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Adaptive Robust Manipulator Trajectory Control — An Application of Unfalsified Control Method*

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Abstract

This paper takes manipulator trajectory control as an example to demonstrate how the unfalsified control theory as given in [6] is applied to a real world control problem. A new adaptive robust control method is proposed for manipulator trajectory control. With the help of mathematical assumptions, the proposed method can be made to possess global stability property with tracking error proportional to size of external disturbances.

1 Introduction

Unfalsified control, like model invalidation, is motivated by the desire to develop an identification technique that is suitable for use with robust control methods [12], i.e., a technique that can produce from data a plant model with guaranteed model error bounds. However, such a technique does not exist. The best can be obtained from data are models whose assumed model error upper bounds are not invalidated by data (e.g. [2, 3, 5, 11]), since finite time data cannot produce a model with the guaranteed model error bounds that are expected to hold for future measurements which has not yet been collected. Confronted by this fact, we are compelled to reconsider the meaning of robust control when it is put in a real world setting, in which robust claims must be justified by real world data. As a result, by the same reason as nonexistence of the guaranteed model error bounds, guaranteed robust control performance also does not exist in reality; the best we have is a performance that is "not-demonstrably-unrobust," i.e., a robust performance statement that — though not provably correct — cannot be proved to be wrong by available data. This observation leads to the development of the unfalsified control theory [6], which defines a sensible control performance criterion (perhaps the only sensible one) that takes into account the real world data, and describes how *all* the controllers satisfying such criteria – the unfalsified control performance – can be *directly* obtained from data without having to place the controllers in control loop for trial runs.

The unfalsified control method is a model-free approach: without *a priori* prejudicial assumptions about plant behaviors, the *a posteriori* data alone is able to determine whether or not any performance claim of any control law is consistent with or, equivalently, unfalsified by data. Similar to what has been done in control engineering testing, the method suggests: in real time operation, when a control law in use is shown by data not able to achieve specified performances, a new "unfalsified" performance/control law combination should be switched to, and when the newly selected performance/control law combination is again falsified by further coming data, another combination should be switched to, and so on. It is true that one would not be able to know when such switching will stop or whether the switching will stop at all. A controller keeping switching its parameters could result in instability and an unacceptable performance. This violates our basic expectation of a control theory, namely stability and convergence. However, despite our disliking it, the nonexistence of these two properties in reality is an undeniable fact. (As an analogy, think about the introduction of Newton's laws of motion, the later appearance of the theory of relativity, and the challenge of the theory of relativity.) In fact, no control theory can guarantee these two properties as well as any other performance statement when they are applied to real systems. Nonetheless, perhaps comforted by the assurance provided by theories, we tend to think certain control theory results hold in reality, and forget in applying any theory there always requires a leap of faith based on a subject judgment about the closeness between the theory's mathematical assumptions and the reality.

The new control performance notion provided by the unfalsified control theory is not expected to affect the non-adaptive control area, in which data are not involved in synthesizing a controller, except the serving as a reminder that the guaranteed results of control theories are mathematical fictions and that prejudicial engineering judgments are necessary and are used in applying the results to real world problems in bridging the gap between theory and practice, which inevitably undermines the meaning of "guaranteed" results of mathematical theories. However, because of the emphasis on what's knowable from data and how such knowledge can be exploited efficiently in control and decision applications, the unfalsified control notion is expected to affect areas where real world data are taken into account, such as adaptive control or places where measured data are used to construct models for later controller synthesis.

The model-free unfalsified control and the conventional model-based methods are not necessarily mutually exclusive, and they can complement each other. The mathematical knowledge of model-based methods, together with *a priori* observation about the real system to be controlled, helps us produce a (biased) choice of (sequence of) achievable performance, control law; whereas, the unfalsified control provides a method to test and switch within the choice according to the collected data. The biased choice could result in fewer times (even finite times) of switching than that from a choice based on a wild selection of performance/control law combination; on the other side, the test against data and the switching assure satisfactory performance. This paper takes manipulator trajectory control as an example to demonstrate how *a priori* mathematical knowledge and prejudicial judgment can be incorporated with the unfalsified control method to form a complete control procedure that takes into consideration both prior and posterior information. As a result, a new adaptive robust controller for manipulator trajectory control is developed and, with the help of *mathematical* assumptions, is shown to possess the property of global stability.

This paper is organized as follows. Section 2 briefly reviews some mathematical results about robot manipulators and their control. Section 3 reiterates the results of unfalsified control the-

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ory taken from [6] for the completeness of the paper. Section 4 describes how *a priori* mathematical knowledge and prejudices can be incorporated with the unfalsified control method to form a new adaptive robust manipulator trajectory control approach. Mathematical simulation outcomes are provided in Section 5. Finally, discussion and conclusions are given in Section 6.

2 Mathematical Knowledge about Manipulators

The dynamics of a mathematical rigid manipulator can be described by the following equation,

$$H(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) = \tau_a\,, \tag{1}$$

in which q is an $n \times 1$ vector representing the rotational angles of the n links of the manipulator; $H(q)$ is the inertia matrix; $C(q,\dot{q})\dot{q}$ accounts for the coupling Coriolis and centripetal forces; $g(q)$ is the torque caused by gravity; τ_a is an $n \times 1$ vector whose elements are joint torques consisting of actuator outputs and external disturbances.

