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ROBUST SWITCHING MISSILE AUTOPILOT

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Abstract

A robust switching controller is developed by application of the unfalsified control concept. Key ideas and implementation issues are discussed. In particular, a new performance specification and a falsification algorithm are developed. Simulation results are provided using a nonlinear model for the missile.

1 INTRODUCTION

Autopilot design for highly maneuverable missiles is a challenging problem because of their highly nonlinear dynamics, wide variations in plant parameters, and strong performance requirements. Design of missile autopilot control laws remains an open problem from a theoretical point of view because of its nonlinear nature. It is of interest to remark that a recent study gives some new insights into the nonlinear nature of the missile autopilot design [5]. From a practical point of view, many approaches have appeared in literature in the recent years [6-12].

In this context, given the nature of the problem we consider that the unfalsified control approach [1-4] could bypass some of the difficulties because of its learning capability. The unfalsified control is a methodology for direct identification of robust controllers primarily based on experimental information. However, the apriori information is used for selecting the class of possible controllers. In addition, it has the advantage that it does not require one to model the plant and obtain its uncertainty bounds which in this case is a difficult task.

The specific aim of this paper is to design an autopilot based on the unfalsified control approach. To some extent, it demonstrates the applicability of the unfalsified control to real problems. Moreover, we will present and explain in detail some implementation issues. Among these, we will emphasize two aspects: falsification algorithms and controller switching and initialization.

The paper is organized as follows. Section 2 provides a brief review of the unfalsified control. Section 3 presents the autopilot design. The nonlinear simulation results are in section 4. And section 5 concludes with the discussion.

2 UNFALSIFIED CONTROL

In this section we give a brief review of the unfalsified control and discuss some implementation issues such as the falsification condition and switching.

2.1 THEORY

This section reviews the unfalsified control concept [1]. As a brief introduction, we present the main idea behind the unfalsified control that is to apply the hypothesis testing principle to the identification of robust controllers. This means, that it assumes as a hypothesis the ability of a control law to meet the performance specification, and tests it against the experimental information about the plant.

Earlier, we mentioned that the unfalsified control is a methodology for robust identification of controllers. Robustness is understood in the general sense that the controller meets the performance specification at all times. This definition differs from the conventional one in that now robustness is with respect to all uncertainties (linear and nonlinear) that appear in the experimental data used, instead of being with respect to some precalculated bounded uncertainties.

Let us present the framework in which the unfalsified control works. Consider the input-output perspective and define the plant, controller and performance specification as graphs in the signal space $R \times U \times Y$ as follows:

-Plant:

$$P = \left\{(r,u,y) \middle| P_\tau(u - u_{data}) = 0, P_\tau(y - y_{data}) = 0\right\} \subset R \times U \times Y$$

$$P_{data} = \mathrm{P}_\tau P \qquad (\mathrm{P}_\tau \text{ is the time truncation operator})$$

-Controller:

$$K = \left\{(r,u,y) \middle| k(r,u,y) = 0\right\} \subset R \times U \times Y$$

-Performance Specification:

$$T_{spec} = \left\{(r,u,y) \middle| J(r,u,y) \geq 0\right\} \subset R \times U \times Y$$

The concept of controller falsification is defined as follows:

Definition: *A controller, K, is said falsified if the measurement information, P_{data}, is sufficient to deduce that the performance specification, T_{spec}, would have been violated if the data had been generated with the controller in the loop. Otherwise, the controller is said unfalsified.*

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In this context, the problem that the unfalsified control attempts to solve is the following:

Problem: *Given the measurement information, P_{data}, the performance specification, T_{spec}, and a set of admissible controllers,* ***K****, find the subset,* ***K****$_{ok}$, of unfalsified controllers.*

A solution to this problem is given in the following theorem (Theorem 1 in [1]):

Theorem 1: [1] *Given the problem mentioned above, then a control law K in* ***K****, is unfalsified by the measurement information P_{data} if for each triple $(r_0, u_0, y_0) \in P_{data} \cap K$, there is at least one pair (u_1, y_1) such that:*

$$(r_0, u_1, y_1) \in P_{data} \cap K \cap T_{spec}. \quad (1)$$

The main idea of the unfalsified control is to use the measurement information to eliminate controllers, which do not satisfy the defined performance criteria. This method guarantees that the unfalsified controllers satisfy the criteria insofar as data up to the current time shows.

