COST DETECTABILITY AND SAFE MCAC

by

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Dedication

To

my family

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Abstract

Hysteresis switching adaptive control systems designed using certain types of $\mathfrak{L}_{2e}$-gain type cost functions are shown to be robustly stabilizing if and only if certain plant-independent conditions are satisfied by the candidate controllers. These properties ensure closed-loop stability for the switched multi-controller adaptive control (MCAC) system whenever stabilization is feasible. The result is a *safe adaptive control system* that has the property that closing the adaptive loop can never induce instability provided only that at least one of the candidate controllers is capable of stabilizing the plant.

Notation

$P_\tau(\cdot)$: Projection operator that truncates a signal after $t = \tau$

$\Re_+$: $(0, \infty)$

$\Re$: $(-\infty, \infty)$

x: a general signal

$x(t)$: the value of signal x at time t

$x_{(a,b)}$: x's truncation over a time interval (a, b)

x_τ: x's truncation over a time interval $(0, \tau)$

v : the input to a general system

w: the output of a general system

r: reference signal

$\tilde{r}$: fictitious reference signal

u: control output/plant input

y: system output/plant output

d: (u, y)

$\mathbb{D}$: set of all possible d

d_τ: (u_τ, y_τ)

D_τ: the set of all possible (r, u, y) given d_τ

K: a controller

K^f: the final controller in the closed-loop

$\mathbb{K}$: controller set

$\mathbb{K}_{unf}$: unfalsified controller set

V: a scalar cost function

$\mathfrak{L}_{2\mathfrak{e}}$: an extended norm space

Chapter 1

INTRODUCTION

1.1 Brief Review on Switching Adaptive Control Methods

Generally, an adaptive control system is defined by three essential elements: goals, information and a set of candidate controllers. An adaptive control algorithm for such systems is the scheme to select/choose/order/tune/switch among candidate controllers by using real-time and prior information to achieve specified goals. Among all these goals, to achieve stability is the minimum goal of an adaptive control system. Whenever an active controller can not stabilize the system, a 'safe' algorithm [SWS04, WS05] should be able to 'know' the situation, abandon the active controller, and change to a stabilizing controller if there is one in the candidate controller set. If there is a stabilizing controller for the system in the candidate controller set at any time, *i.e.*, if stabilization by one of the candidate controllers is feasible, then a good adaptive control algorithm should be able to stabilize the system without further assumptions on the plant.

In the literature of adaptive control, there are several methods involving switching among a set of controllers. For example, hysteresis switching reported by Middleton et. al. [MGHM88], Morse et.al. [MMG92], and Weller and Goodwin [WG94], supervisory control produced by Morse [Mor96, Mor97] and also studied by Hocherman-Frommer *et. al.* [HFKR95] and Narendra and Balakrishnan [KJ94], switching adaptive control proposed by Fu and Barmish [FB86, MD91, Fu96],localization approach proposed by Zhivoglyadov *et.al.* [ZMF97a,ZMF00,ZMF97b,ZMF99,ZMF01] and unfalsified adaptive control introduced by Safonov and his co-workers [PS03, SWS04, ST97, Tsa94, PAMS04, CS96, JS99, SC01].

Martensson [Mar86] showed how to achieve adaptive goals using only the feasibility assumption via a pre-routed switching among the candidate controllers until one controller was found which could achieve the control objective. Other pre-routed based switching scheme can be found in [FB86, MD91, Fu96, MMG92] and [WG94]. However, though pre-routed switching schemes generally require only feasibility for convergence, they give poor transient response and switching to a stabilizing controller can take a long time, especially when the number of candidate controllers is large. Feasibility is the weakest assumption on the plant under which adaptive stabilization can be assured.

A slightly different switching control approach, called hysteresis switching, is also reported. The basic idea in the hysteresis switching by Morse *et.al* [MMG92] and Weller and Goodwin [WG94] involves two steps. The first step is to construct an estimator for each subfamily of plants with the same McMillian degree, the same relative degree, the same high-frequency gain sign and the same 'permutation' of outputs. Subsequently, a classical model reference adaptive controller is designed for each such subfamily. The second step is to use a so-called hysteresis switching algorithm based on Middleton *et. al.* to adaptively select a correct controller.

One serious problem with the conventional switching adaptive control schemes is that the number of candidate controller could be excessively large [Fu96]. It is shown in Weller and Goodwin [WG94] that this number is $2^m \times m! \times mn_{max}$ for $m-$input $m-$output plants. For example, even for $m = 5$ and $n_{max} = 10$ which are very moderate, this number is equal to 192,000. The excessively large number of candidate controllers means that it may potentially take an extremely long dwell time before a correct controller can be found.

To alleviate this problem, several methods have been reported recently. One of them is the so called *localization approach* by Zhivoglyadov *et.al.* [ZMF97a, ZMF97b, ZMF99, ZMF01] where a fast algorithm is introduced to prune 'bad' candidate controllers, and therefore a 'good' set of candidate controller is quickly localized.

Another approach to speeding up the switching process is the *supervisory control approach* proposed by Morse [Mor96, Mor97] to improve the transient response. The main idea of the supervisory control approach is to apply an 'optimal' candidate controller based on certain on-line plant model estimation rather than directly sequentially eliminating invalid controllers. The basic assumption is that there is at least one plant model in the candidate model set which is sufficiently close to the actual process to be controlled.

One of the problems with supervisory control is that the notion of 'optimal' control may be very ambiguous. In [ABLM01], they try to solve this problem by introducing a new monitoring signals(*i.e.* , new cost function) based on the properties of one kind of gap metric, the δ_v metric, to measure the closeness of plant models and thus controllers to the 'optimal' ones. Unfortunately, It is stated in [HS93] that gap metric may be unsuitable for evaluating the closeness of systems having uncertain poles and zeros on or near the imaginary axis. Even if this metric does work under any circumstances, such two-step model validation approach may still fail to find a stabilizing controller when there is one in the candidate controller set ([SWS04, BTS01, WSS04]), because the uncertainty-bounds associated with plant model may unnecessarily and mistakenly ignore some stabilizing controllers.

Unfortunately, safe adaptive control algorithms are rare. Aside from the impractical pre-routed switching methods, the above modern adaptive methods have for the most part continued to rely on additional plant modelling assumptions, compromising the robust performance properties that adaptive control is intended to enhance. See, for example, [GS84] and [SB89] for overviews. These recent efforts to partially relax some, but not all, plant assumptions [ZMF99, ZMF01, Mor96, Mor97, ABLM01] fall short of solving the safe adaptive control problem mentioned above where the only assumption is feasibility. A notable exception is [ST97, SWS04, WS05], where we showed that *unfalsified adaptive control* can overcome the poor transient response associated with the earlier pre-routed schemes by doing direct validation of candidate controllers very fast by using experimental data only, without making any assumptions on the plant beyond feasibility, and thus can potentially provide a practical solution to the safe adaptive control problem. But, not all unfalsified adaptive controller are safe. Preliminary results in [SWS04, WSS04] suggested that, to be safe, the unfalsification criterion used in unfalsified adaptive control must have a property known as *cost detectability*.

A useful concept in unfalsified control is the *fictitious reference signal* [ST97], which facilitates validation of candidate controllers from open-loop experimental data, or even from closed-loop data acquired while another controller is in the feedback loop. This leads to fast validation and relatively better transient response compared with prerouted adaptive switching methods like Martensson's [Mar86], while avoiding the robustness pitfalls associated with other recent methods that rely on additional plant assumptions.

1.2 Motivation

In the recent paper [WS05], stability of unfalsified adaptive control systems is reexamined from the perspective of the hysteresis switching lemma of Morse, Mayne and Goodwin [MMG92] in order to address the above problem. Sufficient conditions for the stabilization of adaptive control using multiple controllers are provided. We found that if a system is cost-detectable, it can always be stabilized if the problem is feasible.