Example 1 (A Planar Two-Link Manipulator [10]) *The dynamics of a planar, two-link manipulator can be written in form of (1):*

$$\begin{bmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{bmatrix} \begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{bmatrix} + \begin{bmatrix} -h\dot{q}_2 & -h(\dot{q}_1+\dot{q}_2) \\ h\dot{q}_1 & 0 \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} = \begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix} \tag{2}$$

with

$$\begin{aligned} H_{11} &= a_1 + 2a_3\cos q_2 + 2a_4 \sin q_2 \\ H_{12} &= H_{21} = a_2 + a_3\cos q_2 + a_4\sin q_2 \\ H_{22} &= a_2 \\ h &= a_3\sin q_2 - a_4\cos q_2 \\ a_1 &= I_1 + m_1 l_{c1}^2 + I_e + m_e l_{ce}^2 + m_e l_1^2 \\ a_2 &= I_e + m_e l_{ce}^2 \\ a_3 &= m_e l_1 l_{ce}\cos\delta_e \\ a_4 &= m_e l_1 l_{ce}\sin\delta_e \end{aligned}$$

where parameters with subscript 1 are related to link 1 and paramters with subscript e are related to the combination of link 2 and end effector; for example, I_1 is the inertia of link 1, m_1 is the mass of link 1, and l_{c1} indicates location of link 1 mass center, etc. (cf. [10]) The dynamic equation (2) can be expressed linearly in terms of the parameter vector $[a_1, a_2, a_3, a_4]^T \triangleq \theta$ as shown in the following

$$\tau_a = Y(q,\dot{q},\ddot{q})\theta \tag{3}$$

in which Y is a 2×4 matrix with elements

$$\begin{aligned} &Y_{11} = \ddot{q}_1, \quad Y_{12} = \ddot{q}_2, \quad Y_{21} = 0, \quad Y_{22} = \ddot{q}_1 + \ddot{q}_2 \\ &Y_{13} = (2\ddot{q}_1 + \ddot{q}_2)\cos q_2 - (2\dot{q}_2\dot{q}_1 + \dot{q}_2^2)\sin q_2 \\ &Y_{14} = (2\ddot{q}_1 + \ddot{q}_2)\sin q_2 + (2\dot{q}_2\dot{q}_1 + \dot{q}_2^2)\cos q_2 \\ &Y_{23} = \ddot{q}_1\cos q_2 + \dot{q}_1^2 \sin q_2 \\ &Y_{24} = \ddot{q}_1 \sin q_2 - \dot{q}_1^2\cos q_2 \end{aligned}$$

In addition to the above two-link example, the dynamics of a general n-link manipulator also can expressed linearly in terms of the parameters involved, as (3).

For manipulator trajectory control, "computed torque" control method (e.g. [4]) is commonly used to deal with the nonlinearity of the dynamic equation. The following control law is an example of the computed torque method

$$\tau = H(q)[\ddot{q}_d - 2\lambda\dot{\tilde{q}} - \lambda^2\tilde{q}] + C(q,\dot{q})\dot{q} + g(q) \tag{4}$$

where q_d is desired trajectory, $\ddot{q}_d$ is its second derivative, $\tilde{q} = q - q_d$ is tracking error, and τ is actuator input signal. $\lambda > 0$ is a design parameter which determines the speed at which the tracking error converges to zero. When $\tau_a = \tau$ (τ_a is joint torque, τ is actuator input), the application of control law (4) to a robotics system described by (1) gives

$$H(q(t))[\ddot{\tilde{q}}(t) + 2\lambda\dot{\tilde{q}}(t) + \lambda^2\tilde{q}(t)] = 0 \tag{5}$$

Because the inertia matrix $H(q)$ is strictly positive definite for all q, the above equality implies that tracking error $\tilde{q}$ decreases to zero as fast as $e^{-\lambda t}$. When external disturbance exists and $\tau_a = \tau + d$, d being the disturbance, (5) becomes

$$H(q(t))[\ddot{\tilde{q}}(t) + 2\lambda\dot{\tilde{q}}(t) + \lambda^2\tilde{q}(t)] = d(t) \tag{6}$$

The above implies that the tracking error will eventually fall into a region of size proportional to the magnitudes of $d(t)$; which means the control law (4) is robust against external disturbance, when parameters are exactly known.

3 Unfalsified Control

Consider the feedback control system in Figure 1.The goal of feedback control theory is to describe a methodology for determining a control law K for a plant P so that the closed-loop system response, say T, satisfies certain given specifications. The need for learning arises when the plant is either unknown or is only partially known and one wishes to extract information from measurements which will be helpful in selecting a suitable control law K. To understand the issues in developing learning control laws, it is helpful to follow the lead of Zames [15, 16] and adopt the input/output perspective. A very simple learning control problem can be formulated as follows:

Problem 1 (Simple Unfalsified Control) *Given*

a). Data: A measured pair of signals $(u_0, y_0) \in \mathcal{U} \times \mathcal{Y}$.

b). Performance specification:

$$(r,y) \in T_{spec} \subset \mathcal{R} \times \mathcal{Y}.$$

c). Control laws: A class K *of candidate controllers sysK*

Determine the subset of $\mathsf{K}_{OK} \subset \mathsf{K}$ *of those control laws* K *whose ability to meet the specification* T_{spec} *is not invalidated (i.e., is unfalsified) by the data point* (u_0, y_0). □

The set K_{OK} is called the *unfalsified controller set* associated with Problem 1.