By implementing a switching policy that selects a controller from the subset of unfalsified ones when the one in the loop is falsified we obtain a robust learning controller. In fact, this is what we implement in this paper.

For further details on the unfalsified control we encourage the reader to go to references [1-4] and references therein.

2.2 FALSIFICATION ALGORITHM

For the falsification we will use an equivalent condition to the one given in Theorem 1. It is formally stated in the following observation, which hold for any reasonable T_{spec}.

Given the assumptions of Theorem 1, if in addition, the controller is causally-left-invertible, i.e. given K $P_\tau \tilde{r}$ (called fictitious reference signal) is uniquely determined by $P_\tau u$ and $P_\tau y$, i.e., $P_\tau \tilde{r} = K^{-1}(P_\tau u, P_\tau y)$ (observe that we abuse the notation by using the inverse), then, the controller K is unfalsified if and only if

$$P_\tau(\tilde{r}, u, y) \in P_\tau T_{spec} \quad (2)$$

At this point, it is of interest to emphasize an important implication of Theorem 2. It allows one to falsify a controller without the need to put it in the loop. In fact, this is how the falsification test will be carried out. In addition, since the controller parameterization is causal-left-invertible, we will use condition (2). If condition (2) is not satisfied, then we falsify that controller. To evaluate condition (2) the first thing is to calculate the fictitious reference signal for each controller. This process may need a large number of computations since it requires to invert each controller (a dynamic system) in the unfalsified set. At this point, the chosen controller parameterization helps to reduce the number of computations. Such reduction is a consequence of the representation of the fictitious reference signal obtained by inverting the controller, as a scalar product of two vectors. One vector contains nonlinear static functions of the controller parameters, and the other vector contains filtered values of the input and output signals and does not depend on the controller parameters. That is,

$$\tilde{r}(t) = V(\theta)^T S(u, y_1, y_2, t)$$

This separation into two terms is very useful since the dynamic term does not depend on the controller parameters, and therefore it is common to all reference signals. Thus, *S* will only need to be calculated once. After we have calculated the fictitious reference signal for each controller we need to substitute it with the input and output signals in the cost function and check whether it is positive or negative. If it result negative then it means that the controller does not satisfy the performance specification, and therefore is falsified.

In some sense, this falsification process can be interpreted as a conceptual experiment for each candidate controller. This experiment consists of considering that each candidate controller is in the loop and is driven by its fictitious reference signal. Note that this in fact, would produce the same input and output signals as the one measured by definition of the fictitious reference signal.

2.3 CONTROLLER SWITCHING

There are two issues related to the controller switching the definition of the switching policy, and the implementation of the switching.

The switching policy is as follows: when the controller in the loop becomes falsified, switch it for the controller with smallest average tracking error (note this is with respect to each controller’s fictitious reference signal since this is the only reference signal for which we can obtain information about the new controller’s performance from the available data).

The switching implementation requires us to change the controller parameters and initialize the new controller. We can initialize the controller in several different ways. In one way, we can initialize it with the value that it will have at the switching time if it were on-line and were driven by its fictitious reference signal. For this case, the calculation of the initial follows from the observation made in the previous section. In particular, the initial condition for the new controller is given by $(\tilde{r}(T), u(T), y(T))$ where T is the switching time.

However, this initialization presents the following problem. There will be a perturbation (jump) in the reference signal because the initial condition corresponds to the fictitious reference signal that is different in general from the real reference signal. Since the controller is biproper this perturbation will directly affect the controller output producing a jump in its value. This in turn can affect the overall performance.

To solve this problem we propose the following solution: initialize the states of the controller to eliminate the jump in the controller output. For this reason, we will perturb the states of the controller to cancel the effect of the perturbation in the control signal. In doing so, the current value of the associated criteria does not correspond to the current controller. To avoid this problem, we consider this change in the initial condition as the effect of an external disturbance on the controller. This disturbance is not included in the calculation of the criteria, we consider it as a part of the measurement data and therefore it will probably affect the value of the criteria. This problem is bypassed in the implementation by using the fictitious reference signal for the current controller in the loop.

3 MISSILE AUTOPILOT DESIGN

The objective is to design a longitudinal autopilot for a tail-governed missile. More specifically, use the tail deflection to track an acceleration maneuver with a time constant of less than 0.35s, a steady state error of less than 5%, and a maximum overshot of 20% for the step response. The autopilot is to provide such performance over the operation range of +/- 0.35 rad angle of attack.

