Special Issue on Multivariable Stability Margin.
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EDITORIAL

The seven papers in this issue reflect traces of a revolution in thought that began more than twenty years ago when the dominant focus of control theory research shifted from optimality to robustness. The multivariable stability margin concept has become such an integral part of present day control theory that it is difficult to imagine the time not so very long ago when the concept lacked a mathematical representation and the tools of multivariable stability margin analysis were yet to be identified. Therefore, it seems desirable to revisit that time and to examine the events that facilitated, and necessitated, this remarkable paradigm shift.

It began in 1975 at the MIT Electronic Systems Laboratory headed by Michael Athans. Linear multivariable control had quietly arrived at a state of crisis. Theorists who had had the highest expectations for optimal LQG feedback design theory were being jolted by disappointing results from initial attempts to apply the theory to apply the theory to realistic problems. In one classified design study carried out by Systems Control Inc. under the aegis of D. L. Kleinman with Michael Athans as consultant, and LQG controller for a Trident submarine caused the vessel to unexpectedly surface in nonlinear simulations involving moderately rough seas (see the comments of Athans cited on page 40 of Reference 1). In another example, a paper describing the disappointing results of an LQG control design study for the F-8C Crusader aircraft[2] concluded euphemistically with the observation that “The study has pinpointed certain theoretical weaknesses. . . as well as the need for using common sense pragmatic techniques to modify the design based on ‘pure’ theory.” The problem — yet to be clearly identified and labeled — was of course inadequate attention to multivariable stability margin issues.

Prior to 1975, mathematical formulations of linear control problems had not yet captured the concept of stability robustness. Augering things to come, G. Zames had made the following suggestive remark in 1966.[3]

> “One of the broader implications of the theory here concerns the use of functional analysis for the study of poorly defined systems. It seems possible, from only coarse information about a system, and perhaps even without knowing details of internal structure, to make useful assessments of qualitative behavior.”

Even earlier, Popov[4] had coined the term “hyperstability” to describe the kind of nonlinear robustness which is implicit in the inequalities of Lyapunov stability theory. But by 1975 the suggestive remarks of Zames and Popov had had little impact. The robustness implications of nonlinear stability theory had yet to be developed. The small-gain theorem which was soon to emerge as a core concept in robust control theory was still regarded as an exotic plaything for nonlinear system theorists. There was no notation and no terminology to help focus the powerful mathematical minds of control theorists on the fundamental practical issue of robustness, though there was a remarkably prescient 1967 paper by Medanic[44] applying LQ game theory to the design of uncertainty tolerant control laws.

To be sure, there had been warnings of the impending crisis for linear multivariable control theory. As early as 1971, Athans[47] had ominously remarked, ‘It appears that the most pressing need is related to... modeling inaccuracy.' And Rosenbrock and McMorran[5] had observed that the much touted LQG theory, though optimal, had failed to address the "essential requirement... that changes of loop gains... in all combinations, should leave the system with an adequate stability margin." Rosenbrock had anticipated the multiloop stability margin problem and had already developed a crude multiloop extension of classical frequency-response design methods based on diagonal dominance which, for nearly decoupled feedback loops, allowed one to retain the simplicity of classical design one-loop-at-a-time design methods, while giving one a tool for quantitatively assessing tolerance of simultaneous gain variations in several feedback loops. Unfortunately, others who followed Rosenbrock's lead in developing multiloop extensions of classical frequency-response design methods (e.g., References 6 and 7) had lost sight of the *simultaneous* gain-variation issue. Engineers were still designing multiloop feedback control systems one-loop-at-a-time, oblivious to the simultaneous gain variation issue. Rosenbrock's warning about the importance of *simultaneous* loop gain variations had gone unheeded.

By 1975, the much lamented gap between academic theory and engineering practice in the control field had grown to prodigious proportions. Despite the fact stability margin was an integral part of the classical Bode-Nyquist theory for single-loop control, post-1950's textbooks (with the notable exception of Horowitz[8]) had reduced the phase-margin concept to little more than an aid for estimating closed-loop dominant-pole locations. Since 1959, introspective academic control theorists had been at work developing elegant, but fanciful control design techniques which were guaranteed to result in stable and even optimal feedback controllers — provided that one began with a sufficiently accurate plant model. But, as had been well known to classical control theorist such as I. Horowitz[8], the trouble with simply supposing that a model is sufficiently accurate is that the quantitative accuracy of a plant model is in fact a key determinant of what can, and what cannot, be achieved with feedback. Alas, in 1975 classical methods were no longer even studied by a majority of students pursuing the doctoral degree in control. And the theories of Horowitz were read by few. In early 1975, the stability margin concept had not yet found its way into the mathematical problem formulations of modern control theorists.

