The Design of Strictly Positive Real Systems Using Constant Output Feedback

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Abstract

In this paper we present a Linear Matrix Inequality (LMI) approach to strictly positive real (SPR) synthesis problem: find an output feedback K such that the closed-loop system $T(s)$ is SPR. We establish that if no such constant output feedback K exists, then no dynamic output feedback with a proper transfer matrix exists to make the closed-loop system SPR.

The existence of K to guarantee the SPR property of the closed-loop system is used to develop an adaptive control scheme that can stabilize any system of arbitrary unknown order and unknown parameters.

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1 Introduction

The notion of a passive system is one of the oldest in system, circuit and control theory. Within control theory, a well-known result is that a negative feedback connection of a passive dynamic system and a stable strictly passive uncertainty is internally stable. For finite-dimensional linear, time-invariant (LTI) systems, passivity is equivalent to positive realness.

Recently the positive real synthesis problem has been investigated by several researchers (e.g., [1, 2, 3, 4, 5]). In [6], it has been shown that the strongly positive real synthesis problem is equivalent to a Bilinear Matrix Inequality (BMI) feasibility problem. However, because BMI problems are in general nonconvex and hence difficult to solve [7, 8], there has been much interesting in identifying special cases in which the BMI problem can be reduced to a Linear Matrix Inequality (LMI) feasibility problem. So far, this has been possible only in the cases of (1) full-order control [5] and (2) full-state feedback [3]. A main result of the present paper is the addition of the special case of constant output feedback to the list of positive real synthesis problems that can be solved via LMI's. The result, it turns out, has an interesting application to a problem in adaptive control theory.

We consider the configuration in Figure 1. This is a special case of that in [1, 2, 3, 4, 5] in which the original plant matrices $B_1 = B_2$, $C_1 = C_2$ and $D_{ij} = 0$ for $i, j = 1, 2$. We derive an LMI necessary and sufficient conditions for the existence of a constant output feedback matrix K for a closed-loop system in Figure 1 to be SPR. We also develop a formula for all such K that solves the problem. The derivation leads to a parameterization of all solutions K with only one free matrix which is positive definite. Further, we show that if no constant feedback can lead to an SPR closed-loop system then no dynamic feedback with proper feedback transfer matrix can do it either. Hence, there exists an output feedback such that the closed-loop system is SPR if and only if there exists a constant output feedback rendering the closed-loop system SPR. Finally, we demonstrate the use of the results by developing an adaptive control scheme that can stabilize and regulate the plant output to zero of any plant with arbitrary and unknown order and unknown parameters.

2 Preliminaries and Notation

Consider the system $T(s)$ shown in Figure 1. In this figure, K is a constant feedback and $G(s)$ is the transfer function of the system

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= C^T x \end{aligned} \tag{1}$$

where $x \in R^n$, $u \in R^q$, $y \in R^q$, and $A \in R^{n\times n}$, $B \in R^{n\times q}$, $C \in R^{n\times q}$ are constant matrices.

The equation of the closed-loop system of Figure 1 can be expressed as

$$\begin{aligned} \dot{x} &= A_k x + Bv \\ y &= C^T x \end{aligned} \tag{2}$$

where $A_k = A - BKC^T$.

The following definitions and lemmas are referred to in our main result.

Definition 2.1 *[9, 10] A square transfer function matrix $X(s)$ is strictly positive real (SPR) if:*
1. *$X(s)$ is analytic in the closed right half complex plane,*
2. $\mathrm{herm}\{X(j\omega)\} > 0$ *for all* $\omega \in (-\infty, \infty)$,

I, I_n	The identity matrix, the $n \times n$ identity matrix.
X^T	Matrix transpose.
X^*	Complex conjugate transpose.
$X^\dagger$	The Moore-Penrose pseudo-inverse of X
$\mathrm{herm}\{\cdot\}$	Hermitian part; $\mathrm{herm}\{X\} := \frac{1}{2}(X + X^*)$
$X_\perp$	Orthonormal null space of X, $X_\perp^T X = 0$, $[X, X_\perp]$ invertible and $X_\perp^T X_\perp = I$.
$X^{\frac{1}{2}}$	Square root matrix of X such that $X^{\frac{1}{2}T} X^{\frac{1}{2}} = X$
$\stackrel{ss}{=}$	State space realization $G(s) = C(Is - A)^{-1}B + D \stackrel{ss}{=} \begin{bmatrix} A & B \\ C & D \end{bmatrix}$