In a recent paper [SWS04], we studied the stability and convergence of unfalsified adaptive control system using multiple controllers. More explicitly, assuming that the candidate controller set C contains at least one robustly stabilizing and robustly performing controller, it is proved that, if a well-designed data-driven cost function which is monotonically non-decreasing in time and uniformly bounded above for all conceivable plant data, is used, and if the system with such a cost function is cost-detectable, unfalsified adaptive algorithms can consistently and reliably identify controllers that can achieve stability and performance specifications based on cost-minimization. Thus two questions arise:

1. What are the the necessary and sufficient conditions to guarantee cost-detectability?
2. How to extend the result on unfalsified stability of a fictitious loop to the guaranteed stability of the real switching system?

In this thesis, the above questions are addressed. More precisely, given an unknown plant, a set of finite candidate controllers, a reference signal and a reference plant model, we need to choose a controller such that a suitably defined performance specification will be satisfied and the plant is stabilized. The main contribution of this thesis is that, given a set of controllers and a cost function, an adaptive control system with such a cost function is proved to be cost-detectable, and its stability is achieved while relaxing the monotonically-non-decreasing property on the chosen cost function in [SWS04]. Furthermore, the necessary and sufficient conditions for cost-detectability are given.

The hysteresis switching lemma proposed in [MMG92] by Morse, Mayne and Goodwin holds a similar but different idea from this thesis. Its cost function is more general and stability property can not be derived without more assumptions on the plant, while in this thesis without requiring more assumptions on the plant, stability can be proved using a new cost functions.

Starting with the same set of candidate controllers, and comparing with the recently proposed switching adaptive schemes [Nar97, NB93] whose stability is assured by assuming that at least one plant model is sufficiently close to the true plant so that it is within the robustness margin of its associated controller model, our assumption that there is at least one stabilizing controller in the candidate controller set is easier to satisfy. Such relaxation on the assumptions is very helpful for controller design of highly nonlinear systems, such as controller design for spacecrafts and autopilot crafts [BFS98].

This thesis is organized as follows. In chapter 2, the formulation of a safe adaptive control problem is given. In chapter 3, important definitions in unfalsified adaptive control method are introduced. Some properties of adaptive control is examined in chapter 4 from the perspective of unfalsified adaptive control. A multiple controller adaptive control(MCAC) algorithm is proposed in chapter 5; Stability analysis of this algorithm and examples are provided in chapter 5 too. Simulations in chapter 6 shows that the MCAC algorithm is efficient and stable. Conclusion and future work follow in chapter 7.

Chapter 2

SAFE ADAPTIVE CONTROL PROBLEM

In this chapter, firstly several definitions related to stability of a general control system are given. These general definitions serve as the basis for the definitions in section 3.1. Secondly, a general structure of adaptive control system is stated, based on which we present the safe adaptive control problem formulation.

2.1 General Definitions

Let $\Re_+ = (0, \infty)$. Define the truncation of a signal over a time interval (a, b) as

$$x_{(a,b)}(t) = \begin{cases} x(t), & \text{if } t \in (a,b) \\ 0, & \text{otherwise.} \end{cases}$$

and x_τ denotes the time truncation over the interval $(0, \tau)$

$$x_\tau(t) = \begin{cases} x(t), & \text{if } t \in (0,\tau) \\ 0, & \text{otherwise.} \end{cases}$$

We say $x \in \mathfrak{L}_{2\mathfrak{e}}$ if $\|x_\tau\|$ exists for all $\tau < \infty$ where

$$\|x_\tau\| = \sqrt{\int_0^\tau \|x(t)\|^2 \, dt}$$

Our definition of stability of a system is related to the $\mathfrak{L}_{2\mathfrak{e}}$-gain, which pertains to the ratio of the norm of the output to the input.

Definition 1 *(Stability and Gain) We say a system G with input v and output w is stable if for every input $v \in \mathfrak{L}_{2\mathfrak{e}}$ there exist constants $\beta, \alpha \geq 0$ such that*

$$\|w_\tau\| < \beta\|v_\tau\| + \alpha, \forall\, t > 0; \tag{2.1}$$

otherwise, it is said to be unstable. Furthermore, if (2.1) holds with a single pair $\beta, \alpha \geq 0$ for all $v \in \mathfrak{L}_{2\mathfrak{e}}$, then the system G is said to be finite-gain stable, in which case the gain of G is the least such β. □

Definition 2 *(Incremental Stability and Incremental Gain) We say that G is incrementally stable if, for every pair of inputs v_1, v_2 and outputs $w_1 = Gv_1, w_2 = Gv_2$, there exist constants $\tilde{\beta}, \tilde{\alpha} \geq 0$ such that*

$$\|[w_2 - w_1]_\tau\| < \tilde{\beta}\|[v_2 - v_1]_\tau\| + \tilde{\alpha}, \forall\, t > 0; \tag{2.2}$$

and the incremental gain of G, when it exists, is the least $\tilde{\beta}$ satisfying (2.2) for some α and all $v_1, v_2 \in \mathfrak{L}_{2\mathfrak{e}}$. □

Following Willems ((cf. Willems [Wil73,Wil76]), our definitions of stability e admit non-zero values for the parameters $\alpha, \tilde{\alpha}$. These parameters allow for consideration of systems with non-zero initial state, and would depend on the initial state (e.g., $\alpha = \alpha(x_0)$ for an initial state were x_0). This slight generalization of classic input-output stability definitions of Zames [Zam66] turns out to be useful in analyzing switching controllers, since switching from one controller to another generally leaves a system in a non-zero state.

2.2 Adaptive Control System

We are examining the stability of a switching adaptive control system in this thesis. An *adaptive control system* is a control system with an adaptive controller. An *adaptive controller* is a controller with adjustable parameters/structures and a mechanism for adjusting the parameters/structures [AW95]. A set composed by time-invariant controllers with any of these possible parameters/structures is called *candidate controller set*.

In this thesis we consider a general adaptive control system $\Sigma(P, \widehat{K})$ mapping $(r, v) \mapsto (u, y)$ whose block diagram depicted is shown in Fig.2.1. The system is defined on $\mathfrak{L}_{2\mathfrak{e}}$ [Zam66], which is to say that the signals r, s, u, y are all assumed to be

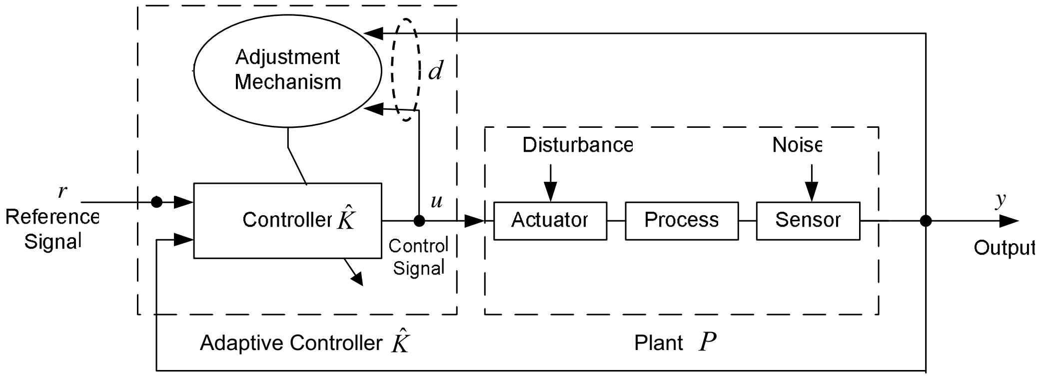

Figure 2.1: Adaptive control system $\Sigma(P, \hat{K})$

square-integrable over every *bounded* interval $[0, \tau]$, $(\tau \in \Re_+)$. The adaptive *adjustment mechanism* has input signal

$$d := \begin{bmatrix} u \\ y \end{bmatrix}.$$

and output $K \in \mathbb{K}$ so that the *adaptive control law* has the general form

$$u = \widehat{K}(t, d) \begin{bmatrix} r \\ y \end{bmatrix}.$$

We limit our consideration in this thesis to the case in which the *candidate controller set* $\mathbb{K}$ has finitely many elements, where $\widehat{K}(t, d) \in \mathbb{K}$ is piecewise constant. The adaptive control law $K(t, d)$ 'switches' among the controllers K in the candidate set according to the measured data d. If switching stops, then final controller and final switching time are denoted respectively, $K^f(d)$ and $t^f(d)$.