To apply the unfalsified control we need to define the candidate controller set and the performance specification such that we can use the results of section 2.

3.1 CANDIDATE CONTROLLER SET

There are two important issues in the definition of the candidate controller set. To begin with, the set should be general enough to include good controllers, that is, controllers that are able to meet the performance because otherwise the algorithm will falsify the whole set. On the other hand, the candidate controller set should be chosen to facilitate the falsification procedure. To meet both these requirements we select a controller set that has enough structure to guarantee a simple falsification procedure, and at the same time, includes as a special case the PID controller, which is a proven good controller. Indeed it is the most commonly used in the industry because of its simplicity and good performance, which in fact have been recently explained theoretically [5]. See figure 1 for a typical PID missile autopilot structure.

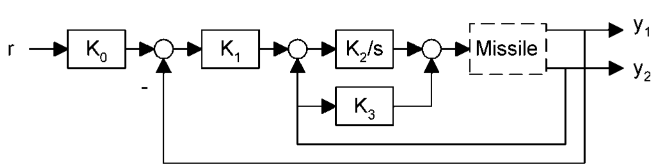

Figure 1: PID missile autopilot.

The controller structure for the unfalsified control that includes the PID as a special case is shown in figure 2.

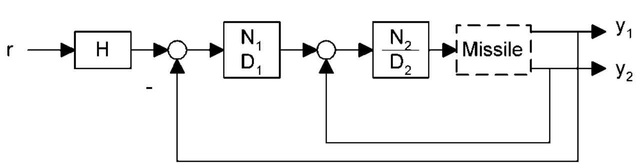

Figure 2: Controller Structure.

where $N_1(s)/D_1(s)$, and $N_2(s)/D_2(s)$ are stable minimum-phase biproper transfer functions, and H is a pure gain.

The set of candidate controllers is generated by allowing the coefficients of $D_1(s)$ and $D_2(s)$ and the gain H to vary in some intervals around some nominal values. In order to reduce the number of computation we let each coefficient take a finite number of values in its interval. The nominal values are calculated beforehand by a classical control technique based in simplified model [e.g. 4].

This definition leads to the set of candidate controllers

$$K = \left\{ (r,u,y) \in R \times U \times Y \middle| u = \frac{N_2(s)}{D^{\theta}{}_2(s)} \left[\frac{N_1(s)}{D^{\theta}{}_1(s)} \left(H^{\theta} r - y_1 \right) + y_2 \right], \theta \in \Theta \right\}$$

3.2 PERFORMANCE SPECIFICATION

The performance specification set has the form:

$$T_{spec} = \{ (r,u,y) | J(r,u,y,\tau) \geq 0, \forall \tau \geq 0 \}$$

where $J(r,u,y,\tau)$ is the cost function.

In the unfalsified control literature it is suggested to use an L_2 cost of the mixed sensitivity type because of its frequency response shaping ability. However, in the general context an L_2 gain criteria does not necessarily provide the desired properties concerning the behavior of the system. This is especially the case for nonlinear systems as is shown in reference [5]. For this reason we choose the cost

$$J(r,u,y,\tau) = \left[-|r - y_1| + |E_{ss} r| + \left| \frac{Ks}{s+N} r \right| \right] \Bigg|_{t=\tau}$$

This cost is defined to shape the time response such that defines upper and lower bounds for the tracking error. The value of the bound depends on the reference signal that is being track. In particular the bounds are the sum of the E_{ss} % of the reference signal and a term proportional to the derivative of the reference signal. The first term signifies the steady state requirement (E_{ss} stands for steady state error), and the second one signifies the transient response requirement. Note that this second term can be written as

$$\frac{Ks}{s+N} r = K \left(1 - \frac{1}{\frac{1}{N} s + 1} \right) r$$

which is proportional to the error of a first order system. Moreover, this cost can be interpreted as the model of the worse closed loop performance acceptable, such that the controller has to provide a performance that is at least as good as this model.

4 SIMULATIONS

For the simulations we use a nonlinear mathematical model of a missile [6]. The controller design follows the steps described in the previous sections.

4.1 MISSILE MODEL

We use the pitch axis model of a missile for the simulations, which flies at Mach 3 and at an attitude of 20,000 ft. This mathematical model is extracted from reference [6], and due to its realistic representation of the dynamics it has being extensibly used as a benchmark for missile analysis and controller design [5,8-10].

