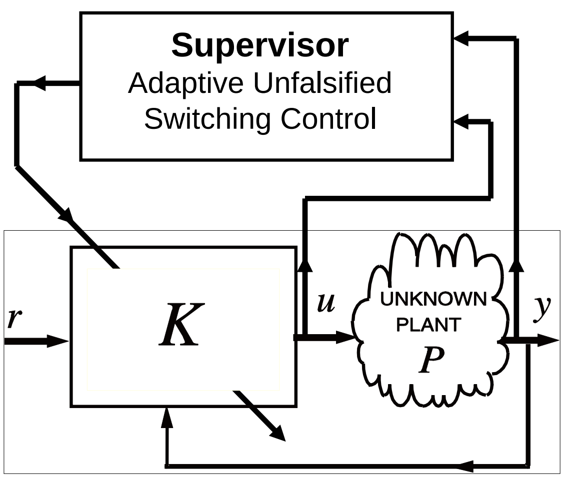

Figure 1: An unfalsified adaptive controller has a two-tier structure, consisting of a adaptive supervisor and a conventional controller K. The supervisor monitors plant data (u, y) for evidence that would falsify candidate controllers. If the currently active controller K becomes falsified by data, then an as yet unfalsified controller is switched into the loop to replace it.

Theory: Validation and Unfalsification

Unfalsified control is essentially a data-driven adaptive control theory that permits learning based on physical data via a process of elimination, much like the candidate elimination algorithm of Mitchell [57, 58]. The theory concerns the feedback control configuration in Figure 1. As always in control theory, the goal is to determine a control law K for the plant P such that the closed-loop system response, say T, satisfies given specifications. Unfalsified control theory is concerned with the case in which the plant is either unknown or is only partially known and one wishes to fully utilize information from measurements in selecting the control law K. In the theory of unfalsified control, learning takes place when new information in measurement data enables one to eliminate from consideration one or more candidate controllers.

As indicated in Fig. 2 three elements that define the unfalsified control problem are (1) plant measurement data, (2) a class of candidate controllers, and (3) a performance specification, say T_{spec}, consisting of a set of admissible 3-tuples of signals (r, y, u). More precisely, we have the following.

Definition [31] *A controller K is said to be* **falsified** *by measurement information if this information is sufficient to deduce that the performance specification $(r, y, u) \in \mathsf{T}_{spec}\ \forall r \in \mathcal{R}$ would be violated if that controller were in the feedback loop. Otherwise, the control law K is said to be* **unfalsified**. ■

To put plant models, data and controller models on an equal footing with performance specifications, these like T_{spec} are regarded as sets of 3-tuples of signals (r, y, u) — that is, they are regarded as relations in $\mathcal{R} \times \mathcal{Y} \times \mathcal{U}$. For example, if $P : \mathcal{U} \to \mathcal{Y}$ and $K : \mathcal{R} \times \mathcal{Y} \to \mathcal{U}$ then

$$\begin{aligned} \mathsf{P} &= \left\{ \ (r, y, u) \ \middle| \ y = Pu \ \right\} \\ \mathsf{K} &= \left\{ \ (r, y, u) \ \middle| \ u = K \begin{bmatrix} r \\ y \end{bmatrix} \ \right\}. \end{aligned}$$

And, if $J(r, y, u)$ is a given loss-function that we wish to be non-positive, then the performance specification T_{spec} would be simply the set

$$\mathsf{T}_{spec} = \left\{ \ (r, y, u) \ \middle| \ J(r, y, u) \le 0 \ \right\}. \qquad (1)$$

On the other hand, experimental information from a plant corresponds to *partial* knowledge of the plant

P. Loosely, data may be regarded as providing a sort of an "interpolation constraint" on the graph of P — i.e., a 'point' or set of 'points' through which the infinite-dimensional graph of dynamical operator P must pass.