In this thesis, the plant P in Figure 2.1 is a *completely unknown plant*, which is to say that we have no prior knowledge of P other than that its input and output are in $\mathfrak{L}_{2\mathfrak{e}}$. At time $t = 0$ prior to collecting data d, we know neither the plant structure, its parameters, its order, its operating environment, nor the noises and disturbances. The plant may be unstable, non-minimum phase, non-linear and infinite order. We say that an adaptive control problem is *feasible* if at least one stabilizing controller K is available in the candidate controller set $\mathbb{K}$, even though which controllers are stabilizing is not known *à priori*. Given a completely unknown plant and a candidate controller set, a *safe adaptive control* law is one that never fails to stabilize whenever adaptive stabilization is feasible.

2.3 Safe Adaptive Control Problem

Problem 1 *(Safe adaptive control problem). Find, when feasible, an adaptive control law* $\widehat{K}(t,d)$ *that switches among the candidate controllers* $K \in \mathbb{K}$ *in such a way that the resultant closed-loop adaptive system* $\Sigma(P, \widehat{K}(t,d))$ *is stable.* □

It should be emphasized that a safe adaptive system is more than just robustly stable for plant uncertainty in a given uncertainty set. A safe adaptive system stabilizes the plant whenever a stabilizing controller exists in its candidate controller set, without regard to other prior knowledge of the plant model or its uncertainty. In this sense, a *safe* adaptive controller is a maximally robust adaptive controller.

Chapter 3

UNFALSIFIED ADAPTIVE CONTROL

In this chapter, several important definitions, such as *cost-detectability*, *causal left invertible controller* and $\mathfrak{L}_{2e}$*-gain-related cost*, in unfalsified control approach are presented. Then, after we exam the properties of a general adaptive control system from the perspective of unfalsified adaptive control theory, the stability of $r \mapsto \tilde{r}$ is proved and the necessary and sufficient conditions for cost-detectability are provided.

3.1 Definitions in Unfalsified Adaptive Control Theory

Let $d = (u, y)$ denote one possible experimental plant data over the time 0 to ∞, and $\mathbb{D}$ the set of all possible d. Denote by d_τ the time-truncation of d. Thus, d_τ represents past experimental plant data up to current time τ. Given past data d_τ, we denote by D_τ the set of signals in $\mathcal{R} \times \mathcal{Y} \times \mathcal{U}$ that interpolate (*i.e.*, are consistent with) d_τ:

$$D_\tau \triangleq D(d_\tau) = \left\{ \ (r, y, u) \ \middle| \ (y_\tau, u_\tau) = d_\tau \ \right\}. \tag{3.1}$$

The set of candidate controllers K is denoted as $\mathbb{K}$. In this thesis, we call the scalar valued function, $V : \mathbb{K} \times \mathbb{D} \times \mathfrak{R} \to \mathfrak{R}$, a *cost function*. It is used to evaluate candidate controllers K based on past data d_τ in [MMG92] and is also closely related to the cost functions employed in unfalsified control methods [SSP05]. The cost $V(K, d, \tau)$ is assumed to be causally dependent of d; that is, for all $\tau > 0$ and all $d \in \mathfrak{L}_{2\mathfrak{e}}$,

$$V(K, d, \tau) = V(K, d_\tau, \tau)$$

Definition 3 *(Unfalsification at a cost level) Given V, $\mathbb{K}$ and a scalar $\gamma \in \Re$, we say that a controller $K \in \mathbb{K}$ is falsified at time τ with respect to cost level γ by past measurement information d_τ if $V(K, d, \tau) > \gamma$. Otherwise the control law K is said to be unfalsified by d_τ. The least value of γ for which K is unfalsified by data d_τ is the unfalsified cost level of K at time τ. The set of unfalsified controllers having an unfalsified cost level of γ or less at time τ is called unfalsified controller set $\mathbb{K}_{unf}(\gamma, \tau)$.* □

The foregoing definition of unfalsification at a cost level is a minor extension of the 'unfalsification' definition in [ST97], where only falsification with respect to cost level $\gamma = 0$ was considered. Clearly, for all time $\tau \geq 0$,

$$\mathbb{K}_{unf}(\gamma_1, \tau) \in \mathbb{K}\} \subset \mathbb{K}_{unf}(\gamma_2, \tau)$$

if $\gamma_1 < \gamma_2$.

Definition 4 *(Fictitious reference signal). Given plant data d and a candidate controller K, a fictitious reference signal for K, when it exists, is a hypothetical signal $\tilde{r}$ that would have produced exactly the same d had the controller K with noise $s = 0$ been*

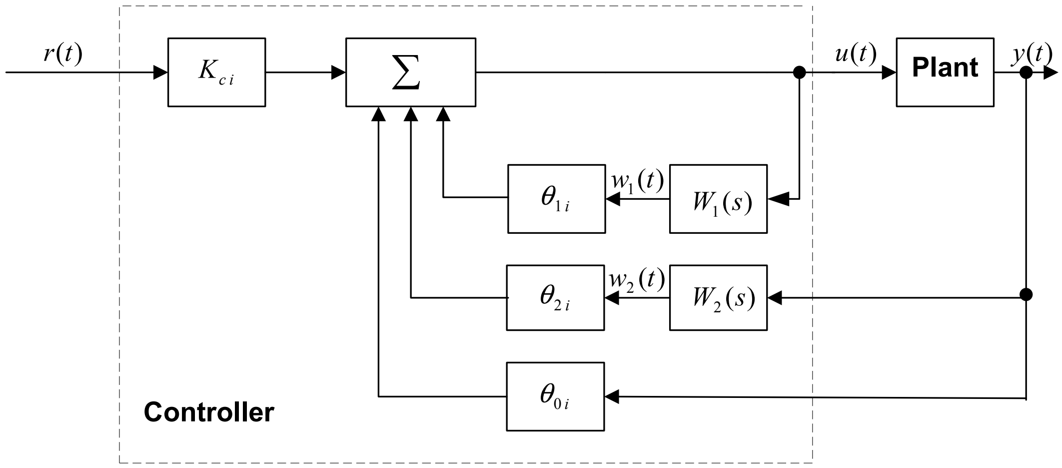

Figure 3.1: A controller with a minimum-phase subsystem from r to u

in the feedback loop with the completely unknown plant during the entire time period over which d were collected. □

When it exists and is unique, the fictitious reference signal is denoted as $\tilde{r}(K, d)$ or as $\tilde{r}$ when no confusion will arise. When $\tilde{r}(K, d)$ exists, the induced mapping $d \mapsto \tilde{r}(K, d)$ is called the *fictitious reference signal generator* of that controller, denoted by $\tilde{G}(K)$ or as $\tilde{G}$ when no confusion will arise.