If all that is known about a plant is the result (u_0, y_0) of an experiment, the question arises "Can Problem 1 be solved?" The answer, it turns out, is affirmative. In this setting, the following is immediate

Theorem 1 (Simple Unfalsified Control) *Consider Problem 1. Let*

$$\widetilde{T}_{spec} = \left\{ (r,y,u) \mid (r,y) \in T_{spec} \right\} \subset \mathcal{R} \times \mathcal{Y} \times \mathcal{U}.$$

A control law K *is unfalsified by the measurement data* (u_0, y_0) *if and only if*

$$\Big(K \cap \{\, (r,y,u) \in \mathcal{R} \times \mathcal{Y} \times \mathcal{U} \mid r \in \mathcal{R}, y = y_0, u = u_0 \,\}\Big) \subset \widetilde{T}_{spec} \;\square$$

Like Zames [15, 16], we consider the plant P and the controller K to be relations in appropriate function spaces:

$$P \subset \mathcal{U} \times \mathcal{Y}, \quad K \subset \mathcal{R} \times \mathcal{Y} \times \mathcal{U}$$

and, each control law K is a surface in the space $\mathcal{R} \times \mathcal{Y} \times \mathcal{U}$

$$K(r, y, u) = 0.$$

In Problem 1 the assumption is made that data consisting of a pair of entire signals (u_0, y_0) is measured. A more general situation arises when incomplete information about (u_0, y_0) is available, e.g., only the past values of the signals (u_0, y_0) or perhaps only their values at certain sampling instants. To accommodate such situations, let us suppose that all that can be deduced from the available measurements of the signal pair (u_0, y_0) is a set, say M, containing the (u_0, y_0); i.e.,

$$(u_0, y_0) \in M \subset \mathcal{U} \times \mathcal{Y}.$$

Definition (Measurement) *The set M is called the* **measurement information**. □

Additionally, to add flexibility, let us generalize the specification set T_{spec} to include constraints on u as well as on r, y. This leads to the following generalization of Problem 1.

Problem 2 (General Unfalsified Control) *Given*

a). A measurement information set $M \subset \mathcal{U} \times \mathcal{Y}$ containing an otherwise unknown plant input/output pair (u_0, y_0).

b). A performance specification set

$$\widetilde{T}_{spec} \subset \mathcal{R} \times \mathcal{Y} \times \mathcal{U}. \quad (7)$$

c). A class K of admissible control laws.

Determine the subset of $\mathsf{K}_{OK} \subset \mathsf{K}$ of those control laws K whose ability to meet the specification $\widetilde{T}_{spec}$ is not invalidated (i.e., is unfalsified) by the measurement information M. □

Definition *The set K_{OK} is called the* **unfalsified set** *and the control laws $K \in \mathsf{K}_{OK}$ are said to be* **unfalsified control laws**. □

The following theorem is immediate from the definitions.

Theorem 2 (General Unfalsified Control) *Consider Problem 2. A control law K is unfalsified by the knowledge that*

$$(u_0, y_0) \in M$$

if and only if, for each r_0, y_0, u_0 such that $(u_0, y_0) \in sysM$ and $r_0, y_0, u_0) \in K$, there exists at least on pair $(u_1, y_1) \in M$ such that

$$(r_0, y_1, u_1) \in K \cap \widetilde{T}_{spec}$$

Note that in the case where an ensemble of experiments has produced partial knowledge M_α for each element of a family of input/output data pairs (u_α, y_α), the unfalsified controller set K_{OK} would be simply the intersection of the unfalsified controller sets, say $\mathsf{K}_{OK\alpha}$, associated with each M_α; i.e.,

$$\mathsf{K}_{OK} = \bigcap_{\alpha} \mathsf{K}_{OK\alpha}. \quad (8)$$

The dynamical case presents no special conceptual difficulties. It turns out that the dynamical learning control problem is really just a special case of the General Unfalsified Control Problem addressed previously. The necessary machinery for embedding the dynamical case in the framework of Theorem 2 is provided by the time truncation operator P_τ of input/output stability theory.

Definition (Time Truncation Operator [7, 8, 15, 16]) *For any $\tau \in \mathbb{R}$, the* **time truncation operator** *P_τ is a mapping of time signals into time signals defined by*

$$[\mathsf{P}_\tau \mathsf{x}](\mathsf{t}) \triangleq \begin{cases} x(t), & \text{if } t \leq \tau \\ 0, & \text{if } t > \tau \end{cases}$$

□

The goal of learning in the dynamical control context is to use *past* plant input/output data to assess the *future* potential of each control law $K \in \mathsf{K}$ to meet the performance specification

$$(r, y, u) \in \widetilde{T}_{spec}.$$

Definition *A control law $K \in \mathsf{K}$ is said to be* **falsified** *by past data $(\mathsf{P}_\tau \mathsf{u}_0, \mathsf{P}_\tau \mathsf{y}_0)$ if it can be proved that the control law K could not have met the performance specification $\widetilde{T}_{spec}$ if that control law had been in place when the plant generated the data. Otherwise, the control law K is said to be* **unfalsified**. □

The key observation is that a measurement of *past* input/output data $(\mathsf{P}_\tau \mathsf{u}_0, \mathsf{P}_\tau \mathsf{y}_0)$ corresponds to *partial* knowledge of a plant input/output pair (u_0, y_0); i.e., it corresponds to the case of Theorem 2 in which the measurement information set is

$$M = \left\{ (u, y) \in \mathcal{U} \times \mathcal{Y} \mid \mathsf{P}_\tau \begin{bmatrix} u \\ y \end{bmatrix} = \mathsf{P}_\tau \begin{bmatrix} u_0 \\ y_0 \end{bmatrix} \right\} \quad (9)$$

Thus, Theorem 2 specializes to the following.

Theorem 3 (Dynamical Unfalsified Control) *Consider Problem 2. A control law K is unfalsified by past plant input/output data $(\mathsf{P}_\tau \mathsf{u}, \mathsf{P}_\tau \mathsf{y})$ if and only if for each triple (r_0, y_0, u_0) such that $(\mathsf{P}_\tau \mathsf{u}_0, \mathsf{P}_\tau \mathsf{y}_0) = (\mathsf{P}_\tau \mathsf{u}, \mathsf{P}_\tau \mathsf{y})$ and $(r_0, y_0, u_0) \in K$, there exists at least one pair (u_1, y_1) such that $(\mathsf{P}_\tau \mathsf{u}_1, \mathsf{P}_\tau \mathsf{y}_1) = (\mathsf{P}_\tau \mathsf{u}, \mathsf{P}_\tau \mathsf{y})$ and*

$$(r_0, y_1, u_1) \in K \cap \widetilde{T}_{spec}$$

4 Combination of Mathematical Knowledge, Prejudice, and Unfalsified Control Method

This section takes manipulator trajectory control as an example to demonstrate how the *a priori* mathematical knowledge and prejudice and the *a posteriori* data can be combined by using the unfalsified control concept to form an adaptive control procedure.