Typically, the available measurement information will depend on the current time, say τ. For example, if we have complete data on (u, y) from time 0 up to time $\tau > 0$, then the *measurement information* is characterized by the set [31]

$$P_{data} \triangleq \left\{ (r, y, u) \in \mathcal{R} \times \mathcal{U} \times \mathcal{Y} \;\middle|\; P_\tau \begin{bmatrix} (u - u_{data}) \\ (y - y_{data}) \end{bmatrix} = 0 \right\} \tag{2}$$

where P_τ is the familiar time-truncation operator of input-output stability theory (cf. [59, 60]), viz.,

$$[P_\tau x](t) \triangleq \begin{cases} x(t), & \text{if } 0 \le t \le \tau \\ 0, & \text{otherwise.} \end{cases}$$

The main result of unfalsified control theory is the following theorem which gives necessary and sufficient conditions for past open-loop plant data P_{data} to falsify the hypothesis that controller K can satisfy the performance specification T_{spec}.

Unfalsified Control Theorem [31] *A control law K is unfalsified by measurement information P_{data} if, and only if, for each triple $(r_0, y_0, u_0) \in P_{data} \cap K$, there exists at least one pair $(\hat{u}_0, \hat{y}_0)$ such that*

$$(r_0, \hat{y}_0, \hat{u}_0) \in P_{data} \cap K \cap T_{spec}. \tag{3}$$

Proof: *With controller K in the loop, a command signal $r_0 \in \mathcal{R}$ could have produced the measurement information if, and only if, $(r_0, y_0, u_0) \in P_{data} \cap K$ for some (u_0, y_0). The controller K is unfalsified if and only if for each such r_0 there is at least one (possibly different) pair (u_1, y_1) which also could have produced the measurement information with K in the loop and which additionally satisfies the performance specification $(r_0, y_1, u_1) \in T_{spec}$. That is, K is unfalsified if and only if for each such r_0, condition (3) holds.* ■

The **Unfalsified Control Theorem** constitutes a mathematically precise statement of what it means for experimental data and a performance specification to be inconsistent with a particular controller. It has some interesting implications:

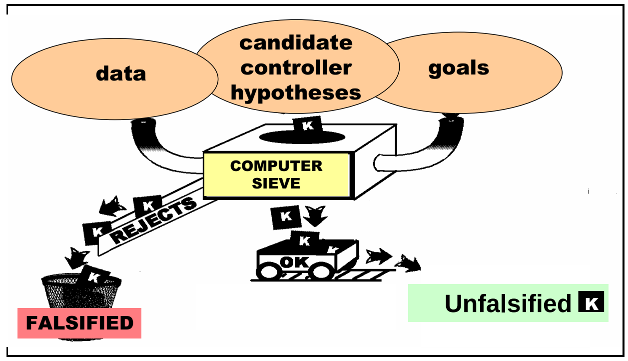

Figure 2: An unfalsification process is used supervisory control design. The process requires three types of input (1) goals, (2) candidate controllers and (3) data. Controllers are sifted to find those that are consistent with both performance goals and physical data. No plant models are required while the process is running, though a plant model can be useful for prior selections of the candidate controllers and the performance goal.

- The **Unfalsified Control Theorem** is nonconservative; i.e., it gives "if and only if" conditions on K. It uses all the information in the past data — and no more. It provides a mathematically precise "sieve" which rejects any controller which, based on experimental evidence, is demonstrably incapable of meeting a given performance specification.

- The **Unfalsified Control Theorem** is "model free". No plant model is needed to test its conditions. There are no assumptions about the plant.

- Information P_{data} which invalidates a particular controller K need not have been generated with that controller in the feedback loop; it may be open loop data or data generated by some other control law (which need not even be in $\mathbf{K}$).

- When the sets P_{data}, K and T_{spec} are each expressible in terms of equations and/or inequalities, then falsification of a controller reduces to a minimax optimization problem. For some forms of inequalities and equalities (e.g., linear or quadratic), this optimization problem may be solved analytically, leading to procedures for direct identification of controllers — as the example in [34].