Fictitious reference signals are in general not the true signals([ST97,PS03]), hence the name fictitious. Given data $d = (u_0, y_0)$ and a controller K having graph K, the fictitious reference signals are the $\tilde{r}(K, d)$ that satisfy $(\tilde{r}(K, d), y_0, u_0) \in D_\tau \cap \mathsf{K}$. As noted in [SSP05], fictitious reference signals allow unfalsified control performance goals

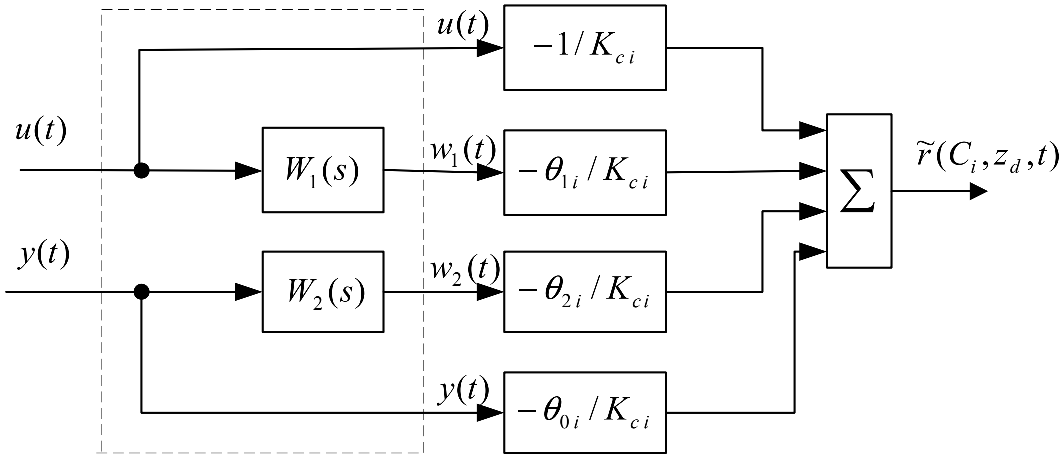

Figure 3.2: Fictitious reference signal of the controller in Fig.3.1

of the form $J(r, y, u, \tau) \leq \gamma$ to be expressed in a form suitable for use in conjunction with the convergence lemma of [MMG92]:

$$V(K, d, \tau) = J(\tilde{r}(K, d), d, \tau). \tag{3.2}$$

Example 1 *For example, a controller, K, with the structure in Fig. 3.1 is such a controller. Its fictitious reference signal would be*

$$\begin{aligned} \tilde{r}(K, d) &= \tilde{G}(K)d \\ &= \frac{1}{k}[u - \theta_1 W_1(s)u - \theta_2 W_2 y - \theta_0 y], \end{aligned} \tag{3.3}$$

which is shown in Fig.3.2. Of particular interesest is the case when $\tilde{G}(K)$ is incrementally stable, in which case K has the following property. □

Definition 5 *(CLI and SCLI Controllers). A controller K with input (r, y) and output u is said to be Causal Left Invertible (CLI), if $\tilde{G}(K)$ exists and is causal. If additionally $\tilde{G}(K)$ is incrementally stable, then we say it Stable Causal Left Invertible (SCLI).* □

Since the $\tilde{G}(K)$ in (3.3) is the fictitious reference signal generator for the controllers of the standard form Fig.(3.1), it is clear that such controllers are SCLI if and only if $W_1(s)$ and $W_2(s)$ are stable and $k_I \neq 0$.

Definition 6 *(Unfalsified stability). Given an input output pair (v, w) of a system, we say the stability of the system is unfalsified by (v, w) if there exist $\beta, \alpha \geq 0$ such that (2.1) holds; otherwise, we say the stability of the system is falsified by (v, w).* □

Unfalsified stability is determined from (2.1) based on the data from one infinite-duration experiment for one input, while 'stability' requires additionally that (2.1) hold for the data from every possible input.

Definition 7 *(Cost-detectability).*

Let r denote the input and let $d := \begin{bmatrix} u \\ y \end{bmatrix}$ denote the resulting plant data collected while $\widehat{K}(t, d)$ is in the loop. Consider the adaptive control system $\Sigma(P, \widehat{K})$ of Figure 2.1 with input r and output $d := \begin{bmatrix} u \\ y \end{bmatrix}$. The pair $(V, \mathbb{K})$ is said to be cost detectable if, without any assumptions on the plant P and for every $\widehat{K}(t, d) \in \mathbb{K}$ with finitely many switching times, the following statements are equivalent:

1). $V(K^f, d, \tau)$ is bounded as τ increases to infinity;

2). The stability of the system $\Sigma(P, \widehat{K}(\tau, d))$ is unfalsified by (r, d). □

Cost-detectability is controller-dependent, but plant-independent. -detectability is in a sense the dual of the plant-dependent, but controller-independent property of tunability introduced by Morse *et al.* [Mor90,MMG92]. Cost-detectability can substituted for tunability in adaptive stability proofs, allowing one to remove the need for most plant assumptions. The key ideas is that when we have cost-detectability, then we can use the cost $V(K, d, \tau)$ to reliably detect any instability exhibited by the adaptive system $\Sigma(P, \widehat{K}(\tau, d))$, *even when initially the plant is completely unknown.* The implication is that one can completely circumvent "model-mismatch" instability problems that would otherwise arise in cases where the plant turns out not to conform to assumptions.

Definition 8 *($\mathfrak{L}_{2\mathfrak{e}}$-gain-related cost). Given a cost/candidate controller-set pair $(V, \mathbb{K})$, we say that the cost V is $\mathfrak{L}_{2\mathfrak{e}}$-gain-related if for each $d \in \mathfrak{L}_{2\mathfrak{e}}$ and $K \in \mathbb{K}$,*

1. *$V(K, d, \tau)$ is monotone in τ,*
2. *the fictitious reference signal $\tilde{r}_\tau(K, d) \in \mathfrak{L}_{2\mathfrak{e}}$ exists and,*
3. *For every $K \in \mathbb{K}$ and $d \in \mathfrak{L}_{2\mathfrak{e}}$, $V(K, d, \tau)$ is bounded as τ increases to infinity if, and only if, stability is unfalsified by the input-output pair $(\tilde{r}_\tau(K, d)), d)$.* □

The third condition in Definition 8 requires that the cost $V(K, d, \tau)$ be bounded uniformly with respect to τ if and only if $\mathfrak{L}_{2\mathfrak{e}}$-stability is unfalsifed by $(\tilde{r}(K, d), d)$; this is the motivation for the choice of terminology '$\mathfrak{L}_{2\mathfrak{e}}$-gain-related'. Clearly, cost detectability implies $\mathfrak{L}_{2\mathfrak{e}}$-gain-relatedness. In fact, $\mathfrak{L}_{2\mathfrak{e}}$-gain-relatedness is simply cost detectability of V for the special case where $\widehat{K}(t,d) = K \in \mathbb{K}$ is a constant, unswitched non-adaptive controller .

$\mathfrak{L}_{2\mathfrak{e}}$-gain-related cost functions are easily identified. For example, consider weighted 'mixed sensitivity' cost functions of the form (cf. Tsao and Safonov [ST97])

$$V(K,d,\tau) \triangleq \max_{t\leq\tau} \frac{\left\| \begin{bmatrix} [W_1\ (\tilde{r}(K,d)-y)]_t \\ [W_2 * u]_t \end{bmatrix} \right\|}{\|\tilde{r}_t(K,d)\|}. \tag{3.4}$$

provided that the 'weights' W_1 and W_2 are stable operators with stable inverses. If the controllers K in $\mathbb{K}$ is SCLI, then clearly the cost (3.4) is $\mathfrak{L}_{2\mathfrak{e}}$-gain-related.

Like cost-detectability, the $\mathfrak{L}_{2\mathfrak{e}}$-gain-relatedness is a plant independent concept. It turns out, as we shall show, that for some broad classes of candidate controllers $\mathbb{K}$, the $\mathfrak{L}_{2\mathfrak{e}}$-gain-relatedness of $V(K,d,\tau)$ implies cost-detectability of $V(K,d,\tau)$. As we shall show, this together with the Morse-Mayne-Goodwin convergence lemma [MMG92] will lead to a solution to the safe adaptive control problem.

