Fitting Controllers to Data: The MIMO Case

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***Abstract*— The problem of optimally fitting controllers to data is examined for the identification of a controller from a given class of MIMO controllers used in model reference adaptive control. The problem of identifying a MIMO controller from this class is formulated. This formulation leads to an optimization problem where a best $2nm \times m$ matrix of parameters is looked for. It is proved that the solution to this problem can be given in terms of the solution of an optimization problem where a best $2nm^2 \times 1$ vector of parameters is looked for, which has already been solved for the SISO case. Simulations are provided for an example which has appeared a few times in the model reference adaptive control literature. The formulation and solution of this problem illustrates a unifying link between the design of model reference adaptive control for SISO and MIMO linear systems.**

Keywords: multivariable systems; fitting; behavioral approach; model reference; adaptive control.

I. Introduction

The design of model reference adaptive control for MIMO linear systems has been considered in several works for the past few years ([1],[2],[3],[4],[5]). However, differently from what happens with SISO linear systems, where we have “a parameter estimation perspective for model reference adaptive control” [6], the identification step for MIMO linear systems has been dealt with, in many situations, through the application of arbitrary identification algorithms.

In this work, we show that it is possible to deal with MIMO controllers in a way analogous to the way SISO controllers were dealt with in [7]. More, specifically, we examine the problem of fitting controllers to data for one class of MIMO controllers used in model reference adaptive control.

In the fitting controllers to data approach we take advantage of the the division of the adaptive control problem in two parts according to [9]: an algebraic part and an analytic part. From the analytic part we formulate our class of candidate controllers. The analytic part, however is substituted by an optimization problem where we try to find the controller that fits a performance criterion in an optimal way.

For the MIMO problem, the class of candidate controllers and performance criterion are related to the algebraic part of the multivariable adaptive control problem presented in ([3],[4],[5]).

The formulation of the problem of fitting MIMO controllers to data leads to an optimization problem where a best $2nm \times m$ matrix of parameters is looked for. It is proved that the solution to this problem can be given in terms of the solution of an optimization problem where a best $2nm^2 \times 1$ vector is looked for, which has already been solved for the SISO case [7].

II. Basic Definitions

The problem of optimally fitting controllers to data was defined in [7] according to the Willems' behavioral approach to dynamical systems [8]. Two basic definitions of the behavioral framework were used, namely the definition of a mathematical model and the definition of a data set.

Definition 2.1: A mathematical model is a pair $(\mathbf{U}, \mathscr{B})$, with $\mathbf{U}$ the universum — its elements are called outcomes — and $\mathscr{B} \subseteq \mathbf{U}$ the behavior.

Definition 2.2: A data set is a nonempty subset $\mathscr{D}$ of $\mathbf{U}$.

Using these two definitions, defining a controller as a mathematical model, noticing that the intersection of behaviors is a way of additional restrictions on a system, an optimization problem was formulated with the goal of finding a best controller from a given class.

Definition 2.3: A controller is a mathematical model.

Problem 2.1: Given a class of controllers $\{(\mathbf{U}, \mathscr{B}_c(\theta)) \mid \theta \in \Theta\}$, where Θ is a set of parameter vectors, the performance (cost) index $\mathscr{J}_\tau$, the operator $\mathscr{E}$, the time truncation operator $\mathbf{P}_\tau$, $\tau \in \mathbb{R}_+$, and a data set $\mathscr{D}_\tau \subset \mathbf{P}_\tau \mathbf{U}$, find the set of parameters Θ^* such that

$$\Theta^*(\tau) = \arg\min_{\theta \in \Theta} \mathscr{J}_\tau(\theta) \tag{1}$$

where

$$\mathscr{J}_\tau(\theta) \stackrel{\Delta}{=} \mathscr{E}(\{\mathscr{I}_\tau(b) \mid b \in \mathbf{P}_\tau(\mathbf{P}_\tau^{-1}(\mathscr{D}_\tau) \cap \mathscr{B}_c(\theta))\}).$$