For a real world manipulator, if its characteristics were exactly described by (1) with known parameters, and joint torque τ_a and actuator input τ had exactly the same magnitude, then the application of the control law (4) would yield satisfactory performance. However, a real world manipulator control system have many other factors that cannot be characterized by (1) and (4), such as link flexibility and the effects of actuator dynamics, saturation, friction, gear train backlash etc. In fact, a mathematical model is never able to describe every detail of a physical system, there is always a gap in between. Such a gap has to be overcome by a subject judgement (a prejudice) based on a prior observation of the real system. For example, one has to judge from the prior observations whether the aforementioned factors are "negligible" (e.g. its actuator dynamics are "fast" or not, etc.) to determine the appropriateness of the application of control law (4) on a given physical manipulator.

Now, assume the following conditions occur:

C1. prior mathematical knowledge and prejudice: our mathematical knowledge about manipulator and *a priori* observation of the manipulator's constituents has helped us make the biased decision that the use of control law (4) could result in the performance described by (6), and that (1) and (3) should hold

C2. parameters cannot be known correctly: parameters such as inertia, location of mass center etc. cannot be correctly known in advance, due to change of working conditions or other unknown reasons

C3. measurement: the actuator's input command τ and the manipulator's output angle q, velocity $\dot{q}$, and acceleration $\ddot{q}$ are measurable

For this scenario, the unfalsified control method described in Section 3 can be applied in the manner described below by taking the reference signal, plant output and input as $r = [q_d, \dot{q}_d, \ddot{q}_d]^T$, $y = [q, \dot{q}, \ddot{q}]^T$, and $u = \tau$, respectively.

Control Law and Unfalsified Controller Parameter Set

Based on condition C1, a subset K_1 of the set of admissible control laws K and an achievable performance set $\widetilde{T}_1$ can be selected as a first candidates for falsification test, as given below:

$$\mathsf{K}_1 = \left\{(r, y, u) \in \mathcal{R} \times \mathcal{Y} \times \mathcal{U} \mid K_{\hat{\theta}}(r, y, u) = 0, \hat{\theta} \in \mathbb{R}^m \right\}$$

$$\widetilde{T}_1 = \{(r, y, u) \in \mathcal{R} \times \mathcal{Y} \times \mathcal{U} \mid T(r, y, u) < 0\}$$

with

$$K_{\hat{\theta}}(r, y, u) \triangleq \hat{H}(\hat{\theta}, q)[\ddot{q}_d - 2\lambda\dot{\tilde{q}} - \lambda^2\tilde{q}] + \hat{C}(\hat{\theta}, q, \dot{q})\dot{q} + \hat{g}(\hat{\theta}, q) - \tau \quad (10)$$

$$T(r, y, u) \triangleq |H(q)[\ddot{\tilde{q}} + 2\lambda\dot{\tilde{q}} + \lambda^2\tilde{q}]| - \bar{d} \quad (11)$$

for some function $\bar{d}$, and for some $\hat{H}(.)$, $\hat{C}(.)$, and $\hat{g}(.)$ satisfying $\hat{H}(\theta^*, q) = H(q)$, $\hat{C}(\theta^*, q, \dot{q}) = C(q, \dot{q})$, $\hat{g}(\theta^*, q) = g(q)$ for some θ^*. In (11), $T(r, y, u) < 0$ means each entry of $T(r, y, u)$, a vector, is less than zero at all times. Based on condition C3, the measurement information given in (9) has u_0 and y_0 as the measured τ and $[q, \dot{q}, \ddot{q}]^T$, respectively.

To indicate the time dependence, let

$$M(\tau) = \left\{ (u, y) \in \mathcal{U} \times \mathcal{Y} \mid \mathsf{P}_\tau \begin{bmatrix} u \\ y \end{bmatrix} = \mathsf{P}_\tau \begin{bmatrix} u_0 \\ y_0 \end{bmatrix} \right\}$$

denote the measurement information obtained from measurements up to time τ, and let $\Theta(t)$ denote the parameter set of all the unfalsified control laws given in Theorem 3 with $M = M(t)$, $K = K_1$, and $\widetilde{T}_{spec} = \widetilde{T}_1$ (each element of $\Theta(t)$ corresponds to a control law K).

The following describes how the unfalsified controller parameter set $\Theta(t)$ can be obtained through set intersections. According to (10), a control law $K_{\hat{\theta}}$ with parameter vector $\hat{\theta}$ calculates its control signal by using

$$\tau = \hat{H}(\hat{\theta}, q)[\ddot{q}_d - 2\lambda\dot{\tilde{q}} - \lambda^2\tilde{q}] + \hat{C}(\hat{\theta}, q, \dot{q})\dot{q} + \hat{g}(\hat{\theta}, q) \quad (12)$$

Hence, given the measurement information $M(t)$, the measured data $u = \tau$ and $y = [q, \dot{q}, \ddot{q}]^T$ can be regarded as being generated by any control law with parameter $\hat{\theta}$ by an application of a reference command signal $r_{\hat{\theta}} = q_d^{\hat{\theta}}$ satisfying

$$\ddot{q}_d^{\hat{\theta}} + 2\lambda\dot{q}_d^{\hat{\theta}} + \lambda^2 q_d^{\hat{\theta}}$$
$$= \hat{H}(\hat{\theta}, q)^{-1}\left(\tau + \hat{H}(\hat{\theta}, q)(2\lambda\dot{q} + \lambda^2 q) - \hat{C}(\hat{\theta}, q, \dot{q})\dot{q} - \hat{g}(\hat{\theta}, q)\right)$$