3.2 Properties of Adaptive Control

The coming lemmas and theorems may require the following assumptions:

Assumption 1 *The cost function $V(d,K,\tau)$ is $\mathfrak{L}_{2\mathfrak{e}}$-gain-related.*

Assumption 2 *Each candidate controller $K \in \mathbb{K}$ is SCLI;*

Lemma 1 *(Stability of $r \mapsto \tilde{r}$) Consider the adaptive system $\Sigma(P, \widehat{K}(t,d))$ of Figure 2.1. Suppose that controller switching eventually stops; i.e., for each r, there exists $t^f \geq 0$ such that,*

$$\widehat{K}(t,d) = K^f \in \mathbb{K}, \ \forall t > t^f.$$

If the final controller K^f is SCLI, then the composite map $\tilde{G} * \Sigma : r \mapsto \tilde{r}(K^f)$ is stable with gain $\beta = 1$, i.e.,

$$\|\tilde{r}_\tau\| \leq \|r_\tau\| + \alpha < \infty, \ \forall \tau \geq 0. \tag{3.5}$$

Proof. The proof of the stability of $r \mapsto \tilde{r}$, will be furnished without constraints on linearity or time-invariance of the candidate controllers.

By assumption that controller switching eventually stops; i.e., for each r, there exists $t^f \geq 0$ such that,

$$\widehat{K}(t,d) = K^f \in \mathbb{K}, \ \forall t > t^f.$$

Also by assumption that K^f is SCLI, $\tilde{G}(K^f)$ exists and is causal and incrementally stable.

WLOG, consider the control configuration in Fig. 3.3. The top branch is the serial connection of $\hat{K}$ and $\tilde{G}(K^f)$. It illustrates how we generate fictitious reference signal $\tilde{r}(K^f)$ for K^f with the adaptive controller $\hat{K}$ in the close loop. The middle branch is an imaginary connection supposing that (r, y) is the same as that in the top branch while K^f stays in the close loop from the very beginning. The top branch generates $\tilde{r}(K^f)$ since (y, u) is the input of $\tilde{G}(K^f)$, while the middle branch generates the reference signal r since (y, u^f) is the input of $\tilde{G}(K^f)$. Finally, the bottom branch has the identical structure as the top one, except that an additional input, denoted as ω, is added. ω

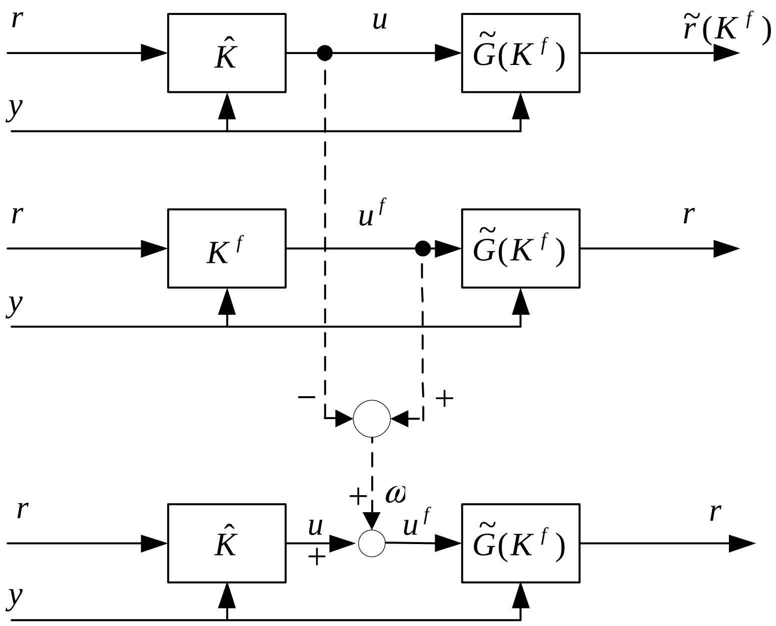

Figure 3.3: Generators of the true and fictitious reference signals.

acts as a compensating (bias) signal, which accounts for the difference between signal u in the top branch and u^f in the middle branch. Hence, the input of $\tilde{G}(K^f)$ in the bottom branch is (y, u^f), and its output becomes r as well. In particular, it can be shown that $\omega \triangleq u^f - u$.

As stated above, $\tilde{G}(K^f)$ is incrementally stable. Then, according to the definition of incremental stability, there exist constants $\tilde{\beta}$ and $\tilde{\alpha}$, such that,

$$||(\tilde{G} * d_1)_t - (\tilde{G} * d_2)_t|| \leq \tilde{\beta}(\tilde{G}) \cdot ||(d_1)_t - (d_2)_t|| + \tilde{\alpha},$$
$$\text{for all } d_1, d_2 \in \mathfrak{L}_{2e} \tag{3.6}$$

Now, choose $d_1 \triangleq (u, y)$ and $d_2 \triangleq (u^f, y) = (u+\omega, y)$ where $\omega = u^f - u$; Due to the fact that $u^f \equiv u$, for all $t \geq t^f$, ω is defined on a finite time interval and thus it has a finite $\mathfrak{L}_{2e}$ norm with finite energy(no finite-time-escape signals considered here). Then, (3.6) reduces to:

$$||\tilde{r}_t - r_t|| \leq \tilde{\beta}(\tilde{G}) \cdot ||\omega_t|| + \tilde{\alpha} < \infty \tag{3.7}$$

$$\Rightarrow (\tilde{r} - r) \in \mathfrak{L}_{2e} \tag{3.8}$$

from where, use the triangle inequality:

$$||\tilde{r}_t|| \leq ||r_t|| + \tilde{\beta}(\tilde{G}) \cdot ||\omega_t|| + \tilde{\alpha} \tag{3.9}$$

where the second item of the right-hand-side of (3.9) is finite. So, there exists a constant α, s.t.,

$$\|\tilde{r}_\tau\| \leq \|r_\tau\| + \alpha < \infty, \ \forall \tau \geq 0. \tag{3.10}$$

□

Theorem 1 *(Cost detectability) Suppose Assumption-1 holds. Then, a sufficient condition for the pair* $(V, \mathbb{K})$ *is cost-detectable is that the candidate controllers be SCLI. If, additionally, the candidate controllers are LTI, then it is also necessary.*

Proof. For sufficiency, by $\mathfrak{L}_{2e}$-gain-relatedness of V, statement 1) in the definition of 'cost-detectable' is valid if, and only if, stability of the system with K^f staying in the loop all the time, is unfalsified by the input-output pair $(\tilde{r}(K^f), d)$. That is, statement 1) is equivalent to: there exists constants β_1 and α_1, s.t.,

$$\|d_\tau\| < \beta_1 \|\tilde{r}_\tau\| + \alpha_1, \forall\, t > 0; \tag{3.11}$$

According to inequality (3.5) in lemma 1, (3.11) becomes

$$\|d_\tau\| < \beta_1 \|r_\tau\| + \beta_1 \alpha + \alpha_1, \forall\, t > 0, \tag{3.12}$$

which is equivalent to say: the stability of the adaptive system is unfalsified by the input-output pair (r, d). So, statement 1) is equivalent to statement 2) in the definition of 'cost detectable'. Therefore, $(V, \mathbb{K})$ is cost-detectable given the candidate controllers are SCLI.