and $\mathscr{E}$ denotes either the mean, the max, or the expectation operator and $\mathscr{I}_\tau$ is a functional

$$\mathscr{I}_\tau : \mathbf{P}_\tau(\mathbf{U}) \to \mathbb{R}.$$

Let us notice that $\mathbf{P}_\tau^{-1}(\mathscr{D}_\tau)$ gives all the information consistent with the data $\mathscr{D}_\tau$. The expression $(\mathbf{P}_\tau^{-1}(\mathscr{D}_\tau)) \cap \mathscr{B}_c$ gives the behavior of the closed loop system.

III. Examination of the MIMO MRAC Problem

The adaptive control problem is usually divided into two parts [9], an algebraic part and an analytic part. The algebraic part deals with a matching problem whereas the analytic part deals with the updating of controller parameters in such a way that the solution of the matching problem is attained.

A. The Algebraic Part

An examination of the algebraic part of the MIMO MRAC problem is useful for the specification of the class of candidate controllers to be used and for the specification of the model to be followed in the formulation of the problem of fitting MIMO controllers to data.

For the MIMO MRAC ([3],[4],[5]), the "controller" structure is parameterized by a variable ν which is an upper bound on the maximum of the observability indices of the plant. For the problem of fitting MIMO controllers to data, on the other hand, since we do not have a priori plant information, the class of candidate controllers will be simply indexed by a parameter n. The data will tell us whether a given controller of this class corresponds to a good or a bad match.

Regarding the model to be followed, for the multivariable model reference control problem ([3],[4],[5]) the model to be followed is given by a matrix

$$H(s) = \begin{bmatrix} \frac{1}{(s+a)^{r_1}} & 0 & \cdot & \cdot \\ \frac{1}{(s+a)^{r_2-1}} & \frac{1}{(s+a)^{r_2}} & 0 & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \frac{1}{(s+a)^{r_m}} \end{bmatrix}.$$

where $\partial H_{ij}(s) < r_i - 1$ and a is arbitrary, but fixed a priori.

The matrix $H(s)$ corresponds to the Hermite normal form *of the* plant $P(s) \in \Re^{m\times m}$, i.e. there exists a plant $P(s) \in \Re^{m\times m}$ such that the following property is satisfied

$$P(s) = H(s)U(s),$$

$$\text{with} \quad U(s) \in \Re^{m\times m} \quad \text{and} \quad \lim_{s\to\infty} U(s) = K_p.$$

For the fitting controllers to data problem, on the other hand, we can specify the above model to be followed, but the above statement should be modified to "…$H(s)$ corresponds to the Hermite normal form *for some* plant $P(s) \in \Re^{m\times m}$."

B. The Analytic Part

For the fitting controllers to data problem, the analytic part of the MIMO MRAC problem is substituted by an optimization problem. The cost to be minimized is different from the cost used in a fitting models to data problem, since it takes in consideration the norm of the reference signal. Notice that this is important in light of the fact that for the MRAC problem, the equations that determine the control law constitute a parameter relation between plant signals and the reference signal. So, given plant data and a control law, the reference input signal is then *implicitly* determined. In particular, this means that any reference signal thusly determined will in general depend on hypothesized control law parameters. That is, a different reference signal is associated, in general, with each candidate controller. The cost to be considered will have the form

$$\frac{\|y - H[r]\|_\tau^2}{\|r\|_\tau^2}, \text{ if } \|r\|_\tau \neq 0.$$

Notice that the size of the reference signal has not been take in consideration in the MIMO MRAC, since a perfect matching is expected due to the hypotheses imposed on the plant. An explanatory reasoning could be that for that case the optimal cost would be equal to zero and, consequently, the size of the denominator in the above expression would be irrelevant. Let us notice however, that the cost function to be minimized is not convex, what might lead to the existence of several local minima, which might be a problem for gradient algorithms.