Michael Athans,[9] who had been of leading proponent of multiloop LQG feedback, was eager to identify and fix the failings of LQG theory that had been so graphically illustrated by the F-8C aircraft and the Trident submarine design studies. Progress was rapid and, unlike many scientific revolutions, acceptance was almost immediate. By June 1975, Athans' student P.K. Wong had completed an MS thesis on the subject[10] which laid much of the groundwork for the emergent theory of multivariable stability margin. Following are some of the early milestones of multivariable stability margin theory:

1975 **Diagonally Structured Uncertainty.** P. K. Wong[10] formulated the multivariable stability margin problem in terms of a matrix simultaneously varying uncertain real gains. As a special case, he examined the case of a real diagonal uncertainty matrix.

LQ multivariable gain margins. Wong[10] also proved that full-state feedback LQ controllers have the remarkable property of a 50% gain reduction tolerance and an infinite tolerant of gain increases in each of the control input channels *even when the gain variations occur simultaneously.*

1976 **LQ phase-margins.** Applying the methods of input/output stability theory along with Parseval's theorem, Safonov and Athans[11, 12] showed that Wong's result could be extended from real uncertainties to frequency-dependent complex uncertainties. Multiloop LQ state-feedback designs were shown to possess a tolerance of simultaneous phase variations of up to ±60° in each control input channel.

Robustness. The term *robust* was introduced into the control theory vocabulary by E. J. Davison[13] who used it to describe asymptotic rejection of certain classes of disturbances despite non-destabilizing plant variations. The usage of the word robustness to describe multivariable stability margins originated with Safonov and Athans[11].

Small-Gain/Positivity. In 1976, the use of the Zames-Sandberg[14, 15, 3, 16] nonlinear techniques for stability margin analysis was clearly demonstrated by Safonov and Athans[11]. Previously, the Zames-Sandberg theory had been generally regarded as a purely nonlinear theory.

1977 H_∞ **Control Theory.** Building on the 1967 game-theory results of Reference 44, an optimal small-gain feedback theory was developed by Mageirou and Ho[17]. Independently rediscovered ten years later,[18] the theory came to be known as the "game theory" or "riccati" approach to H_∞ control. But as of 1977, the term H_∞ had not yet entered the control lexicon.

The 1977 PhD thesis of Safonov[19] (later published as Reference 20) included the following:

Canonical Robust Control Problem. The now standard robust control problem was formulated in which a diagonal operator of uncertain internal gains is "pulled out" as a multivariable feedback around a nominal system (cf. Reference 19, Figure 6.1).

Fundamental Stability Theorem. The problem of stability robustness analysis was shown to be equivalent to computing a *topological separation* of graphs of feedback operators. Lyapunov, conic sector, positivity and small-gain theories were shown to emerge as special cases.

Frequency-Domain Robustness Criteria. Bode plots of eigenvalues were proposed for evaluating the stability margins of diagonally perturbed multivariable feedback systems (see Reference 19, Theorem 5.7).

1978 **LQG Counterexample.** J. C. Doyle[21] demonstrated via a simple counterexample that embedding a Kalman filter in an LQG controller may cause stability margins to become vanishingly small.

Singular Values. After a spending a year as a half-time adjunct professor at MIT, G. Stein assembled a team of consultants consisting of N. R. Sandell, J. C. Doyle, and M. G. Safonov at the Honeywell Systems and Research Center, Minneapolis, MN, USA. Influenced by discussions with MIT's A. J. Laub on the uses of singular values in the theory of matrix computations, the team of Honeywell consultants adopted the now familiar singular-value Bode plot representation of the multivariable stability margin results of Reference 19. Though singular-value stability conditions were new to the control field, very similar representations of small-gain conditions had been proposed for nonlinear stability analysis as early as 1964 by Sandberg[14].

Allerton. The singular-value robustness theory publicly debuted at the 1978 Allerton Conference in a special session[22] devoted to singular value robustness analysis. Featured speakers were A. J. Laub, N. R. Sandell, G. Stein, J. C. Doyle, M. G. Safonov and B. C. Moore.