It remains to establish necessity. Since by hypothesis Assumption 1 holds, it follows that $\tilde{r}(K, d)$ is defined and hence the fictitous reference signal generator $\tilde{G}(K)$ exists and is causal. Suppose $\tilde{G}(K, d)$ is not stable. Then, the dominant pole of $\tilde{G}(K, d)$ has a non-negative real part, say $\sigma_0 \geq 0$. Since by definition cost-detectability is a plant independent property, it must hold for every plant mapping $\mathfrak{L}_{2\mathfrak{e}}$ into $\mathfrak{L}_{2\mathfrak{e}}$. Choose P so that $\Sigma(P, K)$ has its dominant closed loop pole at σ_0. Choose bounded duration inputs $r, s \in L_2$ so that the modes of $\tilde{G}(K, d)$ and $\Sigma(P, K)$ associated with the unstable dominant poles with real part σ_0 are both excited. Then, since the fictititous reference signal $\tilde{r}(K, d)$ is unstable with the same growth rate $e^{\sigma_0 t}$ as the unstable closed loop response $d(t)$, there exists a constant β such that $\|d_\tau\| \leq \beta \|\tilde{r}_\tau(K, d)\| + \alpha \; \forall \tau \geq 0$ holds.

Hence, by Assumption 1, the cost $\lim_{t\to\infty} V(K,d,t)$ is finite. On the other hand, stability of $\Sigma(P,K)$ is falsified by (r,d), which contradicts cost-detectability. Therefore the LTI controller K must be $SCLI$. $\square$

Chapter 4

MULTIPLE CONTROLLER ADAPTIVE CONTROL(MCAC)

A new algorithm is proposed as a solution to the safe adaptive stabilization problem stated in chapter 2 for Multiple Controller Adaptive Control(MCAC).

4.1 Algorithm 1

Consider a deterministic switching adaptive control system in Fig. 4.1, with reference signal r, and measurable control output and system output (u, y). For simplicity, noise $n(t)$, disturbance $d(t)$ and initial conditions x_0 in Fig. 2.1 are assumed to be zeros. The plant, which includes the disturbance/noise signals (d, n), is unknown. We are given a finite set of candidate controllers $\mathbb{K} = \{K_i\}, i = 1, 2, \ldots, N$. At each time instant, say τ, if the current controller's cost exceeds the minimal cost by more than a pre-specific small number ϵ, the task is to identify and switch to the optimal controller $K^\star(\tau)$, *i.e.*

,

$$K^\star(\tau) = arg \min_{K_i \in \mathbb{K}} V(K_i, d, \tau) \text{ and } \widehat{K}(\tau) = K^\star(\tau), \tag{4.1}$$

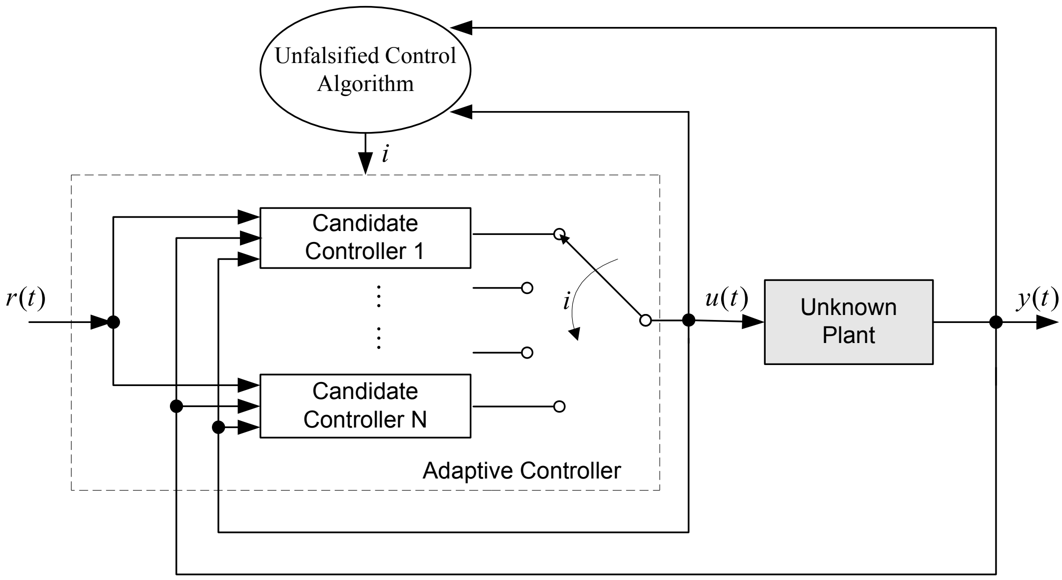

Figure 4.1: Unfalsified adaptive control system using multiple controllers

where $V(K_i, d, \tau)$ is a given cost function. The steps of the algorithm are:

Algorithm 1 *(ϵ-Hysteresis Algorithm [MMG92])*

1. *Initialize: Let* $t = 0$, $\tau = 0$; *choose* $\epsilon > 0$.

   Let $\hat{K}(0) = K_0, K_0 \in \mathbb{K}$, *be the first controller in the loop.*

2. $\tau \leftarrow \tau + 1$.

   If $V(\hat{K}(\tau-1), d, \tau) > \min_{K \in \mathbb{K}} V(K, d, \tau) + \epsilon$ *then* $\hat{K}(\tau) \leftarrow \arg\min_{K \in \mathbb{K}} V(K, d, \tau)$,

   else $\hat{K}(\tau) \leftarrow \hat{K}(\tau - 1)$

3. *go to 2.* □

Suppose the unfalsified controller set at each time τ is denoted by $\mathbb{K}_{unf}(\gamma, \tau)$. If the cost function is chosen so that $V(K, d, \tau)$ is monotone non-decreasing in τ for all

$K \in \mathbb{K}$ and all $d \in \mathfrak{L}_{2e}$, then for each $\gamma \in \Re$ the unfalsified set $\mathbb{K}_{unf}(\gamma, \tau)$ shrinks monotonically as τ increases; that is, if $\tau_1 < \tau_2$, $\mathbb{K}_{unf}(\gamma, \tau_1) \subset \mathbb{K}_{unf}(\gamma, \tau_2)$. Note that candidate controllers can be reused and no controller is ever discarded here. Candidate controllers are grouped into different set according to their different cost levels.

Algorithm 1 is essentially the same as the 'hysteresis algorithm' of [MMG92]. To solve the safe adaptive control problem, we will use Algorithm 1 with an $\mathfrak{L}_{2e}$-gain-related cost function—cf. Assumption assu:two in section 3.2. An important property of Algorithm 1 is the *Hysteresis Switching Lemma* [MMG92], which says essentially.

> *If $V(K, d, \tau)$ is monotone in τ and $\min_K \sup_{\tau,d} V(K, d, \tau) < M < \infty$, then there is a time $t^f < \infty$ beyond which the controller switching in Algorithm 1 stops and, moreover, $V(\hat{K}(\tau), d, \tau) < M + \epsilon \ \forall \tau$.* □

This suggests that if $(V, \mathbb{K})$ is cost-detectable, then Algorithm 1 can be used to solve our safe adaptive control problem.

4.2 Stability of adaptive control system using Algorithm 1

Now consider MCAC using Algorithm 1. To deduce the following lemma 2 and theorem 2, it requires Assumption 1 and 2 to be satisfied, and additionally, the following assumption 3 should be satisified too:

- *Assumption 3*: The safe adaptive control problem is feasible.

Lemma 2 *(Convergence) If (1) the candidate controllers are SCLI, (2) the cost function $V(K, \tau, d)$ is monotone increasing in τ, and (3) the safe adaptive control problem is feasible, then using Algorithm 1 for any input r, there are finitely many switches among candidate controllers before switching stops, and the cost $V(K^f, d, \tau)$ remains bounded as τ increases to infinity.*

Proof. By Assumption, the cost of each candidate controller is monotone increasing with time, so does the minimum of the costs of all candidate controllers. At the same time, since the safe adaptive control problem is feasible, there is at least one stabilizing controller in the candidate controller set. So there exists an upper bound for the costs of that controller. That means there exists an upper bound for the minimum of the costs for all controllers. By Algorithm 1, after each switch, the minimum of the costs rises a finite value ε, therefore, there will be finite switches before the minimum reaches its upper bound and the switching will stop.