IV. Problem Formulation

A. The System

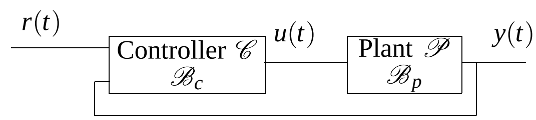

Fig. 1. Feedback control system.

Consider the system in Figure 1. As in [7], we omit any arrows on the block diagram in Figure 1. This illustrates a departure from the usual input/output setting, from the processor point of view. For the fitting controllers to data problem which we are about to formulate,these remarks indicate that we are interested in relations involving signals instead of functions. More specifically, notice that relations define sets and subsets, which may be used to define the feasible region in an optimization problem. Let us notice, however, that this does not preclude the a posteriori use of an identified controller in an input/output setting.

B. The Universum

Let $(r,y,u) \in \mathbf{U}$ where $\mathbf{U} = \mathscr{R} \times \mathscr{Y} \times \mathscr{U} = \mathscr{L}_{2e}^{n_z}$. Here $\mathscr{R} = \mathscr{L}_{2e}^{n_r}$ is the set of reference signals, $\mathscr{Y} = \mathscr{L}_{2e}^{n_y}$ and $\mathscr{U} = \mathscr{L}_{2e}^{n_u}$ are sets of plant signals, and $n_z = n_r + n_y + n_u$. In this paper we focus on the case $n_r = n_y = n_u = m$.

C. The Data Set

The plant information imposes restrictions only on the past values of the signals u and y. Thus the data set is given by

$$\mathscr{D}_\tau = \{(r,y,u) \in \mathbf{P}_\tau(\mathbf{U}) \mid y = y_{data}, u = u_{data}\},$$

for some

$$(y_{data}, u_{data}) \in \mathbf{P}_\tau(\mathscr{Y} \times \mathscr{U}).$$

D. The Performance (Cost) Index

For the multivariable model reference control problem ([3],[4],[5]) the model to be followed is given by a matrix

$$H(s) = \begin{bmatrix} \frac{1}{(s+a)^{r_1}} & 0 & \cdot & \cdot \\ \frac{1}{(s+a)^{r_2-1}} & \frac{1}{(s+a)^{r_2}} & 0 & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \frac{1}{(s+a)^{r_m}} \end{bmatrix}.$$

where $\partial H_{ij}(s) < r_i - 1$ and a is arbitrary, but fixed a priori.

Before introducing our performance index, let us define the norm to be used.

Definition 4.1: Given a constant $\sigma > 0$, we define the exponentially-weighted truncated L_2 inner-products $< x,y >_\tau$ and norm $\|x\|_\tau$ by

$$< x,y >_\tau \stackrel{\Delta}{=} \int_0^\tau e^{-2\sigma(\tau-t)} y^T(t)x(t)\,dt \tag{2}$$

$$\|x\|_\tau \stackrel{\Delta}{=} \sqrt{< x,x >_\tau}. \tag{3}$$

Definition 4.2: Let the performance (cost) index be given by

$$\mathscr{I}_\tau((r,y,u)) \stackrel{\Delta}{=} \begin{cases} \|y - H[r]\|_\tau^2 / \|r\|_\tau^2, & \text{if } \|r\|_\tau \neq 0 \\ 0, & \text{if } \|r\|_\tau + \|y\|_\tau = 0 \\ \infty, & \text{otherwise.} \end{cases}$$