Table 1: Notation

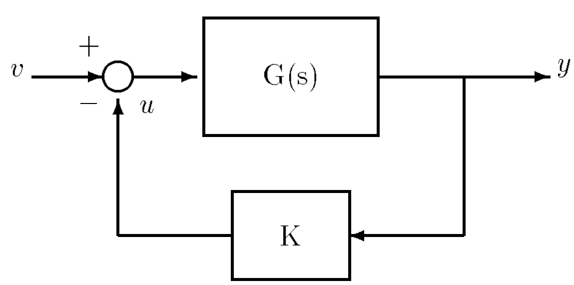

Figure 1: Closed-loop system $T(s)$

3. $\text{herm}\{X(\infty)\} \geq 0$,
4. $\lim_{\omega\to\infty} \omega^2 \, \text{herm}\{X(j\omega)\} > 0$ *if* $\text{herm}\{X(\infty)\}$ *is singular.*

Lemma 2.1 (Strictly Positive Real Lemma) *[11, 12, 13, 14] The closed-loop transfer function matrix* $T(s) = C^T(sI - A_k)^{-1}B$ *is SPR if and only if there exists a matrix* $P = P^T > 0$ *such that*

$$PA_k + A_k^T P < 0 \tag{3}$$

$$PB = C \tag{4}$$

Lemma 2.2 (Positive Real Version of the Parrott's Theorem) *Let* R, U, V *and* P *be given matrices with appropriate dimensions where* U, V^T *are full column rank and* P *is invertible. Then there exists a matrix* Q *such that*

$$\text{herm}\Big\{R + UQV^T\Big\} > 0 \tag{5}$$

if and only if

$$\text{herm}\Big\{U_\perp^T R U_\perp\Big\} > 0, \qquad \text{herm}\Big\{V_\perp^T R V_\perp\Big\} > 0.$$

Moreover, the matrix Q *in (5) is given by the equation*

$$Q = (I - YV^T(I+R)^{-1}U)^{-1}Y \tag{6}$$

where
$Y = -\bar{U}^\dagger(I-L)^{-1}\bar{R}V^{\dagger T}$
$L = \bar{R}\bar{V}_\perp \bar{V}_\perp^T \bar{R}^T \bar{U}_\perp \bar{U}_\perp^T$
$\bar{R} = (I-R)(I+R)^{-1}$
$\bar{U} = -\sqrt{2}(I+R)^{-1}U$
$\bar{V} = -\sqrt{2}(I+R^T)^{-1}V.$

Proof:
See[15, 16, 17]. *Q.E.D.*

In [18], a formula for all symmetric matrices P satisfying (4) have been introduced. In the following lemma, we develop a formula for all positive definite matrices P satisfying (4).

Lemma 2.3 *Suppose* B *and* C *are full rank. Then there exists a matrix* $P = P^T > 0$ *that satisfies (4) if and only if*

$$B^T C = C^T B > 0 \tag{7}$$

Furthermore, when (7) holds, all solutions of (4) are given by

$$P = C(B^T C)^{-1} C^T + B_\perp X B_\perp^T \tag{8}$$

where $X \in R^{n-q \times n-q}$ *is an arbitrary positive definite matrix.*

Proof: A matrix P satisfies (4) if and only if P can be expressed as

$$P = CB^{\dagger} + YB_{\perp}^{T} \tag{9}$$

for some $n \times n - q$ matrix Y.

Premultiplying (9) by $\begin{bmatrix} B^T \\ B_{\perp}^T \end{bmatrix}$ and postmultiplying by its transpose, we obtain

$$\begin{bmatrix} B^T \\ B_{\perp}^T \end{bmatrix} P \begin{bmatrix} B & B_{\perp} \end{bmatrix} = \begin{bmatrix} B^TC & B^TY \\ B_{\perp}^TC & B_{\perp}^TY \end{bmatrix}.$$

Therefore, $P = P^T$ and (4) holds if and only if

$$B^TC = C^TB, \tag{10}$$

$$B_{\perp}^TC = Y^TB, \tag{11}$$

and

$$B_{\perp}^TY = Y^TB_{\perp}. \tag{12}$$

Now, the matrix Y satisfies (11) if and only if Y can be expressed as

$$Y = B_{\perp}Z^T + (B^{\dagger})^TC^TB_{\perp} \tag{13}$$

for some $n - q \times n - q$ matrix Z.