Assume $V(K^f, d, \tau)$ becomes unbounded during $(0, t^f(d))$. Then K^f would not be switched into the closed-loop at all. If $V(K^f, d, \tau)$ becomes unbounded during $(t^f(d), \infty)$, then there exists a finite time t_1 when K^f's cost pasts the upper bound of the costs of a stabilizing controller. Consequently, another controller(or the stabilizing controller) will be switched to the closed-loop, which contradicts with the fact that the final controller K^f is 'final'. So, $V(K^f, d, \tau)$ remains bounded as τ increases to infinity. □

Theorem 2 *(Stability Theorem) If assumption 1, 2 and 3 hold, the unfalsified MCAC system is stable.*

Proof. With Assumption 1 and 2, $(V, \mathbb{K})$ is cost-detectable for any r according to theorem 1. Also by lemma 2, the cost $V(K^f, d, \tau)$ is bounded as τ increases to infinity for any r and its corresponding K^f. According to the definition of 'cost-detectable', the last statement is equivalent to: the stability of the system $\Sigma(P, \widehat{K}(\tau, d))$ is unfalsified by any possible (r, d), *i.e.*, the system is stable. □

4.3 Simulations and Discussion

It is important in designing cost functions for adaptive systems that careful attention be paid to the issue of cost-detectability. For example as we showed in [WSS04], model mismatch instability occurs despite feasibility of the adaptive problem using fixed multiple plant models adaptive algorithm of [Nar97], which uses the cost function

$$\begin{aligned} V(K, d, t) &= \alpha e^2(K, d)(t) \\ &\quad + \beta \int_0^t exp(-\lambda(t-\tau)) e^2(K, d)(\tau)\, d\tau \end{aligned} \tag{4.2}$$

where $e(K, d) = \hat{y}(K, d) - y$ is the identification error associated with an *assumed* i-th plant model for which the i-th controller $K_i \in \mathbb{L}$ would be optimal if there was no residual model mismatch. Here, $\hat{y}(K, d)(t)$ is the output of the i-th candidate plant model at time t when the past input is the measured plant input data u_t. As shown by us in [WSS04], a danger with the method in [Nar97] is that the cost function

(4.2) fails the cost-detectability requirement of [SWS04, Proposition 1]. If it happens that there is substantial model mismatch and none of the assumed candidate plant models is sufficiently close to the unknown true plant, simulation results show that adaptive control law of [Nar97] can incorrectly discard a stabilizing controller and instead lock on to a destabilizing controller. In contrast, with a'cost-detectable' cost-function the stabilizing controller is correctly identified by cost minimization and safe adaptive control is robustly achieved, as predicted by Theorem 2 and as shown by the simulation results in [WSS04], which uses the following cost-detectable cost function:

$$V(c_i, D, \tau) \triangleq \begin{cases} \frac{\|\tilde{e}(c_i,D,t)\|_\tau + \epsilon \|u\|_\tau}{\|\tilde{r}(c_i,D,t)\|_\tau}, \text{if } \| \tilde{r}(c_i, D, t) \|_\tau \neq 0 \\ 0, \qquad\qquad\qquad \text{if } \| \tilde{r}(c_i, D, t) \|_\tau = 0. \end{cases}, \tag{4.3}$$

where ϵ is some positive constant, $\tilde{r}(c_i, D, t)$ the fictitious reference signal and $\tilde{e}(c_i, D, t)$ the fictitious error of the i-th controller, defined as:

$$\tilde{e}(c_i, D, t) \triangleq W_m \tilde{r}(c_i, D, t) - y(t). \tag{4.4}$$

W_m is a reference model.

In the following paragraph, we compare the method in [Nar97, NB93] and the proposed MCAC algorithm in this thesis closely, under the situation that enough closeness of plant models to the true plant is not guaranteed. Starting with the same set of candidate controllers, and comparing with the recently proposed switching adaptive schemes in [Nar97] and [NB93] whose stability is assured by assuming that at least one plant model is sufficiently close to the true plant so that it is within the robustness margin of its associated controller model, our assumption that there is at least one stabilizing controller in the candidate controller set is easier to satisfy. Such relaxation on the assumptions is very helpful for controller design.

1). Example Setup

The adaptive method using fixed multiple plant models in [Nar97] is used to compare with unfalsified adaptive control algorithm 1 here. Suppose the true plant P^* has parameter vector $p^* = (\beta_0, \beta_1^T, \alpha_0, \alpha_1^T) = (1, 0, -2, 0)$ and its corresponding controller parameter vector $\theta^* = (k, \theta_1^T, \theta_0, \theta_2^T) = (1, 0, 2, 0)$. Two plant models, P_1 and P_2, are designed so that their parameter vectors, p_1 and p_2, are far from p^*. (i.e. sufficient closeness assumption for theorem 2 in [Nar97] is violated). So, let $p_1 = (2, 0, 4, 0)$ and $p_2 = (1, 0, -6, 0)$. Denote the corresponding controllers for each plant model as c_1 and c_2, and the parameter vectors of the controllers as θ^1 and θ^2. Thus, $\theta^1 = (.5, 0, -2, 0)$ and $\theta^2 = (1, 0, 6, 0)$. c_1 is designed to be stabilizing for the true plant, while c_2 is not. Reference model is $W_m(s) = 1/(s+3)$ and its output is denoted as y_m. Unknown plant is $W_p(s) = 1/(s+5)$ and its output is denoted as y_p. The input signal $r(t)$ is a pulse signal. The initial active controller is the stabilizing controller, i.e. $\widehat{c}(0) = c_1$, (if $\widehat{c}(0) = c_2$, which is destabilizing, the final result is the same). All initial conditions are zero. The same cost function $J(t)$ as in [Nar97] is used for each pair of plant and controller, *i.e.* , cost function (4.2) is used, where $\alpha = \beta = 1$ and $\lambda = 0.05$. Here $i = 1, 2$ since we have two plant models and their corresponding controllers.

For comparison, all initial conditions, parameters and structures of candidate controllers, reference model and input reference signal are set to be the same in the above method (method A) and in unfalsified adaptive control algorithm 1 (method B) using multiple controllers. The difference is cost function (4.3) is used with $\varepsilon = 0$.

2). Simulation Results and Discussion

Fig. 4.2 and 4.3 are the system output and desired output , and the values of cost function (4.2) for both pairs when method A is applied. According to these figures, because cost value of the unstabilizing controller c_2 is smaller than that of the stabilizing controller c_1 and this keeps being true, is switched into the loop and stays there and thus the system becomes unstable. The wrong decision of switching to an unstablizing controller and of keeping it in loop after the system becomes unstable may come from one fact, that is making decisions of switching only based on output signals, while stability is associated with input-ouput relationship. Out of this consideration, the new cost function (4.3) in method B is used.

Fig. 4.4 and 4.5 demonstrate the system output and desired output, and the values of cost function (4.3) for both pairs when method B is applied. In these figures, cost values of c_1 are smaller than that of c_2 and are bounded. So, c_1 is switched into the loop and stays in it. This demonstrates that a stabilizing controller can be successfully chosen and system can be stabilized with the method proposed in this thesis.