E. The Class of Candidate Controllers

The class of candidate controllers is similar to the one used in multivariable model reference adaptive control ([3],[4],[5]) and also similar to the one used in the [9] and [10] for the SISO case. In order to define the class of candidate controllers, we first define a vector of filters as in ([7],[11]). Notice that we define a vector of filters, and not just a vector of time-domain filtered signals as in [9] and [10]. Let us define $v : \mathscr{L}_{2e}^m \rightarrow \mathscr{L}_{2e}^{m(n-1)}$ by

$$\begin{aligned} \dot{v}(q) &= \begin{bmatrix} \Lambda & 0 & \cdot & 0 \\ 0 & \Lambda & \cdot & 0 \\ \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & \cdot & \Lambda \end{bmatrix} v(q) \\ &\quad + \begin{bmatrix} l & 0 & \cdot & 0 \\ 0 & l & \cdot & 0 \\ \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & \cdot & l \end{bmatrix} q \qquad (4) \\ (v(q))(0) &= 0 \qquad (5) \end{aligned}$$

where (Λ, l) is an asymptotically stable system in controllable canonical form, with

$$\lambda(s) = det(sI - \Lambda) \tag{6}$$

for some monic Hurwitz polynomial $\lambda(s)$ of degree $n-1$. Let us also define

$$w(u,y) = (u^T, v^T(u), y^T, v^T(y))^T. \tag{7}$$

The class of controllers considered is given by $\{(\mathbf{U}, \mathscr{B}_c(\theta)) \mid \theta \in \Theta\}$, where

$$\mathscr{B}_c(\theta) = \{(r,y,u) \mid r = \theta^T w(u,y)\} \quad \text{and}$$

Θ is the set of $2nm \times m$ matrix of real-valued constant parameters.

V. Problem Solution

A. The Set of Optimal Controllers

Theorem 5.1: The set of parameter matrices $\Theta^*(\tau)$,

$$\Theta^*(\tau) = \arg \min_{\theta \in \mathbb{R}^{2nm \times m}} \mathscr{E}\{\mathscr{I}_\tau(b) \mid b \in P_\tau(P_\tau^{-1}(\mathscr{D}_\tau) \cap \mathscr{B}_c(\theta))\}$$

is given by

$$\Theta^*(\tau) = \left\{ \begin{bmatrix} \overline{\theta^*}_1 & \cdot & \cdot & \overline{\theta^*}_m \end{bmatrix} \mid \overline{\theta^*} = \begin{bmatrix} \overline{\theta^*}_1 \\ \cdot \\ \cdot \\ \overline{\theta^*}_m \end{bmatrix} = \right.$$

$$\left. \arg \min_{\overline{\theta} \in \mathbb{R}^{2nm^2}} \left\{ \frac{\overline{\theta}^T A(\tau)\overline{\theta} - 2\overline{\theta}^T B(\tau) + C(\tau)}{\overline{\theta}^T D(\tau)\overline{\theta}} \right\} \right\},$$

where

$$A(\tau) = \begin{bmatrix} A_{11}(\tau) & A_{12}(\tau) & \cdot & A_{1m}(\tau) \\ A_{21}(\tau) & A_{22}(\tau) & \cdot & A_{2m}(\tau) \\ \cdot & \cdot & \cdot & \cdot \\ A_{m1}(\tau) & A_{m2}(\tau) & \cdot & A_{mm}(\tau) \end{bmatrix},$$

$$B(\tau) = \begin{bmatrix} B_1(\tau) \\ B_2(\tau) \\ \cdot \\ B_m(\tau) \end{bmatrix},$$

$$D(\tau) = \begin{bmatrix} D_{11}(\tau) & 0 & \cdot & 0 \\ 0 & D_{22}(\tau) & \cdot & 0 \\ \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & \cdot & D_{mm}(\tau) \end{bmatrix},$$

with

$$A_{jl}(\tau) =$$

$$\int_0^\tau e^{-2\sigma(\tau-t)} \sum_{i=\max\{j,l\}}^m H_{ij}[w_{data}](H_{il}[w_{data}])^T dt,$$

$$\begin{aligned} B_j(\tau) &= \int_0^\tau e^{-2\sigma(\tau-t)} \sum_{i=j}^m y_{data,i} H_{ij}[w_{data}] dt, \\ C(\tau) &= \int_0^\tau e^{-2\sigma(\tau-t)} y_{data}^T y_{data} dt, \\ D_{ij}(\tau) &= \int_0^\tau e^{-2\sigma(\tau-t)} w_{data} {w_{data}}^T dt \text{ for } i = j, \\ &= 0 \text{ otherwise, and} \end{aligned}$$

$$w_{data} = w(u_{data}, y_{data}) \tag{8}$$

provided that $\|u_{data}\|_\tau + \|y_{data}\|_\tau \neq 0$.