It is important to remark that the knowledge of the model is not used in the controller design, but only for the simulation.

The state equations are:

$$\dot{\alpha} = \cos(\alpha) K_{\alpha} M C_n(\alpha, \delta, M) + q$$

$$\dot{q} = K_q M^2 C_m(\alpha, \delta, M)$$

and the output equation is:

$$\eta = \frac{K_z}{g} M^2 C_n(\alpha, \delta, M)$$

where,

η is the acceleration in g's,

δ is the tail deflection in radians,

α is the angle of attack in radians,

q is the pitch rate in the plane (Gx,Gz) in rad/s,

M Mach number,

and the stability derivatives are:

$$C_n(\alpha, \delta, M) = a_n \alpha^3 + b_n |\alpha| \alpha + c_n (2 - M/3) \alpha + d_n \delta$$

$$C_m(\alpha, \delta, M) = a_n \alpha^3 + b_m |\alpha| \alpha + c_m (-7 + 8M/3) \alpha + d_m \delta$$

The actuator is model as a second order linear transfer function:

$$A(s) = \frac{\varpi_a^2}{s^2 + 2\xi_a \varpi_a s + \varpi_a^2}$$

For the simulation the parameters take the values below that correspond to a missile flying at Mach 3 at 20,000 ft.

$K_a = 0.7 P_0 S / m / V$	$S = 0.44$	$a_n = 1.0286 * 10^{-4}$
$K_q = 0.7 P_0 S d / I_y$	$m = 13.98$	$b_n = -0.94457 * 10^{-2}$
$K_z = 0.7 P_0 S / m$	$V = 1036.4$	$c_n = -0.1696$
$P_0 = 973.3$	$d = 0.75$	$d_n = -0.034$
$a_m = 2.1524 * 10^{-4}$	$g = 32.2$	
$b_m = -1.9546 * 10^{-2}$	$\varpi_a = 150$	
$c_m = 0.051$	$\xi_a = 0.7$	
$d_m = -0.206$		

4.2 CONTROLLER PARAMETERIZATION

As explained in section 2 we use the following controller parameterization:

$$u = \frac{N_2(s)}{D^{\theta}{}_2(s)} \left[\frac{N_1(s)}{D^{\theta}{}_1(s)} \left(H^{\theta} r - y_1 \right) + y_2 \right]$$

which for simplicity of the simulation we choose to have 5 parameters as follows:

$$N_1(s) = (s/25 + 1) \qquad D^{\theta}{}_1(s) = (\theta_1 s + \theta_2)/\theta_5$$

$$N_2(s) = (s/5 + 1) \qquad D^{\theta}{}_2(s) = (\theta_3 s + \theta_4)$$

$$H^{\theta} = 1/\theta_5$$

We generate the candidate controller set by allowing the parameters to vary up to 20% around a nominal value, which is calculated by a classical technique based on a linearized missile model. In particular, we allow each controller to take discrete values in the intervals. For simplicity of simulation we consider 5 values for each parameter which gives 3125 controllers.

The nominal controller used in the simulations corresponds to the one presented in reference [5], which has the following values: $\theta_1 = 0.4, \theta_2 = 0.01, \theta_3 = 10.2, \theta_4 = 2, \theta_5 = 0.9$.

4.3 PERFORMANCE SPECIFICATION

The performance specification is:

$$\left| \frac{Ks}{s+N} r \right| + |E_{ss} r| - |r - y_1| \geq 0$$

From the missile steady state error requirement we select E_{ss} to be 0.05 (5%), and from the transient response requirements on overshoot and time constant we choose K and N to be 1.2 and 4 respectively. Note that this selection allows some non-minimum phase type behavior.

4.4 FALSIFICATION TEST

The falsification test is carried out by checking whether or not each controller in the unfalsified controller subset satisfies the performance specification. We check for every 0.01 seconds in our simulation. Since the controller satisfies the causal-left-invertibility property we use the condition in the observation made in section 2.2.

4.5 SWITCHING RULE

The switching rule used is as described in section 2.3. When the existing controller is falsified, we switch it with the controller that has the smallest average error.

4.6 RESULTS

The following figures show the nonlinear simulation results of the unfalsified control autopilot. Figure 3 shows the reference, the output signal, and the bounds on the response. Figure 4 shows the control signal. Figure 5 shows the evolution of the controller parameter set. And figure 6 shows the evolution of the number of controllers.