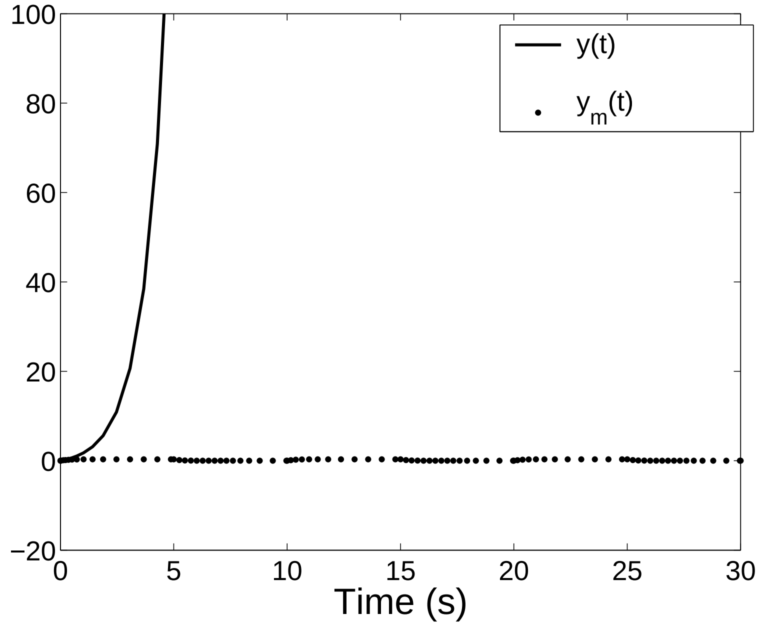

Figure 4.2: System output y and desired output y_m with cost (4.2)

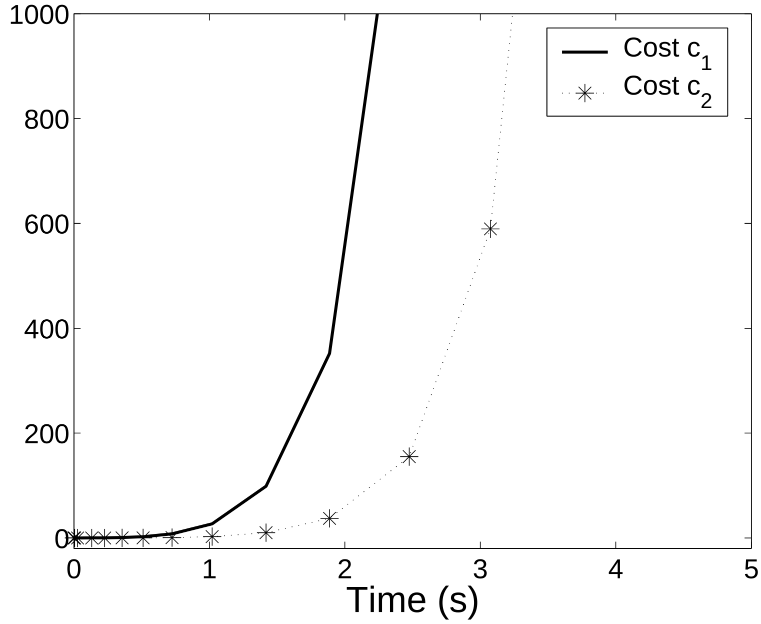

Figure 4.3: Cost (4.2) for both controllers

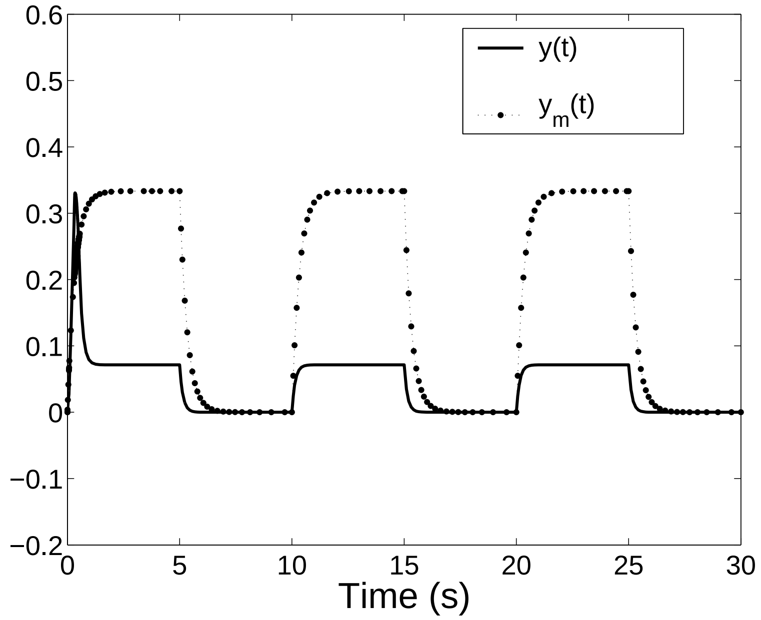

Figure 4.4: System output y and desired output y_m with cost (4.3)

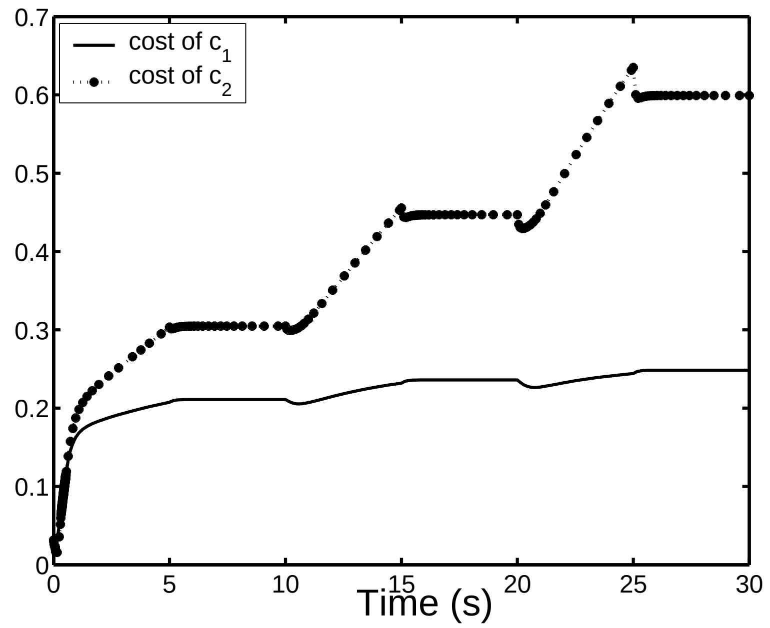

Figure 4.5: Cost (4.3) for both controllers

Chapter 5

SUMMARY AND FUTURE RESEARCH

5.1 Dissertation Summary

In this thesis, we studied the safe adaptive stabilization problem. Unfalsified adaptive control method is used to solve this problem. The statbility of unfalsified adaptive control system method using multiple controllers based on the Morse-Mayne-Goodwin hysteresis algorithm [MMG92] is proved under the assumption that at least one controller is stabilizing. It is shown that SCLI property of candidate controllers is a necessary and sufficient condition to ensure cost-detectability. And, the hysteresis algorithm for cost-detectable systems yields *safe adaptive control* that is guaranteed stable without plant model assumptions, subject only to the feasibility requirement that there exists at least one stabilizing controller amongst the candidate controllers. Theoretical analysis and simulation results show that the proposed method is efficient.

5.2 Future Directions

There are several theoretical and practical issues which might be addressed in future:

1. Although up to now, unfalsified adaptive control literature gives little instructions on how to choose a candidate controller set such that there is at least one stabilizing controller in it under different situations, this is still a very interesting, important and urgent question, both for practical application and theoretical completeness, and thus should be addressed later in more details by doing more simulations and practical design using unfalsified adaptive control method;

2. We gave the definition and properties of a cost-detectable cost function in this thesis and provide the sufficient conditions for a function to be cost-detetable, what's more, for LTI controllers, we gave the necessary and sufficient conditions for that. Even further, we showed in algorithm 1 that the particular cost function (4.3) is cost-detectable. Some related questions are: are there other cost functions that can also catch the input-output relationships of the plant and solve the safe adaptive stabilization problem? What are the necessary and sufficient conditions that these cost functions should satisfy to be cost-detectable?

3. Up to now, unfalsified adaptive control literature focuses on finite candidate controller set. Examining the properties of the adaptive system which has infinite candidate controllers will be desirable. Preliminary work has been tried on gradient-based myopic unfalsified adaptive algorithms and theoretical analysis is still under going.

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