Note that, given only the measured plant input-output data and a parameter vector $\hat{\theta}$, the right hand side of the above can be determined. With this "fictitious" reference signal $q_d^{\hat{\theta}}$, the performance of every control law can be tested even if the data is not produced by the control law being tested. By Theorem 3, the choices of K_1 and $\widetilde{T}_1$, and (3), the unfalsified controller parameter set at time t can be expressed as the intersection

$$\Theta(t) = \bigcap_{0 \le \tau \le t} (\cap_{k=1}^n \Omega_k(\tau)) \quad (13)$$

where

$$\Omega_k(t) \triangleq \{\theta \in \mathbb{R}^m \mid |\theta^T\psi_k(t) - \tau_k(t)| < \bar{d}(t)\}$$

is the "information set" of the $k-$th joint at time t, in which $\psi_k(t)$ represents the k-th column of the matrix $Y^T(q(t), \dot{q}(t), \ddot{q}(t))$ defined in (3). The set Θ_0 represents the *a priori* prejudice about the range of controller parameter vectors that could achieve performance specification $\widetilde{T}_1$.

Parameter Update Law

Only one controller parameter vector can be used at a time in the control loop of a manipulator trajectory control system. When the controller in use is falsified by data, a new controller parameter vector has to be selected from the unfalsified controller parameter set to replace the controller. The are many ways to select a new controller parameter vector; the following parameter update law is used here to produce new controller parameters (for simplicity, $\hat{\theta}_i$ is used for $= \hat{\theta}(t_i)$)

$$\begin{aligned} \hat{\theta}_i &= \hat{\theta}_i(\xi_{i-1}, \hat{\theta}_{i-1}, \nu_{i-1}) \\ &= \begin{cases} \xi_{i-1}, & \text{when } t_i > t_{i-1} + \nu_{i-1} \text{ and} \\ & \hat{\theta}_{i-1} \notin \bigcap_{t_{i-1} < \tau \le t_i} \cap_{1 \le k \le n} \Omega_k(\tau) \\ \hat{\theta}_{i-1}, & \text{otherwise} \end{cases} \\ & i = 1, 2, \cdots, \end{aligned} \quad (14)$$

where t_i is the first moment after $t_{i-1} + \nu_{i-1}$ when $\hat{\theta}_{i-1}$ is not within the intersection $\bigcap_{t_{i-1} < \tau \le t_i} \cap_{1 \le k \le n} \Omega_k(\tau)$ (assume t_i is possible to obtain); ν_{i-1} is the computation time required to calculate ξ_{i-1}; ξ_i is the center of the maximum volumn inner ball within the intersection; i.e.

$$\xi_i = \arg \max_{\theta \in \tilde{\Theta}_i} dist(\theta, \partial\tilde{\Theta}_i) ; \quad (15)$$

in which $\tilde{\Theta}_i = \bigcap_{0 \le j \le i} \cap_{1 \le k \le n} \Omega_k(t_j)$, $dist(a, A)$ is the smallest distance between a and A, and $\partial\tilde{\Theta}_i$ is the set of boundary points of $\tilde{\Theta}_i$. The parameter vector $\hat{\theta}_i$ will be used for control law (12) within the time interval $[t_{i-1} + \nu_{i-1}, t_i + \nu_i)$. As may have been noticed, $\tilde{\Theta}_i$ is not the unfalsified controller parameter set $\Theta(t_i)$; the former is larger. However, the set $\tilde{\Theta}_i$ can be regarded as the unfalsified controller parameter set obtained based on the partial information set $\{(u, y) \in \mathcal{U} \times \mathcal{Y} \mid u(t_j) = u_0(t_j), y(t_j) = y_0(t_j), j = 1, 2, \cdots, i\}$, a set containing $M(t_i)$. The use of $\tilde{\Theta}_i$ in place of $\Theta(t_i)$ is based on implementation consideration.

Assume $\hat{\theta} \in \mathbb{R}^m$. The optimization problem (15) can be transformed to an m-variable linear programming problem with its computational complexity proportional to $i + 1$ as [14]:

$$\max \delta \quad (16)$$

subject to

$$\delta, \xi_i : \begin{cases} \delta \ge 0 \\ (\psi_k^T(t_j)\xi_i - \bar{d}(t_j))/R_j - \delta \ge 0 & j = 0, 1, 2, \cdots, i \\ (\psi_k^T(t_j)\xi_i - \bar{d}(t_j))/R_j - \delta \ge 0 & j = 0, 1, 2, \cdots, i \end{cases}$$

where $R_j = \sqrt{\psi_k^T(t_j)\psi_k^T(t_j)}$ and $\psi_k(t_0)$ can be used to define Θ_0. δ is the radius of the maximal inner ball and ξ_i is its center. Besides the batch-typed approach of linear programming method (16), a recursive algorithm for (14) is also possible because the unfalsified controller parameter set $\tilde{\Theta}_i$ is the intersection of degenerated ellipsoids (regions between "parallel" hyper planes), the recursive algorithm of minimal-volume outer approximation by Fogel and Huang [1] may prove to be useful for the calculation of the intersections.

Previous parameter update law (14) has the following finite time convergence property.