Proof. Let us prove, first, that $\mathbf{P}_\tau(\mathbf{P}_\tau^{-1}(\mathscr{D}_\tau)\cap\mathscr{B}_c(\theta))$ is a unitary set (i.e., it has one and only one point):

$$\begin{aligned}
\mathscr{D}_\tau &= \{(r,y,u)\in\mathbf{P}_\tau(\mathbf{U})\mid \\
&\quad y=y_{data}, u=u_{data}\}\\
\mathbf{P}_\tau^{-1}(\mathscr{D}_\tau) &= \{(r,y,u)\in\mathbf{U}\mid\\
&\quad \mathbf{P}_\tau(y)=y_{data},\mathbf{P}_\tau(u)=u_{data}\}\\
\mathbf{P}_\tau^{-1}(\mathscr{D}_\tau)\cap\mathscr{B}_c(\theta) &= \{(r,y,u)\in\mathbf{U}\mid\\
&\quad M_0[r]=\theta^T w(u,y),\\
&\quad \mathbf{P}_\tau(y)=y_{data},\mathbf{P}_\tau(u)=u_{data}\}\\
\mathbf{P}_\tau(\mathbf{P}_\tau^{-1}(\mathscr{D}_\tau)\cap\mathscr{B}_c(\theta)) &= \{(r,y,u)\in\mathbf{P}_\tau(\mathbf{U})\mid\\
&\quad M_0[r]=\theta^T w_{data},\\
&\quad y=y_{data},u=u_{data}\}.
\end{aligned}$$

where w_{data} is defined by the equation (8) and by the equations (4) to (5).

Thus $\mathbf{P}_\tau(\mathbf{P}_\tau^{-1}(\mathscr{D}_\tau)\cap\mathscr{B}_c(\theta))$ is a unitary set, which implies that we can restrict ourselves to the problem of finding the set of parameters $\Theta^*(\tau)$ such that

$$\begin{aligned}
\Theta^*(\tau) &= \arg\min_{\theta\in\Theta}\{\mathscr{I}_\tau(b)\mid b\in P_\tau(P_\tau^{-1}(\mathscr{D}_\tau)\cap\mathscr{B}_c(\theta))\}\\
&= \arg\min_{\theta\in\Theta}\{\frac{\|y_{data}-H[\theta^T w_{data}]\|_\tau{}^2}{\|\theta^T w_{data}\|_\tau{}^2}\}. \qquad (9)
\end{aligned}$$

Let us define $\theta_1,\ldots,\theta_m$ by

$$\theta^T=\begin{bmatrix}\theta_1^T\\ \cdot\\ \cdot\\ \theta_m^T\end{bmatrix}.$$

Then

$$r=\theta^T w_{data}=\begin{bmatrix}\theta_1^T w_{data}\\ \cdot\\ \cdot\\ \theta_m^T w_{data}\end{bmatrix}$$

and, consequently,

$$r=\begin{bmatrix}\sum_{j=1}^m H_{1j}[\theta_j^T w_{data}]\\ \cdot\\ \cdot\\ \sum_{j=1}^m H_{mj}[\theta_j^T w_{data}]\end{bmatrix},$$