Substituting (13) into (12), we obtain

$$Z = Z^T = B_{\perp}^TY. \tag{14}$$

Since we can always choose $Z = Z^T$ so that (10)–(12) are satisfied, we conclude that $P = P^T$ and therefore (4) holds if and only $B^TC = C^TB$. Now, substituting (13) into (9), all solutions $P = P^T$ of equation (4) are given by

$$P = CB^{\dagger} + (B^{\dagger})^TC^TB_{\perp}B_{\perp}^T + B_{\perp}ZB_{\perp}^T \tag{15}$$

where Z is an arbitrary symmetric matrix. Note that (15) has also been introduced in [18].

Further, $P > 0$ if and only if

$$\begin{bmatrix} B^T \\ B_{\perp}^T \end{bmatrix} P \begin{bmatrix} B & B_{\perp} \end{bmatrix} = \begin{bmatrix} B^TC & B^TY \\ B_{\perp}^TC & B_{\perp}^TY \end{bmatrix} = \begin{bmatrix} B^TC & C^TB_{\perp} \\ B_{\perp}^TC & Z \end{bmatrix} > 0.$$

Applying a Schur complement argument, it can be shown that

$$\begin{bmatrix} B^TC & C^TB_{\perp} \\ B_{\perp}^TC & Z \end{bmatrix} > 0$$

if and only if (7) holds and

$$X \triangleq Z - B_{\perp}^TC(B^TC)^{-1}C^TB_{\perp} > 0. \tag{16}$$

Since it is always possible to choose Z so that (16) holds, we can conclude that there exists $P = P^T > 0$ such that (4) holds if and only if (7) holds.

Moreover, substituting (16) into (15), all solutions $P = P^T > 0$ to equation (4) are given by

$$P = CB^{\dagger} + B_{\perp}B_{\perp}^TC(B^TC)^{-1}C^TB_{\perp}B_{\perp}^T + (B^{\dagger})^TC^TB_{\perp}B_{\perp}^T + B_{\perp}XB_{\perp}^T \tag{17}$$

where X is an arbitrary positive definite matrix.

Substituting $B^{\dagger} = (B^TB)^{-1}B^T$ and $B_{\perp}B_{\perp}^T = I - BB^T$ into (17), we obtain (8). *Q.E.D.*

3 All Solutions to Strictly Positive Real Synthesis Problem

In this section, we develop the necessary and sufficient conditions for the existence of the constant feedback K rendering the closed-loop system with transfer function matrix $T(s)$ in Figure 1 SPR. Once the Lyapunov matrix P in Lemma 2.1 is determined, a formula for all solutions K to the SPR synthesis problem is presented. Further, we study the SPR synthesis problem where instead of constant output feedback we use a dynamic one i.e., the transfer function of the controller is a proper transfer matrix.

Without loss of generality, we assume B and C are full rank.

Theorem 3.1 *There exists a constant matrix K such that the closed-loop transfer function matrix $T(s)$ in Figure 1 is SPR if and only if*

$$B^T C = C^T B > 0 \tag{18}$$

and there exists a positive definite matrix X such that

$$C_\perp^T \operatorname{herm}\Big\{B_\perp X B_\perp^T A\Big\} C_\perp < 0 \tag{19}$$

Furthermore, when (18) and (19) hold, all such solutions K are given by

$$\begin{aligned} K = \;& C^\dagger \operatorname{herm}\{PA\}\,(I - C_\perp(C_\perp^T \operatorname{herm}\{PA\}\,C_\perp)^{-1} C_\perp^T \operatorname{herm}\{PA\})C^{\dagger T} \\ & + S \end{aligned} \tag{20}$$

where $P = C(B^T C)^{-1} C^T + B_\perp X B_\perp^T$ and S is an arbitrary positive definite matrix.