Lemma 1 (Finite Time Convergence lemma) *If Θ_0 is a bounded set and Θ_∞ contains an open ball, then there exists $N \in \mathbb{N}$ such that $\hat{\theta}_i = \hat{\theta}_N$ $\forall i \ge N$; $\Theta_\infty = \lim_{t \to \infty} \Theta(t)$.*

Proof: Let ϵ_1 be the radius of an open ball contained in $\Theta_\infty \subset \tilde{\Theta}_\infty$. According to the definition of ξ_i, whenever $\hat{\theta}_i \neq \hat{\theta}_{i-1}$, the intersection of $B(\hat{\theta}_i, \epsilon_1/2)$ and $B(\hat{\theta}_j, \epsilon_1/2)$ is an empty set $\forall j \leq i-1$. Hence, when $\hat{\theta}_i \neq \hat{\theta}_j$, $B(\hat{\theta}_i, \epsilon_1/2)$ does not intersect with $B(\hat{\theta}_j, \epsilon_1/2)$. But, $\cap_{j=1}^{\infty} B(\hat{\theta}_j, \epsilon_1/2) \subset \Theta_0$, if the N mentioned in the lemma statement does not exist, then Θ_0 will contain infinitely many mutually exclusive open ball, each with radius $\epsilon_1/2$. This contradicts with Θ_0 being a bounded set. *Q.E.D.*

Performance The control law (12) and parameter update law (14) form an adaptive controller. It is true that there is no guarantee that the application of this adaptive controller would result in the expected robust performance (6); the unfalsified controller parameter set may even become an empty set at a finite time. When the unfalsified parameter set becomes empty at a finite time, another choice of control law and achievable performance combination, other than K_1 and $\widetilde{T}_1$, has to be selected for falsification test in order to determine a suitable control law to be used. A proper choice of sequence of control law/achievable performance combinations could result in earlier or even finite time achievement of expected performance. Such a choice also requires a subject judgement of its fitness, similar to the application of a non-adaptive controller to a real system. The following two mathematical theorems serve for that purpose. With the help of mathematical assumptions, the theorems guarantee finite time achievement of the performance $\widetilde{T}_1$ and the global stability w.r.t. initial states, and that tracking errors are proportional to the size of disturbance. A similar performance result for a different control law/achievable performance combination can be found in [13].

Theorem 4 (Ideal Situation) *Let (12) and (14) be the adaptive controller with $\bar{d}_k(t) \equiv \epsilon$, $\forall k = 1, \cdots, m$, where $\bar{d}_k$ is the k-th element of $\bar{d}$. Assume the manipulator structure as follows.*

- *the manipulator system is described by (1), in which the angle q, angular velocity $\dot{q}$, and angular acceleration $\ddot{q}$ are measurable.*
- *the expressions of $H(q)$, $C(q, \dot{q})$, and $g(q)$ are known, but the involved parameters are not known; furthermore, the joint torques are linear functions of parameters, i.e., (3) holds with θ as the unknown parameter vector.*
- *external disturbance does not exist, the only torques acting on the manipulator joints are the actuator outputs, and actuators do not contain dynamics; i.e., $\tau_a = \tau$.*

Assume further that

- *Θ_0 is a bounded set*
- *Θ_∞ contains an open set*
- *$\hat{H}(\theta, q) > \delta I$, $\forall q$, $\forall \theta \in \Theta_0$, for some unknown positive number δ*

Then, for any initial conditions $q(t_0)$ and $\dot{q}(t_0)$, the following hold

1. *there exists $T \in \mathbb{R}$ such that $\hat{\theta}(t) = \hat{\theta}(T)$, $\forall t \geq T$*
2. *q, $\dot{q}$, $\ddot{q}$, and τ are uniformly bounded*
3. *$\limsup_t \|\tilde{q}(t)\| < c\epsilon$,c is a positive constant*

Proof: The first result of this theorem can be obtained directly from lemma 1.

For the second result, because $\hat{\theta}(t)$ no longer changes after $t > T$, the following performance is reached after $t > T$:

$$|\hat{H}(\hat{\theta}(T), q(t))(\ddot{\tilde{q}}(t) + 2\lambda\dot{\tilde{q}}(t) + \lambda^2\tilde{q}(t))| < \epsilon$$

i.e.,

$$\ddot{\tilde{q}}(t) + 2\lambda\dot{\tilde{q}}(t) + \lambda^2\tilde{q}(t) = \hat{H}^{-1}(\hat{\theta}(T), q(t))\tilde{\epsilon}(t), \quad \forall t > T$$

for some $\tilde{\epsilon}$ satisfying $|\tilde{\epsilon}_k(t)| < \epsilon, \forall t > T$. From the assumption of $\hat{H}(.)$, the following bounded condition is obtained.

$$||\hat{H}^{-1}(\hat{\theta}(T), q(t))\tilde{\epsilon}(t)|| < \frac{\epsilon}{\delta} \quad (17)$$

So, after time $t > T$, $\tilde{q}$, $\dot{\tilde{q}}$, and $\ddot{\tilde{q}}$ can be regarded the output of the filters $1/(s+\lambda)^2$, $s/(s+\lambda)^2$, and $s^2/(s+\lambda)^2$, respectively, with the same bounded input $\hat{H}^{-1}(\hat{\theta}(T), q(t))\tilde{\epsilon}(t)$. Hence $\tilde{q}$, $\dot{\tilde{q}}$, and $\ddot{\tilde{q}}$ are uniformly bounded. But $\tilde{q} = q - q_d$ and q_d, $\dot{q}_d$, $\ddot{q}_d$ are uniformly bounded, so q, $\dot{q}$, $\ddot{q}$ are uniformly bounded $\forall t > T$. The uniform boundedness of τ can be obtained from (12) and the uniform boundedness of q, $\dot{q}$, and $\ddot{q}$.

The third result can be obtained from (17). *Q.E.D.*

Remark: In this ideal case without external disturbance, the limit of upper bound of tracking error $\tilde{q}$ can be made arbitrary small by choosing arbitrarily small ϵ.