Data-Driven Learning and Adaptive Control

The unfalsified control theorem says simply that controller falsification can be tested by computing an intersection of certain sets of signals. A noteworthy feature of the unfalsified control theory is that a controller need not be in the loop to be falsified. Broad classes of controllers can be falsified with open-loop plant data or even data acquired while other controllers were in the loop. Adaptive control is achieved within the this framework by using the unfalsification process as the key element of a supervisory controller (cf. [61, 62]). The supervisor switches an unfalsified controller into the feedback loop whenever the current controller in the loop is amongst those falsified by observed plant data — see Fig. 2. The supervisor chooses as the current control law one that is not falsified by the past data, resulting in a control law that is adaptive in the sense that it learns in real time and changes based on what it learns.

Like the controllers of [63, 64], this approach to adaptive real-time unfalsified control leads to a sort of "switching control." Controllers which are determined to be incapable of satisfactory performance are switched out of the feedback loop and replaced by others which, based on the information in past data, have not yet been found to be inconsistent with the performance specification. However, adaptive unfalsified controllers generally would not be expected to exhibit the poor transient response associated with switching methods such as [63]. The reason is that, unlike the theory in [63], unfalsified control theory efficiently eliminates broad classes of controllers before they are ever inserted in the feedbackunfalsified control and other adaptive methods is that in unfalsified control one evaluates candidate controllers objectively based on experimental data alone, without prejudicial assumptions about the plant.

While, in principle, the unfalsified control theory allows for the set K to include continuously parameterized sets of controllers, restricting attention to candidate controller sets K with only a finite number of elements can simplify computations. Further simplifications result by restricting attention to candidate controllers that are "causally-left-invertible" in the sense that, given a $\mathsf{K} \in \mathsf{K}$, the current value of $r(t)$ is uniquely determined by past values of $u(t), y(t)$. When (2) holds, these restrictions on T_{spec} and K are sufficient to permit the unfalsified set to be evaluated in real-time via the following conceptual algorithm.

Algorithm (Recursive Adaptive Control)

INPUT:

- *A finite set K of m candidate dynamical controllers $K_i(r,y,u) = 0$, $(i = 1,\ldots,m)$ each having the causal-left-invertibility property that $r(t)$ is uniquely determined from $K_i(r,y,u) = 0$ by past values of $u(t), y(t)$.*
- *Sampling interval Δt and current time $\tau = n\Delta t$;*
- *Plant data $(u(t), y(t))$, $t \in [0,\tau]$;*
- *Performance specification set T_{spec} consisting of the set of triples (r,y,u) satisfying for all $k = 1,\ldots,n$*

$$\int_0^{k\Delta t} \tilde{T}_{spec}(r(t), y(t), u(t), t)\, dt \leq 0.$$

INITIALIZE:

set $k = 0$, set $\hat{i} = m$;

for $i = 0 : m$, set $s(i) = 1$, set $\tilde{J}(i) = 0$, end.

PROCEDURE:

while $\hat{i} > 0$;
$k = k + 1$;
for $i = 1 : m$;
if $s(i) > 0$;
for each $t \in [(k-1)\Delta t, k\Delta t]$;
solve $K_i(r,y,u) = 0$ for $r(t)$;
(note that $r(t)$ exists and is unique since K_i has the causal-left-invertibility property)
end;
$\tilde{J}(i) = \tilde{J}(i) + \int_{(k-1)\Delta t}^{k\Delta t} \tilde{T}_{spec}(r(t), y(t), u(t), t)\, dt$;
if $\tilde{J}(i) > 0$, set $s(i) = 0$,
end;
end;
end;
$\hat{i} = \max\{\ i \mid s(i) > 0\ \}$;
end.

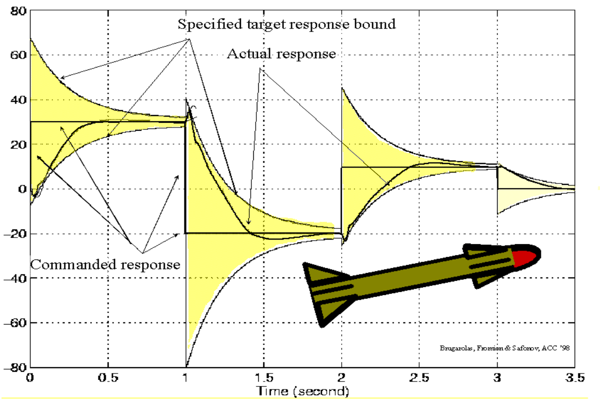

Figure 3: A data-driven unfalsified missile controller would have abilities to adaptively discover solutions in real-time to compensate for sudden in-flight changes and damage.