Proof: From Lemma 2.3, there exists a matrix $P = P^T > 0$ satisfying $PB = C$ if and only if

$$B^T C = C^T B > 0$$

Further, a formula for P satisfying $PB = C$ is given by

$$P = C(B^T C)^{-1} C^T + B_\perp X B_\perp^T \tag{21}$$

where X is an arbitrary positive definite matrix.

From Lemma 2.2, there exists a matrix K such that

$$\operatorname{herm}\Big\{P(A - BKC^T)\Big\} < 0$$

if and only if

$$C_\perp^T \operatorname{herm}\{PA\}\,C_\perp < 0 \;\text{ and }\; (PB)_\perp^T \operatorname{herm}\{PA\}\,(PB)_\perp < 0.$$

Since $PB = C$, $\operatorname{herm}\Big\{P(A - BKC^T)\Big\} < 0$ if and only if

$$C_\perp^T \operatorname{herm}\{PA\}\,C_\perp < 0. \tag{22}$$

Substituting (21) into (22), we obtain (19).

Now we prove equation (20). Suppose $B^TC > 0$ and there exists a positive definite matrix X satisfying condition (19), then we can generate P by (21). Further,

$$\begin{aligned}
&\text{herm}\Big\{P(A - BKC^T)\Big\} \\
&= \text{herm}\Big\{PA - CKC^T\Big\} \\
&= \begin{bmatrix} C_\perp & C(C^TC)^{-\frac{1}{2}} \end{bmatrix} \begin{bmatrix} C_\perp^T \\ (C^TC)^{-\frac{1}{2}}C^T \end{bmatrix} \text{herm}\Big\{PA - CKC^T\Big\} \\
&\quad \begin{bmatrix} C_\perp & C(C^TC)^{-\frac{1}{2}} \end{bmatrix} \begin{bmatrix} C_\perp^T \\ (C^TC)^{-\frac{1}{2}}C^T \end{bmatrix} \\
&= \begin{bmatrix} C_\perp & C(C^TC)^{-\frac{1}{2}} \end{bmatrix} \begin{bmatrix} C_\perp^T \,\text{herm}\{PA\}\, C_\perp & C_\perp^T \,\text{herm}\{PA\}\, C(C^TC)^{-\frac{1}{2}} \\ (C^TC)^{-\frac{1}{2}}C^T \,\text{herm}\{PA\}\, C_\perp & W \end{bmatrix} \begin{bmatrix} C_\perp^T \\ (C^TC)^{-\frac{1}{2}}C^T \end{bmatrix}
\end{aligned} \tag{23}$$

where

$$W = (C^TC)^{-\frac{1}{2}}C^T \,\text{herm}\{PA\}\, C(C^TC)^{-\frac{1}{2}} - (C^TC)^{\frac{1}{2}} \,\text{herm}\{K\}\, (C^TC)^{\frac{1}{2}}.$$

Applying the Schur complement argument, we can verify that

$$\begin{bmatrix} C_\perp^T \,\text{herm}\{PA\}\, C_\perp & C_\perp^T \,\text{herm}\{PA\}\, C(C^TC)^{-\frac{1}{2}} \\ (C^TC)^{-\frac{1}{2}}C^T \,\text{herm}\{PA\}\, C_\perp & W \end{bmatrix} < 0$$

if and only if

$$C_\perp^T \,\text{herm}\{PA\}\, C_\perp < 0$$

and

$$\begin{aligned}
&(C^TC)^{-\frac{1}{2}}C^T \,\text{herm}\{PA\}\, C(C^TC)^{-\frac{1}{2}} - (C^TC)^{\frac{1}{2}} \,\text{herm}\{K\}\, (C^TC)^{\frac{1}{2}} \\
&-(C^TC)^{-\frac{1}{2}}C^T \,\text{herm}\{PA\}\, C_\perp (C_\perp^T \,\text{herm}\{PA\}\, C_\perp)^{-1} \\
&C_\perp^T \,\text{herm}\{PA\}\, C(C^TC)^{-\frac{1}{2}} < 0.
\end{aligned} \tag{24}$$

From (24), we obtain (20). *Q.E.D.*

Remark: In the SISO case, the necessary condition $B^TC > 0$ implies the relative degree of $G(s)$ is one.