Theorem 5 (Robustness Against Disturbance) *Let (12) and (14) be the adaptive controller. Assume the manipulator structure be the same as that stated in Theorem 4 except the joint torque now consists of actuator outputs and external disturbances, $\tau_a = \tau + d$. Assume further that*

- *Θ_0 is a bounded set*
- *Θ_∞ contains an open ball*
- *$\hat{H}(\theta, q) > \delta I$, $\forall q$, $\forall \theta \in \Theta_0$, for some unknown positive number δ*

Then, for any initial conditions $q(t_0)$ and $\dot{q}(t_0)$, the following results hold

1. *there exists $T \in \mathbb{R}$ such that $\hat{\theta}(t) = \hat{\theta}(T)$, $\forall t \geq T$*
2. *q, $\dot{q}$, $\ddot{q}$, and τ are uniformly bounded*
3. *$\limsup_t |\tilde{q}_k(t)| < c \sup_t \bar{d}_k(t)$, c is a positive number*

Proof: the same as the proof of Theorem 4.

5 Mathematical Simulation

Fictitious mathematical simulations are performed to demonstrate the performance of the unfalsified control method. The two-link manipulator given in Example 1 is used in the simulations. In the simulation, the following parameters are used

$$m_1 = 1,\ l_1 = 1,\ m_e = 2,\ \delta_e = 30°,$$
$$I_1 = .12,\ l_{c1} = .5,\ I_e = 0.25,\ l_{ce} = 0.6$$

so that the exact parameter vector is $\theta^* \triangleq [a_1, a_2, a_3, a_4]^T = [3.34, 0.97, 1.0392, 0.6]^T$. The scenario of the simulation is that the end effector mass m_e changes back and forth between 2 and 20 periodically with period 0.5 *sec*, so does its inertia I_e between 0.25 and 2.5, so that the parameter vector changes between $[3.34, 0.97, 1.0392, 0.6]^T$ and [30.079.710.39236] periodically with period 0.5 *sec* accordingly. The magnitudes of parameter vectors are unknown to the controller. The desired trajectory used is

$$q_{d1}(t) = 30°(1 - \cos 2\pi t), \quad q_{d2}(t) = 45°(1 - \cos 2\pi t)\ .$$

The external torque disturbance acting on the two joints are $\sin 200\pi t$ and $2\sin 133\pi t$, respectively. Initial angles of both joints are set to 0.4 *rad*.

For comparison, two simulations are performed; one uses the unfalsified control method and the other uses the adaptive control method by Slotine *et al.* [9] . The adaptive control law of Slotine *et al.* is

$$\tau = Y\hat{\theta} - K_D s \quad (18)$$

where $\hat{\theta}$ is the estimated parameter vector of θ, K_D is positive definite, and s is defined as

$$s \stackrel{\triangle}{=} \dot{\tilde{q}} + \Lambda \tilde{q} \quad (19)$$

Λ is a positive definite matrix. Similar to (3), Y satisfies

$$H(q)\ddot{q}_r + C(q,\dot{q})\dot{q}_r + g(q) = Y(q,\dot{q},\dot{q}_r,\ddot{q}_r)\theta,$$

in which $\dot{q}_r = \dot{q}_d - \Lambda\tilde{q}$. The parameter update law for $\hat{\theta}$ is

$$\dot{\hat{\theta}} = -\Gamma Y^T s \quad (20)$$

in which $\Gamma > 0$. The parameters used in the simulation are $K_D = 100I_2$, $\Lambda = 20I_2$, and Γ =diag([.03, .05, .1, .3]).

For the unfalsified control method, Θ_0 taken as a solid square box centering at origin with each edge of length 200. To avoid the complexity, the choice of Θ_0 does not take into account the condition $\hat{H}(\theta, q) > \delta I, \forall q, \forall \theta \in \Theta_0$, as required by the threorems of Section 4; simulation results show boundedness of signal can still be obtained without this singularity condition. For simplicity, the computation delay time ν_i is taken as constant for all i with $\nu_i = 10^{-3}$ *sec*. The parameter λ used in the control law (12) is $\lambda = 20$. The linear programming parameter update law (16) is used, in which constant $\bar{d} = [2, 4]^T$ is used to bound the effect of external disturbance. Since the "correct" parameter vector changes periodically, the parameter update law is reset every 0.5 *sec*.

In the simulation, both control methods use [10, 10, 10, 10] as initial guess for the parameter estimate, and the results are shown in Figure 2 to Figure 8. It can seen that the transients of the tracking error resulted from change of manipulator end effector are smaller for the unfalsified method. Figure 4 to Figure 7 show that the estimated parameters of Slotine *et al.*'s method cannot keep track of the "correct" parameters (adjusting the controller parameter Γ does not improve the condition), whereas those of the unfalsified method can reach close to the "correct" values quickly.

Finally, in order to show that the computation complexity of the unfalsified method is actually tractable, the number of floating point operations (flops) used in each run of the linear programming routine (solved by using Matlab's lp function) is plotted in Figure 8.Because previous solution from execution of the linear programming routine serves as a good initial guess for the next execution, the computation amount, although grows with time after each reset, is kept under several hundred kilo flops. This amount of computation should be able to handled by a current high performance computer in real time operation. It is expected that the flops could be further reduced by modifications of the linear programming routine; for example, it is not necessary to find the exact largest inner ball within the intersection, an approximate inner ball is enough for use.

6 Discussion and Conclusion

Section 4 contains the main result of this paper, which, using manipulator trajectory control as an example, describes how the prior mathematical knowledge and prejudice and the posterior data can be combined through the unfalsified control concept to form an adaptive control procedure. It is true that there is no guarantee that the application of the resulted adaptive controller to the real system can achieve the expected performance, the use of adaptive controller also requires a subject judgement for its fitness. Two mathematical theorems are given in Section 4 to serve as a reference for such judgement. With the help of mathematical assumptions, the theorems guarantee finite time achievement of the expected performance and the global stability w.r.t. initial states, and that tracking errors are proportional to the size of disturbance.