This algorithm returns for each time the least index $\hat{i}$ for which $K_{\hat{i}}$ is unfalsified by the past plant data. Real-time unfalsified adaptive control is achieved by always taking as the currently active controller

$$\hat{K} \triangleq K_{\hat{i}}$$

provided that the data does not falsify all candidate controllers. In this latter case, the algorithm terminates and returns $\hat{i} = 0$.

It is important to note that while the above algorithm is geared towards the case of an integral inequality performance criterion T_{spec} and a finite set of causally-left-invertible K_i's, the underlying theory is, in principle, applicable to arbitrary non-finite controller sets $\mathbf{K}$ and to hybrid systems with both discrete and continuous time elements.

Comment *If the plant is slowly time-varying, then older data ought to be discarded before evaluating controller falsification. This may be effected within the context of our Algorithm by* fixing $\tau = \tau_0$ *and regarding* $t - \tau_0$ *as the deviation from the current time. The result is a an algorithm which only considers data from moving time-window of fixed duration* τ_0 *time-units prior to the current real-time. In this case the unfalsified controller set* $\mathbf{K}_{OK}$ *no longer shrinks monotonically as it would if* τ *were increasing in lockstep with real-time.*

Design Studies

Design studies have confirmed the theoretical expectation that supervisory controllers that are designed based on the logic of unfalsification can be effective in closing the outer data-driven loop on the control design process. Unfalsified control theory has proved effective in applications involving both off-line controller gain tuning and in real-time adaptive control design studies. Following is a brief description of some of the design studies that have helped us to better understand the potential of the unfalsified control theory, as well as limitations of the current theory.

Missile Autopilot**:** One design study that we conducted involved using an unfalsified controller to robustly discover PID controller gains for an adaptive missile autopilot 'on the fly' in real-time [36]. Figure 3 summarizes the results of the missile design. In all trials, the response of the adaptive loops was swift and sure-footed — in stark contrast to what would be expected from traditional quasi-static adaptive methods (e.g., standard model reference adaptive control).

Universal PID Controller**:** One application of the theory involved implementing a PID-based adaptive 'universal' controller implemented as MATLAB Simulink block based on the unfalsified theory [37] — see Fig 4. The controller sifts through a bank of candidate controllers in real-time, identifying which of the 30 controllers is unfalsified with respect the performance goal of 'mixed-sensitivity' type, viz.,

$$\|w_1 * (r-y)\|_\tau^2 + \|w_2 * u\|_\tau^2 - \sigma^2\tau \le \|r\|_\tau^2 + \rho$$

$$\text{where } \|f\|_\tau^2 = \int_0^\tau |f(t)|^2 dt$$

where w_1 and w_2 are 'weighting' filters and ρ and *sigma* are constants chosen by the designer based on control bandwidth and robustness considerations – exactly as in standard mixed-sensitivity robust control design. Initially, there are 30 candidate PID gain combinations, indicated on the vertical axis. Unfalsified controllers are indicated by the horizontal traces. When the currently active controller indicated by the bold trace becomes falsified, then one of the as yet unfalsified controllers is switched into the loop to replace it. Notice that adaptive supervisor loop is so fast that the controller is able to stabilize the open-loop unstable plant without prior plant knowledge and without appreciable transients. The unfalsification procedure always discovers a stabilizing controller, The supervisory controller designed via unfalsified control quickly discovers a stabilizing candidate controller for the open-loop-unstable plant. The unfalsified control theory assures the procedure will always converge to a stabilizing con-