Remark: Inequality (19) is essentially a Linear Matrix Inequality (LMI) problem which can be solved using the LMI toolbox [19].

Let us now consider the SPR synthesis problem using dynamic output feedback, i.e., we consider

$$u = -H(s)y$$

where

$$H(s) \stackrel{ss}{=} \begin{bmatrix} A_c & B_c \\ C_c^T & D_c \end{bmatrix}$$

Theorem 3.2 *If no constant K in Theorem 3.1 exists, then there exists no dynamic controller with proper transfer matrix renders the closed-loop system $T(s)$ SPR.*

Proof: In the dynamic controller case, the state space form of the closed-loop system can be expressed as:

$$T(s) \overset{ss}{=} \left[\begin{array}{c|c} \begin{bmatrix} 0 & 0 \\ 0 & A \end{bmatrix} + \begin{bmatrix} I_m & 0 \\ 0 & B \end{bmatrix} Q \begin{bmatrix} I_m & 0 \\ 0 & C^T \end{bmatrix} & \begin{bmatrix} 0 \\ B \end{bmatrix} \\ \hline \begin{bmatrix} 0 & C^T \end{bmatrix} & 0 \end{array}\right]$$

where $Q = \begin{bmatrix} A_c & B_c \\ C_c^T & D_c \end{bmatrix}$ and m is the order of the controller.

From Lemma 2.1, $T(s)$ is SPR if and only if there exists a positive definite matrix P such that

$$P \begin{bmatrix} 0 \\ B \end{bmatrix} = \begin{bmatrix} 0 \\ C \end{bmatrix} \tag{25}$$

and

$$\text{herm}\left\{ P\left(\begin{bmatrix} 0 & 0 \\ 0 & A \end{bmatrix} + \begin{bmatrix} I_m & 0 \\ 0 & B \end{bmatrix} Q \begin{bmatrix} I_m & 0 \\ 0 & C^T \end{bmatrix} \right) \right\} < 0 \tag{26}$$

Applying the same technique as in the proof of Theorem 3.1, we can show that there exists a positive definite matrix P satisfying (25) if and only if

$$B^T C > 0.$$

Moreover, when a solution P exists, then all solutions P to (25) are given as

$$P = \begin{bmatrix} 0 \\ C \end{bmatrix} (B^T C)^{-1} \begin{bmatrix} 0 & C^T \end{bmatrix} + \begin{bmatrix} I & 0 \\ 0 & B_\perp \end{bmatrix} X \begin{bmatrix} I & 0 \\ 0 & B_\perp \end{bmatrix}^T \tag{27}$$

where X is an arbitrary positive definite matrix.

From Lemma 2.2,

$$\text{herm}\left\{ P\left(\begin{bmatrix} 0 & 0 \\ 0 & A \end{bmatrix} + \begin{bmatrix} I_m & 0 \\ 0 & B \end{bmatrix} Q \begin{bmatrix} I_m & 0 \\ 0 & C^T \end{bmatrix} \right) \right\} < 0$$

if and only if

$$\begin{bmatrix} I_m & 0 \\ 0 & B \end{bmatrix}_\perp^T \text{herm}\left\{ \begin{bmatrix} 0 & 0 \\ 0 & A \end{bmatrix} P^{-1} \right\} \begin{bmatrix} I_m & 0 \\ 0 & B \end{bmatrix}_\perp < 0 \tag{28}$$

and

$$\begin{bmatrix} I_m & 0 \\ 0 & C \end{bmatrix}_\perp^T \text{herm}\left\{ P \begin{bmatrix} 0 & 0 \\ 0 & A \end{bmatrix} \right\} \begin{bmatrix} I_m & 0 \\ 0 & C \end{bmatrix}_\perp < 0 \tag{29}$$

Also $\begin{bmatrix} I_m & 0 \\ 0 & C \end{bmatrix}_\perp$ can be represented by the following:

$$\begin{bmatrix} I_m & 0 \\ 0 & C \end{bmatrix}_\perp = \begin{bmatrix} 0 \\ C_\perp \end{bmatrix}.$$

Applying (27) to (29), we find that (29) is satisfied if and only if

$$C_\perp^T \,\text{herm}\left\{ B_\perp X_{22} B_\perp^T A \right\} C_\perp < 0 \tag{30}$$

where X_{22} is a (2,2) block of X and is positive definite.