$$\begin{aligned}
&(H[r])^T H[r]\\
=&\sum_{i=1}^m\sum_{j=1}^m\sum_{l=1}^m H_{ij}[\theta_j^T w_{data}](H_{il}[\theta_l^T w_{data}])^T\\
=&\sum_{i=1}^m\sum_{j=1}^m\sum_{l=1}^m \theta_j^T H_{ij}[w_{data}](H_{il}[w_{data}])^T\theta_l\\
=&\sum_{j=1}^m\sum_{l=1}^m \theta_j^T(\sum_{i=1}^m H_{ij}[w_{data}](H_{il}[w_{data}])^T)\theta_l\\
=&\sum_{j=1}^m\sum_{l=1}^m \theta_j^T(\sum_{i=\max\{j,l\}}^m H_{ij}[w_{data}](H_{il}[w_{data}])^T)\theta_l,
\end{aligned}$$

$$\begin{aligned}
y_{data}^T H[r] &= \sum_{i=1}^m y_{data,i}\sum_{j=1}^m H_{ij}[\theta_j^T w_{data}]\\
&= \sum_{j=1}^m \theta_j^T\sum_{i=1}^m y_{data,i}H_{ij}[w_{data}]\\
&= \sum_{j=1}^m \theta_j^T\sum_{i=j}^m y_{data,i}H_{ij}[w_{data}].
\end{aligned}$$

Let us define

$$\overline{\theta}\triangleq\begin{bmatrix}\theta_1\\ \cdot\\ \cdot\\ \theta_m\end{bmatrix},\quad \text{and } A(\tau),\ B(\tau),\ C(\tau),\ D(\tau),$$

as in the statement of the theorem. We can, then, state that the following equality holds

$$\frac{\|y_{data}-H[\theta^T w_{data}]\|_\tau{}^2}{\|\theta^T w_{data}\|_\tau{}^2} = \frac{\overline{\theta}^T A(\tau)\overline{\theta}-2\overline{\theta}^T B(\tau)+C(\tau)}{\overline{\theta}^T D(\tau)\overline{\theta}}.$$

From this equality and the expression 9, the thesis follows. □

B. Matrices Properties

Property 5.1: The null space of $D(\tau)$ is contained in the null space of $A(\tau)$ and in the null space of $B^T(\tau)$.
Proof. We have that

$$\begin{aligned}
&D(\tau)\overline{\theta}=0\\
&\quad\Rightarrow\quad D_{jj}(\tau)\theta_j=0\\
&\quad\Rightarrow\quad \theta_j^T D_{jj}(\tau)\theta_j=0\\
&\quad\Rightarrow\quad \|\theta_j^T w_{data}\|_\tau{}^2=0\\
&\quad\Rightarrow\quad \|H_{ij}[\theta_j^T w_{data}]\|_\tau{}^2=0
\end{aligned}$$

which implies that

$$\begin{aligned}
&A(\tau)\overline{\theta}=0\ \text{ and}\\
&B^T(\tau)\overline{\theta}=0.
\end{aligned}$$

□

Property 5.2: The matrices $D(\tau)$ and

$$\begin{bmatrix}A(\tau) & -B(\tau)\\ -B^T(\tau) & C(\tau)\end{bmatrix}$$

are symmetric and positive semi-definite.
Proof. A simple inspection reveals that these matrices are symmetric. The positive semi-definiteness of these matrices follows by observing that

$$\begin{bmatrix}\theta\\ 1\end{bmatrix}^T\begin{bmatrix}A(\tau) & -B(\tau)\\ -B^T(\tau) & C(\tau)\end{bmatrix}\begin{bmatrix}\theta\\ 1\end{bmatrix}$$

$$=\|y_{data}-H[\theta^T w_{data}]\|_\tau{}^2\geq 0$$

$$\theta^T D(\tau)\theta=\sum_{j=1}^m\|\theta_j^T w_{data}\|_\tau{}^2\geq 0$$

□

These properties together with the theorem proved imply that the optimization problem derived from the problem of fitting MIMO controllers to data can be reduced to an optimization problem derived from a problem of fitting SISO controllers to data. This problem is solved in [7].