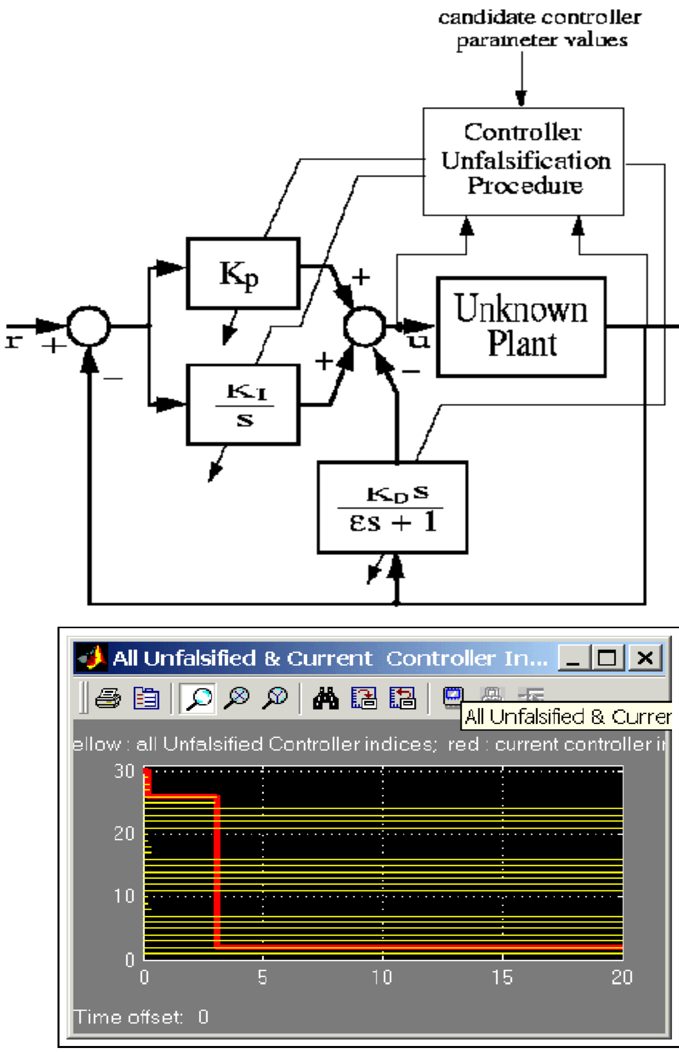

Figure 4: Here we see a MATLAB Simulink time-history for an adaptive 'universal PID controller'.

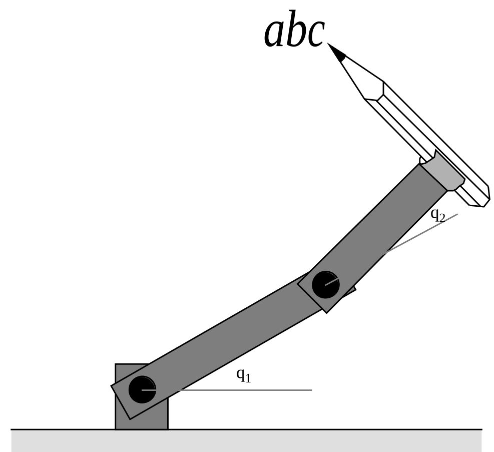

Figure 5: Unfalsified control produced superior results for a nonlinear two-link robot manipulator subject to uncertain dynamics, noisy disturbances and abrupt changes in load mass. The two sluggishly smooth traces large amplitude signals in the plot are with a conventional adaptive controller used to adjust control gain-vector $\theta(t)$, and the two very low amplitude traces are for the unfalsified controller. The unfalsified controller had a much quicker, sure-footed and precise response without increased control effort.

troller that meets the performance goals, provided only that the initial candidate controller set contains at least one such controller; *no other prior knowledge of the plant is necessary to assure convergence.* In particular, the unfalsification logic in the supervisor is guaranteed to stabilize and meet performance goals without any of the 'standard assumptions' on the plant (e.g., [65, 66]) — viz., without minimum phase, linearity, bounds on relative-degree, knowledge of the sign of the plant high frequency gain, or persistence of excitation.