The unfalsified control method provides a new approach to adaptive control and learning. The capability of any control law can be determined either on line or off line without having to put the control law for a trial run, and the roles of *a priori* mathematical knowledge and prejudice and *a posteriori* data are clearly identified within the context of the unfalsified control approach. Since expected non-adaptive results can be hypothesized for falsification test, the unfalsified control performance can be at least as good as that of non-adaptive methods. Furthermore, the possibility of occurrence of an empty unfalsified parameter set provides a mechanism to inform its user the inadequacy in capability of the control law structure in use to achieve expected performance (and the necessity of changing control law structure); on the other side, traditional adaptive controllers do not have a way to tell whether the occurrence of bad transients means the controller would not work as expected, or the transients will diminish eventually.

The unfalsified control approach is "model-free" in the sense that no plant model is required – plant input-output data alone is able to test *all* the controllers to determine which *cannot* achieve an expected performance (it is not possible to determine which one *can*). However, this model-free method and the model-based methods are not necessarily mutually exclusive. As demonstrated in this paper, prior knowledge of model-based methods and observations can help us produce a (biased) choice of (sequence of) achievable performance and control law combination. A proper choice of sequence of control law/achievable performance combinations could result in an earlier achievement of expected performance when possible. In the extreme situation when we are certain about the acuracy of a given plant model, the control law and achievable performance can be determined uniquely according to the model, Θ_0 can be chosen as a single point; in the other extreme situation when no prior knowledge is available, Θ_0 can be chosen as the whole space. The unfalsified control method also applies to time varying and/or nonlinear systems; however, *a priori* knowledge about the time varying/nonlinear structure can help to produce better performance. For example, the manipulator discussed in this paper is a nonlinear system, when its nonlinearity structure is not known, it would not be an easy task to come out with the computed torque control law (4) and the associated achievable performance (6) from blind guesses.

A potential difficulty of the unfalsified control approach is the computation of set intersections. In general, since past data need to be remembered, the amount of computation grows fast. However, for special cases such as the manipulator system discussed in this paper, the computation burden can be relaxed by transforming the intersection problem to a linear programming problem; Figure 8 of our simulation results shows that the required amounts of flops can be manageable. It is expected that similar transformation can be found for each different problem for efficient calculation of set intersections.

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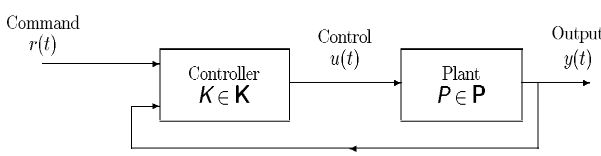

Figure 1: Feedback control system.

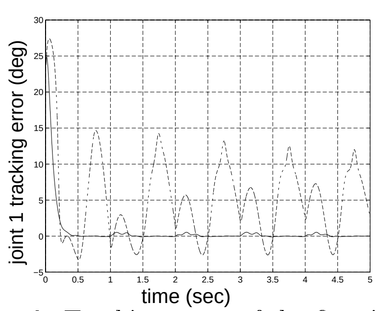

Figure 2: Tracking error of the first joint.

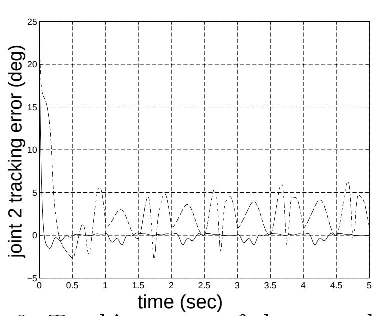

Figure 3: Tracking error of the second joint.

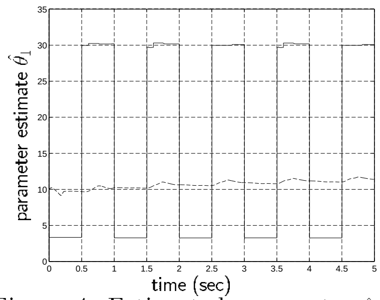

Figure 4: Estimated parameter $\hat{a}_1$.

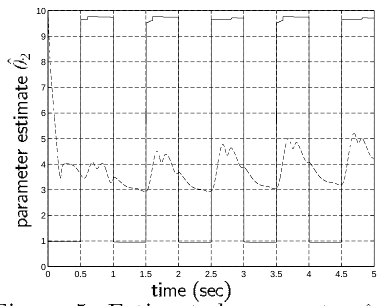

Figure 5: Estimated parameter $\hat{a}_2$.

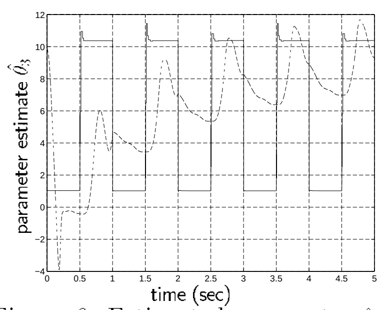

Figure 6: Estimated parameter $\hat{a}_3$.

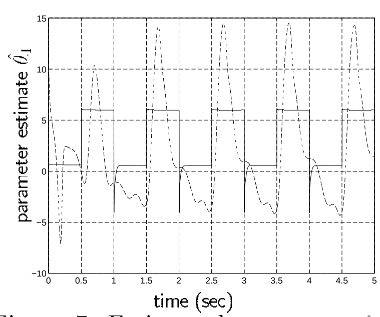

Figure 7: Estimated parameter $\hat{a}_4$.

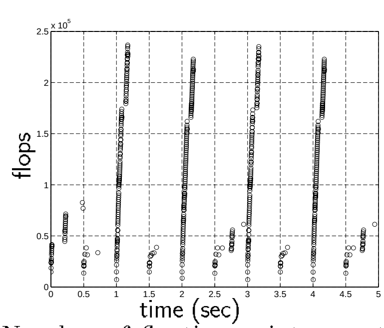

Figure 8: Number of floating point operations of one run of linear programming routine.