VI. Example

Let us choose

$$H(s) = \begin{bmatrix} \frac{1}{s+1} & 0 \\ 0 & \frac{1}{s+1} \end{bmatrix},$$

as in ([3],[4],[5]). Let us also choose $\Lambda = -1$, $l = 1$ and $\sigma = 0.01$. The filter w defined in section IV-E is then given by $w(u,y) = (u^T, \frac{1}{s+1}[u^T], y^T, \frac{1}{s+1}[y^T])^T$ and the class of candidate controllers is given by $\{(\mathbf{U}, \mathscr{B}_c(\theta)) \mid \theta \in \Theta\}$, where $\mathscr{B}_c(\theta) = \{(r,y,u) \mid r = \theta^T w(u,y)\}$ and θ is a 8×2 matrix of constant parameters in $\mathbb{R}$. For purposes of this simulation let "the true but unknown plant" be given by the following noisy state space realization of the plant given in ([3],[4],[5])

$$\begin{aligned} \dot{x} &= A_p x + B_p u + 0.01 n_p \\ y &= C_p x \end{aligned}$$

where

$$A_p = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ -2 & -3 & -3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -2 & -5 & -6 & -4 \end{bmatrix},$$

$$B_p = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix},$$

$$C_p = \begin{bmatrix} 0 & 1 & 1 & -4 & -7 & -7 & -2 \\ 1 & 0 & 0 & 1 & 3 & 3 & 1 \end{bmatrix}.$$

n_p is a vector of uncorrelated normally distributed random signals with mean zero, variance one and standard deviation one. Let the reference signal

$$r(t) = \begin{bmatrix} \sin(5t) + \sin(7t) + \sin(10t) \\ \sin(6t) + \sin(8t) + \sin(9t) \end{bmatrix}, \quad \forall t \geq 0.$$

as in ([3],[4],[5]). We obtain (y_{data}, u_{data}) by closing the loop with the initial controller associated to the matrix of parameters

$$\theta(0) = \begin{bmatrix} -4 & 0 \\ 0 & -1/2 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

([3],[4],[5]) and the initial plant state given by $x = 0$. Thus we are able to use our theory to compute a new controller parameter $\theta(t)$ based on the data available at any given time $t = \tau$. Controller adaptation is achieved by repeating this operation periodically as time τ evolves and (y_{data}, u_{data}) accumulates, in order to update the controller parameter θ. Using this procedure to update the controller parameter vector $\theta(t)$ every 2 seconds, starting at time $\tau = 2$, we obtained the simulation results shown in figures 2 and 3, where $y_m = H[r]$.

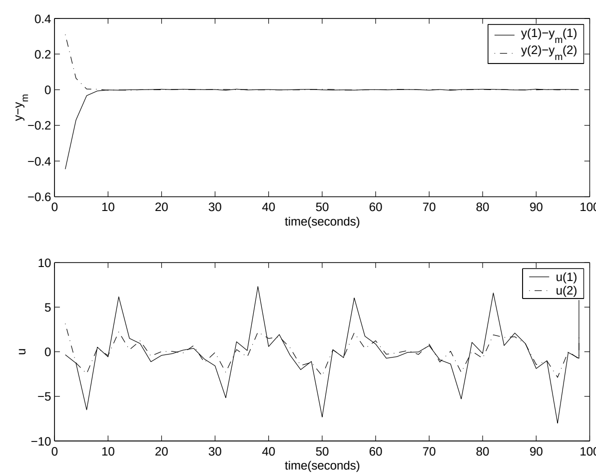

Fig. 2. Simulation Results

VII. Concluding Remarks

The problem of fitting controllers to data was examined for one class of MIMO controllers used in model reference adaptive control ([3],[4],[5]). The formulation of the MIMO problem lead to an optimization problem where a best $2nm \times m$ matrix is looked for. The solution to this problem was shown to be reduced to the solution of a problem where a best $2nm^2 \times 1$ vector is looked for, which is solved in [7] when dealing with the identification of SISO controllers.Thus the problem was solved and an algorithm for fitting MIMO controllers to data was obtained. This algorithm was applied to a noisy realization of a plant and