Robot Manipulator Arm**:** We used to unfalsified methodology to adaptively tune the parameters of a nonlinear 'computed-torque' controller for a robot manipulator arm [67, 32] — see Fig 5. The arm proved to be capable of a quick and reliable control response despite large and sudden variations in load mass. Again, the controller performed with precision, despite noise, dynamical actuator uncertainties and without prior knowledge of the plant model or its parameters. Results for the robot design were surefooted and precise, with the controller maintaining an order of magnitude more precise control than a similar model-reference adaptive controller during widely fluctuating manipulator load variations; the controller was also more robust in that it was capable of maintaining precise control even during load variations that destabilized a similarly structured model-reference adaptive controller.

Industrial Process Control**:** Although very few researchers other than ourselves have as yet examined unfalsified control methods, those who have taken this step have predictably confirmed the effectiveness of unfalsified control methods in several industrial process control applications. For example, Kosut [38] examined unfalsified controller for direct data-driven off-line control gain tuning under the assumption of a noise-free linear-time-invariant plant. Woodley, How and Kosut [68] and used the theory with good result for data-driven discovery of good control gains for a laboratory control problem involving two spring-connected masses. Also, Collins and Fan [39] successfully used the unfalsified control methodology in a run-to-run setting to tune gains off-line in an industrial weigh-belt feeder control design study. More recently, there have been some promising adaptive control applications to machine control by Razavi and Kurfess [40, 69] based on the unfalsified control methodology.

DISCUSSION

In the unfalsified control approach to supervisory control, decisions to adapt are data-driven. Determination of which candidate control laws are suitable are made based on experimental evidence, i.e., the actual values of sensor output signals and actuator input signals. In this process the role, if any, of plant models and of probabilistic hypotheses about stochastic noise and random initial conditions is entirely an a priori role: These provide concepts which are useful in selecting the class K of candidate controllers and in selecting achievable goals (i.e., selecting T_{spec}). The methods of traditional model-based control theories (root locus, stochastic optimal control, Bode-Nyquist theory, H_∞ robust control and so forth) provide mechanizations of this prior selection and narrowing process. Unfalsified control takes over where traditional model-based methods leave off, providing a mathematical framework for determining the proper consequences of experimental observations on the choice of control law. In effect, the theory gives one a model-free mathematical "sieve" for candidate controllers, enabling us (i) to precisely identify what of relevance to attaining the specification T_{spec} can be discovered from experimental data alone and (ii) to clearly distinguish the implications of experimental data from those of assumptions and other prior information.

The **Unfalsified Control Theorem** explains the learning mechanisms of adaptive control theory. It provides an exact characterization of what can, and what cannot, be learned from experimental data about the ability of a given class of controllers to meet a given performance specification. A salient feature of the theory is that the data used to falsify a class of control laws may be either open-loop data or data obtained with other controllers in the feedback loop. Consequently, large classes of candidate controllers are falsified by even a few experimental samples of plant input output data. Candidate controllers need not be actually inserted in the feedback loop to be falsified. This is important because it means that adaptive unfalsified controllers will be significantly less susceptible to poor transient response than adaptive learning algorithms which require inserting controllers in the loop one-at-a-time to determine if they are unsuitable.

A noteworthy feature of the unfalsified control theory is its flexibility and simplicity of implementation. Controller falsification typically involves only real-time integration of algebraic functions of the observed data, with one set of integrators for each candidate controller. The theory may be readily applied to nonlinear time-varying plants, as well as to linear time-invariant ones.

CONCLUSION

As robust control theory has matured, a key challenge has been the need for a more flexible theory that provides a unified basis for representing and exploiting *evolving* real-time data. The role of *unfalsified control* is to close the loop on the adaptive and robust control design processes by developing precise data-driven methods to implement supervisory control schemes that can optimally exploit the information in data to enhance robustness and improve performance. The types of control designs will be better able to compensate for uncertain and time-varying effects, battle damage, equipment failures and other changing circumstances.

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