Kharitonov's Theorem. A simple test for stability in the presence of simultaneous real variations in the coefficients of a systems' characteristic polynomial was developed.[23]

1979 **Linear Robustness.** Aware of the intimidating nature of nonlinear theories, Doyle[24] expanded the audience for the multivariable stability margin concept by specializing the singular-value stability margin results to linear systems and by developing alternative proofs based on Rosenbrock's multivariable Nyquist stability criterion.

Principal Gains. In a parallel development, MacFarlane and Scott-Jones[25] created a theory in which singular values emerged under the name "principal gains."

One-loop-at-a-time Counterexample. Doyle[24] produced a simple but compelling example which clearly demonstrated the pitfalls of traditional one-loop-at-a-time stability margin analysis and the advantages of singular-value Bode plots.

ONR Robustness Workship. To digest and disseminate the rapid progress on multivariable stability margin, N.R. Sandell organized an international workshop[1]. Included among the more than 30 participants were E. Armstrong, M. Athans, C. A. Desoer, J. C. Doyle, I. Horowitz, H. Kwakernaak, A. J. Laub, N. Lehtomaki, B. C. Moore, C. L. Nefzger, I. Postlethwaite, C. Rohrs, M. G. Safonov, R. Sivan, N. R. Sandell, G. Stein, and J. C. Willems.

1980 **Diagonal Scaling.** As was well-known to workers on robustness, diagonal scalings had been used in nonlinear stability theory to reduce the conservativeness of small-gain stability tests — see, for example, the surveys in Reference 26, Section III and Reference 27. In 1980, several authors wrote papers examining the role of diagonal scaling to singular value robustness tests.[20, 28, 29, 30]

k_m **Notation.** Prior to 1980, various upper bounds on the multivariable stability margin had been introduced, but notation for the actual multivariable stability margin itself had not yet been formally introduced. In Reference 29 (later published as References 31, 32), the diagonally-perturbed multivariable stability margin was given the name "excess stability margin" and labeled k_m.

Rohrs counterexample. The turmoil of the robustness revolution spread to adaptive control when studies at MIT showed then standard adaptive control algorithms to have vanishingly small robustness.[45, 46]

1981 **Optimization of Singular Values.** At Berkeley, E. Polak and D. Q. Mayne[33] examined the use of generalized gradient methods to optimize singular values.

Mixed-Sensitivity and H_2 Synthesis. The mixed roles of sensitivity and complementary sensitivity in robust control were identified by Safonov.[34] To manipulate mixed-sensitivity singular value Bode plots, the classical Wiener-Hopf frequency-domain representation of the LQG problem with frequency-dependent weighting matrices was found to be useful. Later the Wiener-Hopf formulation of LQG theory would come to be known as H_2 control.[35]

H_∞ **Optimal Control Synthesis.** G. Zames introduced the the Hardy space H_∞ to tthe control field and solved a very simple SISO H_∞ control problem. Eventually, multivariable versions of this theory would largely supplant the Wiener-Hopf H_2 theory as a tool for manipulating robustness singular value Bode plots.

Neoclassical Control. Solidifying the links to the classical theory of Bode, Lehtomaki and Athans[36] further examined connections between singular values and multiloop gain/phase margins and Doyle and Stein[37] proposed a singular-value loop-shaping formulation of the robust feedback synthesis problem.

1982 **The $n \leq 3$ Result.** Adopting the generalized gradient techniques of Reference 33, Doyle[38] showed for the case in which there are three or fewer complex uncertainties that optimally scaled singular values give the exact value of k_m. Safonov and Doyle[39, 40] subsequently showed that the optimal diagonal scaling problem is convex and therefore solvable.

μ **Notation.** In Reference 38, Doyle introduced the term *structured singular value* for the reciprocal of k_m and associated with it the Greek letter μ. The paper[38] also contained a pithy summary of known results, but relatively few citations.

Performance Robustness. Doyle, Wall and Stein[41] observed that the small-gain stability theorem can be reinterpreted as a *performance robustness theorem.* They pointed out that "fictitious uncertainties" can be used to embed performance specifications within the framework of multivariable stability margin analysis.

After 1982 the field of multivariable stability margin theory blossomed, as may be seen from the many references in References 42 and 43. The innovative contributions of seven papers in this special issue demonstrate the continued vibrancy and richness of the field spawned by the revolution that began in 1975 in the MIT lab of Michael Athans.

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