Figure 3 shows the output signal following the reference signal. During most of the time it stays inside the bound, but at some points it goes outside. At this points is the current controller is falsified and switched for the one with smallest average tracking error from the set of candidate controllers that have not been falsified yet.

This switching process is a learning process. As more data is collected the approach learns which controllers are not robust and throws them away. Therefore, the learning depends on the amount and richness of the data.

In figure 6, we see the evolution of the unfalsified number of controllers. This figure also tells us that there are 198 controller that have not been falsified, and therefore that would have met the performance if were put in the loop.

5 CONCLUSIONS

We have designed a robust switching missile autopilot via the unfalsified control theory. In doing so we have shown the applicability of unfalsified control to a realistic problem. In the development, we have illustrated the key elements and implementation issues of the unfalsified control. In particular, a new performance specification has been defined which is more adequate for problems with strong transient response requirements. Furthermore, a new algorithm for the calculation of the fictitious reference signal is developed that leads to a separation between the controller parameters and controller dynamics and helps to reduce the number of calculations significantly.

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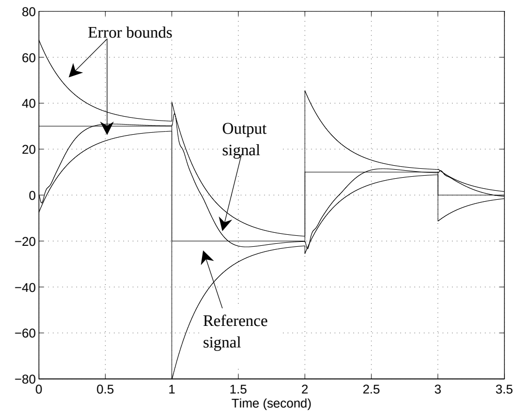

Figure 3: Reference signal, output signal, and error bounds .

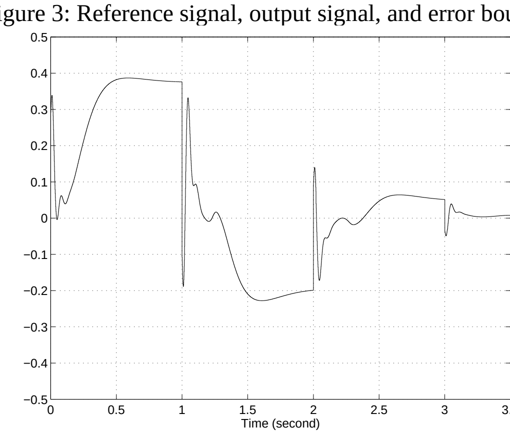

Figure 4: Control signal.

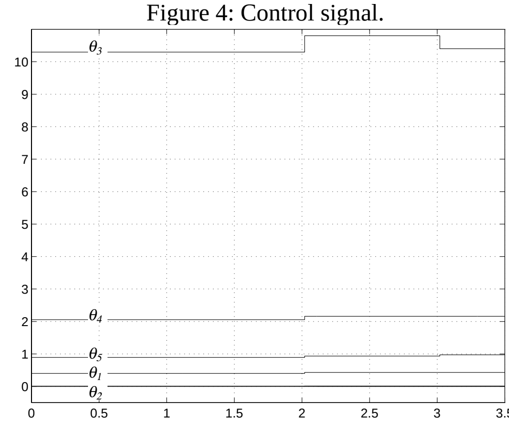

Figure 5: Evolution of Controller parameters.

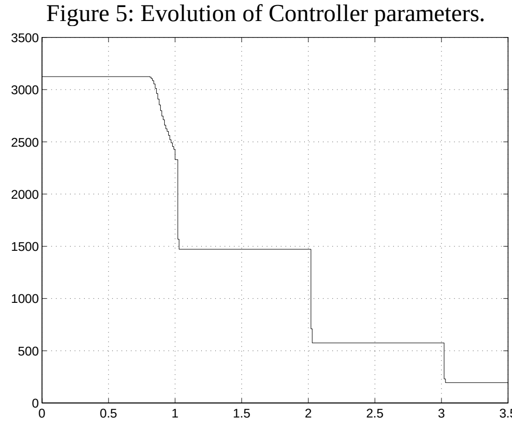

Figure 6: Evolution of the number of unfalsified controllers.