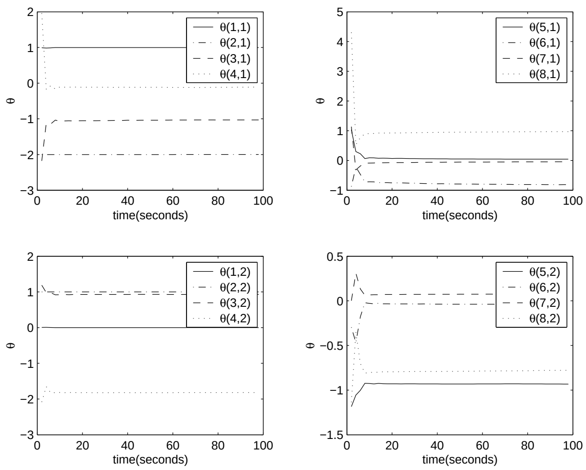

Fig. 3. Simulation Results

a reference model which appeared in the model reference adaptive control literature ([3],[4],[5]). Simulation results were provided which illustrated the applicability of the method. On the theoretical side, it was shown that the formulation of the problem of optimally fitting controllers to data illustrated a unifying link between the design of model reference adaptive control for SISO linear systems and the design of model reference adaptive control for MIMO linear systems.

REFERENCES

[1] L. Hsu, R. Costa, A. K. Imai, and P. Kokotovic. "Lyapunov-based adaptive control of MIMO systems". In *Proc. American Control Conference*, pages 4808–4813, Arlington,VA, USA, June 25-27 2001. IEEE Press, New York.

[2] S. R. Weller and G. C. Goodwin. "Hysteresis Switching Adaptive Control of Linear Multivariable Systems". *IEEE Transactions on Automatic Control*, 39(7):1360–1375, July 1994.

[3] M. de Mathelin and M. Bodson. "Multivariable Model Reference Adaptive Control without Constraints on the High-frequency Gain Matrix,". In *Proceedings of the IEEE Conference on Decision and Control*, pages 2842–2847, Brighton, England, December, 1991.

[4] M. de Mathelin and M. Bodson. "Multivariable Adaptive Control: Identifiable Parametrizations and Parameter Convergence". *IEEE Transactions on Automatic Control*, 39(8):1612–1617, August 1994.

[5] M. de Mathelin and M. Bodson. "Multivariable Model Reference Adaptive Control without Constraints on the High-frequency Gain Matrix,". *Automatica*, 31(4):597–604, 1995.

[6] G. C. Goodwin and D. Q. Mayne. "A Parameter Estimation Perspective of Continuous Time Model Reference Adaptive Control". *Automatica*, 23:57–70, 1987.

[7] M. G. Safonov and F. B. Cabral. "Fitting Controllers to Data". *System Control Letters*, 43(4):299–308, July 2001.

[8] J. C. Willems. "Paradigms and Puzzles in the Theory of Dynamical Systems". *IEEE Transactions on Automatic Control*, 36(3):259–294, March 1991.

[9] K. S. Narendra and A. M. Annaswamy. *Stable Adaptive Systems*. Prentice-Hall, Inc., Englewood Cliffs, NJ, 1989.

[10] K. S. Narendra and J. Balakrishnan. "Improving Transient Response of Adaptive Control Systems using Multiple Models and Switching". *IEEE Transactions on Automatic Control*, 39(9):1861–1866, September 1994.

[11] F. B. Cabral and M. G. Safonov. "A Falsification Perspective on Model Reference Adaptive Control ". In *Proc. IEEE Conf. on Decision and Control*, Kobe, Japan, December 11-13, 1996.