We have shown that $B^T C > 0$ and (30) are the necessary conditions for the existence of the dynamic controller rendering the closed-loop transfer function matrix $T(s)$ SPR. Also $B^T C > 0$ and (30) are the necessary and sufficient conditions given by Theorem 3.1 for the constant output feedback case. *Q.E.D.*

Theorem 3.2 shows that if we can not find a constant output feedback controller to make the closed-loop transfer function matrix SPR, then there is no dynamic feedback controller with a proper transfer function matrix that can make the closed-loop transfer function matrix SPR. The results obtained have interesting applications in adaptive control as demonstrated in the following section.

4 Applications to Adaptive Control Law Design

In this section we apply the results of the previous sections to develop an adaptive control scheme that can stabilize and regulate the output to zero of any plant with arbitrary and unknown order and unknown parameters. The only assumption we use is the existence of a constant output feedback matrix K^* such that the closed-loop transfer function matrix $T(s)$ is SPR. The conditions for existence of K^* are given by Theorem 3.1 This approach with similar assumptions is not new in adaptive control [20, 21, 22] and is included here for demonstrating the usefulness of the results obtained.

We can rewrite the closed-loop system equations (2) as

$$\begin{aligned} \dot{x} &= (A - BK^*C^T)x - B(K - K^*)y \\ y &= C^T x \end{aligned}$$

or

$$\begin{aligned} \dot{x} &= A^* x - B\tilde{K} y \\ y &= C^T x \end{aligned} \tag{31}$$

where $A^* = A - BK^*C^T$, $\tilde{K} = K - K^*$ and $K(t)$ is the estimate of K^* at time t.

We start by considering the quadratic function

$$V = \frac{x^T P x}{2} + \mathrm{trace}(\frac{\tilde{K}^T \Gamma^{-1} \tilde{K}}{2})$$

where P satisfies Lemma 2.1 and Γ is an arbitrary positive definite matrix. The time derivative of V along any trajectory of (31) is given by

$$\dot{V} = x^T (PA^* + A^{*T} P) x - \mathrm{trace}(\tilde{K}^T y y^T - \tilde{K}^T \Gamma^{-1} \dot{\tilde{K}}).$$

If we choose

$$\dot{\tilde{K}} = \Gamma y y^T, \tag{32}$$

we have

$$\dot{V} = x^T (PA^* + A^{*T} P) x \leq 0.$$

Since V is a quadratic function and $\dot{V} \leq 0$, we conclude that V is a Lyapunov function for the system (31), (32).

Since V is a non-increasing function of time, the $\lim_{t\to\infty} V(t)$ exists. Therefore, we obtain x, $\tilde{K} \in L_\infty$ and $x \in L_2$. Since $\dot{K} = \Gamma y y^T$ and $y = C^T x$ where $x \in L_\infty$, we have $\dot{K} \in L_\infty$. Since $\dot{x} \in L_\infty$ due to $\tilde{K}$, x, $y \in L_\infty$ we conclude from $\dot{x} \in L_\infty$ and $x \in L_2$ [23] that $x(t) \to 0$ as $t \to \infty$.

Hence, $u = -K(t)y$ with $\dot{K} = \Gamma y y^T$ can stabilize any system of any order and drive y, x to zero as long as the assumption of the existence of K^* that makes the closed-loop plant transfer matrix SPR is satisfied. Theorem 3.1 gives necessary and sufficient conditions for this assumption to hold.

5 Conclusion

In this paper we developed necessary and sufficient conditions for the plant state space matrices that guarantee the existence of a constant output feedback gain matrix K so that the closed-loop system is SPR. The necessary and sufficient conditions are represented in the form of LMI. In addition we developed a procedure for calculating such K from the knowledge of the system matrices. We established that if no such K exists then no dynamic output feedback with proper transfer function matrix can make the closed-loop system SPR.

We showed that the existence of K for the closed-loop system to be SPR can be used to generate an adaptive control regulator that can stabilize any plant with arbitrary order and unknown parameters and regulate its output vector to zero.

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